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String-Figures

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STRING-FIGURES

by

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PREFACE

In these notes, we discuss four string-figures: Brochos, Osage Diamonds, Crows Feet, Bear. Our discussion and analysis of these four figures is pitched at a fairly technical level, in order to illuminate the distinct, but related, "ideas" in string which they represent. And, to that end, we discuss many figures closely related to each -- the more closely related the figure, the more detailed our discussion of it. Thus, in the case of figures peripheral to the central "idea" behind a given string-figure, only the construction itself will be given, and comparisons drawn therefrom.

The purpose behind these researches is twofold: (1). To explicate the "structure" underlying the set of all string-figures by exploring their interrelations, and (2). to "conserve" string-figures uniquely, in a manner not heretofore possible, through the development of an unambiguous formal language for their discussion. These matters are to be developed in easy stages, using the figures themselves as examples which drive and motivate the evolving theoretical complexity.

Let us discuss topic (1)., above, with a little more particularity. If one imagines the set of all string-figures to be represented by distinct labeled points, or "dots", on a large white sheet of paper, and draws differently colored lines between pairs that a. look alike, b. are made in similar manner, c. share a common opening, d. belong to the same peoples, etc., then the whole "structure" will emerge as a multicolored "tapestry" of dots plus various types of interrelations. One of our purposes is to make explicit those lines of this tapestry that are determined by the structure and construction of the string-figures, themselves. In particular, we shall not seek to investigate those interconnecting lines whose existence is predicated on the grounds of geography, anthropology, or psychology.

And we may use the above "tapestry"-analogy to discuss the topic of (2), above: Conservation of string-figures. If one reads a particular author's construction of a complicated string-figure, usually ambiguity as to the individual steps involved abounds! In most cases, a representation (picture, schema, ideograph) of the finished product prefaces this description, to let you know when you have correctly resolved these ambiguities; and, in a fair percentage of these cases, the representation itself will be incomplete, or incorrect (with respect to internal string-crossings, for example). The hope here is, of course, that one (and only one) of the string-positions whose construction "goes something like this" has a representation "that looks something like this". And for the great authors (e.g. C.F. Jayne, H. Maude) this usually works; for the lesser authors (e.g. F.D. McCarthy) it rarely does. If we circle in red each dot representing a string-position in our tapestry which corresponds to a possible interpretation of an ambiguous construction (and, hopefully, one of these will be the intended object of the description) the resulting red "cloud" of dots is what is really conserved by the author. The second object of these notes is to develop a language of constructions and representations of string-figures which uniquely conserves a given string-figure -- and provably so.

There are about 100 string-constructions given in these notes, the majority of which come from well-recognized sources in the string-figure literature; these are documented by a bibliographical reference which is sufficient to exactly specify an unique entry in the author's earlier "Bibliography" (Storer, T.: STRING-FIGURE BIBLIOGRAPHY. Bulletin of String Figures Association, No. 12, Tokyo (1985). pp. 66). More recent, or peripheral, references appear with complete citation herein. Also, finally, several figures appear without any source-documentation, under the epithet "contemporary". These are figures in current usage in the world, as encountered and recorded by the author from a wide variety of sources. They are included for reasons of comparison, completeness, or general interest not represented in the standard literature on the subject. All figures included herein have a direct bearing on the "ideas" in string represented by the parent four: Brochos, Osage Diamonds, Crows Feet, Bear.

The notes, themselves, may be read at several "levels". The shallowest is to "skim" through, like a novel, eschewing all formalism, to get an idea of what the subject is like, and what demands are to be made of the reader. Next is a reading to learn how to make ~100 pretty string-figures; here the minimal prerequisite is the mastery of the string-figure (constructional) Calculus, introduced in Section 1; "Symbology", and augmented throughout the development of the subject. This is based on the classical "function notation" of Mathematics/Symbolic Logic, and should pose no serious problem to the interested reader, being essentially self-contained. At the next level — what (from the present perspective) string-figures "really" are — attention should be paid to both the Calculus and the associated linear sequences (in the context of the string-figure schemata), from which the beauty and power of the model we have here adopted will emerge. At the deepest level, only the Calculus and the associated linear

sequences are the fundamental constructs, and the Appendices address problems of existence and uniqueness for the formal system so developed.

In conclusion, the author views these notes as "Chapter I" of an extended opus devoted to the formal aspects of string-figures. Our intention, in future "chapters", is to discuss related constructs (e.g. string-tricks) while continuing to illuminate the great ideas, or "resonances" in the string, in conjunction with the development of the formal language prerequisite to their elaboration. We shall also pursue the questions of completeness, consistency, and redundancy for the model at that time, matters whose inception is to be found in the present manuscript.

On a more personal note, in 1958 -- having learned some 20 string-figures from my Grandmother (all she knew!) and, perhaps, another dozen from my friends while growing up -- i discovered the little pamphlet by R. Rohrbough (ed.): FUN WITH FOLKLORE, with its two figures, Takapau and Brush House, taken from J.C. Andersen: MAORI STRING FIGURES. I could hardly believe my good fortune -- that very educated and learned people had actually written about such things -- and i devoured all the literature i could get my hands on. After learning my thousandth or so figure, i began searching for a book or article which spoke to the beautiful "system" which i dimly apprehended underlying these disparate string-figures -- to no avail. The wordy ramblings of collectors were too imprecise to satisfy, and topological Knot-Theorists apparently dismissed the entirety of the string-figures of the world as "trivial". And, although i learned a great deal from both groups of writers, i hungered for an approach which was neither too weak to be effective, nor so powerful that it identified (and as "trivial", at that) all the objects of my insatiable interest. And, since such a work still does not exist, to my knowledge, i have decided to write one, chronicaling my development of such a system over the ensuing years. Most of the theoretical material of the present notes, "Chapter I", was completed by the early 1960's, but not in the context of the present four string-figures discussed herein. These four string-figure "resonances" have here been chosen for the introduction of this formal system because they embody nearly perfect substitution instances of the generalizations fundamental to the emergent theory, at each step of the way. This is the meaning behind the earlier assertion that the string-figures, themselves, are used to "drive and motivate the evolving theory". I feel that, had such a work been available to me, many years ago, i would now know much more about String-figures than i do; and it is for just such a young person as i was then that the present notes are intended. I hope some of the love, joy, and wonder i find in the beautiful world of string-figures will be conveyed to you as you read through the following pages; that you will adopt and develop those ideas which you find fruitful to your viewpoint, and cast aside those that have no relevance to your own interests. Comments are cordially invited, and should be sent to the author at

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TABLE OF CONTENTS

I. SYSTEMOLOGY

String-figures (Cat's Cradles) are constructed in a closed loop of string, the ends being knotted or spliced to form a circle.

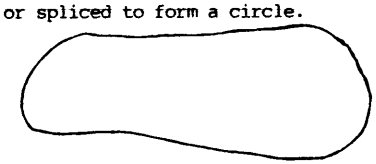


Fig. 1

A good starting-length is 2 to 2.5 meters, but often longer (or shorter) strings will be required for the more complicated figures of certain cultures.

In order to discuss string-figures in an unambiguous way, we must address two problems: 1. Description of the final (and intermediate) positions, and 2. Method of construction, *i.e.* how to proceed from one position to the next. The former, itself, has two aspects: 1A. The string's interrelationships to itself, *i.e.* internal crossings, loopings, *etc.*, and 1B. the string's relationship to the supporting frame; usually, the hands. The latter will be developed *via* "functional" notation; the *operators* being the body-parts (usually the fingers) involved in the manipulations necessary to pass between consecutive positions, and the *arguments* being the actual strings which are to be so manipulated. In the following, we shall address these two problems simultaneously, developing the necessary terminology as we proceed.

Initially, we restrict ourselves to the hands and their constituent parts (*i.e.* *The Frame*), whose names we assign following anatomical taxonomy.

𝔹 -- (both) hands

𝕃 -- Left hand
L1 -- Left thumb
L2 -- Left index
L3 -- Left middle finger
L4 -- Left ring finger
L5 -- Left little finger

ℝ -- Right hand
R1 -- Right thumb
R2 -- Right index
R3 -- Right middle finger
R4 -- Right ring finger
R5 -- Right little finger

These will (initially) be the *functors* in our discussion of Methods of Construction. Later, as needed, we shall add other body-parts to our list of functors; *e.g.*

M -- mouth
W -- wrist
T -- great toe

et cetera.

The reference, or Normal position (#) for the hands in the construction of the string-figures will be palms facing one another, fingers pointing up; **the whole frame is relaxed, fingers moderately spread and just slightly bent,** or curled.

-- hands in normal position

It will be noticed that, in normal position, the geometric distance between the tips of, say, L1 and L2 is about that between L2 and L5; this motivates the following schema for normal position:

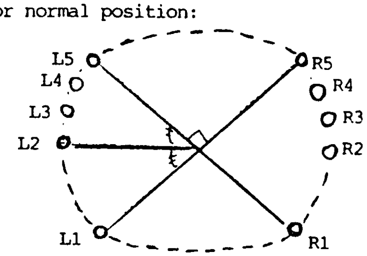


Fig. 2: # - Schema

Here, the view is from directly above the hands -- held in normal position -- as if the frame-owner peeked over and looked directly down thereon, the thumbs being nearest the body, the little fingers being furthest away. Only the "tips" of the fingers appear in the diagram -- as this is all that is required for a-while. Note that, so oriented, the labeling of the individual digits is implicit, and will be omitted in future specific schema.

Now, let's put the string on the hands. The initial position of the string on the hands -- at the start of a string-figure -- is called an Opening, which we shall abbreviate by "O."

O. -- Opening

A particularly useful opening for the construction of string-figures is Opening 1 (O. 1).

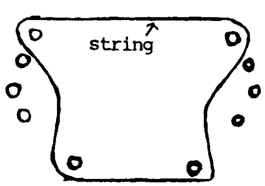


Fig. 3: Opening 1

O.1: Pass the string, from the near side of L1, around behind L1 and across the Left palm; then behind and around L5. The string then passes to the far side of R5, around and to the near side of R5, and directly across the right palm (towards you); then behind and around R1 and then directly across to the near side of L1, and the original point of the string. [Fig. 3]

We remark that the string is usually retained near the base of the fingers -- a fact elided by the schematics -- and is held tightly enough that there is no "slack"; that is, the hands extend slightly to absorb any slack, but not so tightly as to preclude manipulating the strings.

| -- extend hands to absorb slack

Let us now examine the string-frame structure of O.1 with an eye to developing the constructs necessary for the description of string-manipulation. It will be useful in this regard to partition the string into labeled arcs. In the following figure, the index, middle, and ring fingers of both hands have been suppressed for clarity.

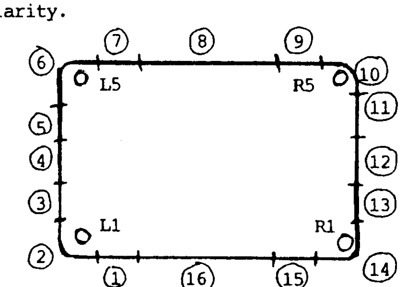


Fig. 4: O.1 - partitioned

We view the position O.1 as inducing a 16 - partition of the string, the elements of which (arcs) have been labeled consecutively from (1), beginning at the near side of L1 and proceeding clockwise around the string. We shall name these arcs according to their geometric proximity to the frame (and, later, to each other):

(1). L1n -- the near L1-string (arc)
(2). L1d -- the dorsal L1-string (arc)
(3). L1f -- the far L1-string (arc)
(4). Lp -- the L palmar string (arc)
(5). L5n -- the near L5-string (arc)
(6). L5d -- the dorsal L5-string (arc)
(7). L5f -- the far L5-string (arc)

⑨. R5f -- the far R5-string (arc)
⑩. R5d -- the dorsal R5-string (arc)
⑪. R5n -- the near R5-string (arc)
⑫. Rp -- the R palmar-string (arc)
⑬. R1f -- the far R1-string (arc)
⑭. R1d -- the dorsal R1-string (arc)
⑮. R1n -- the near R1-string (arc)

The fourteen (of the sixteen) arcs thus designated have been so named on the basis of bilateral specificity; that is, ① through ⑦ appertain to the left half of the frame (i.e., L), while ⑨ through ⑮ appertain to R. The remaining two string (arcs), i.e. ⑧ and ⑯, are connectors of these two halves and, as such, transcend the L/R specifics.

⑧ s; L5f-R5f -- the straight string (arc) running from the far side of L5 to the far side of R5.

⑯ s; R1n-L1n -- the straight string running from the near side of R1 to the near side of L1.

Or, in the absence of possible confusion (e.g. competing strings), as in this case, we simply suppress the bilateral specificity:

⑧ 5f -- the far little finger string

⑯ 1n -- the near thumb string

This should be regarded as a "shorthand" for the previous arc-designation, in the following sense: Initially, we focus on the left half of the frame and the prescribed interlacing of the string thereon; next, we shift our focus to the right half, and the analogous problem. Upon completion, we broaden, or widen, our focus to the whole configuration of frame plus string. It is in this "lifted" perspective that the position emerges as a far 5 string, a near 1 string, plus two palmar strings (on either side). We emphasize that the ability to lift perspective from the bilaterally specific to the wholistic -- and to "delift" back again -- is of paramount importance in the successful construction (and analysis) of string-figures, and often must be accomplished several times in the construction of a single figure . Finally, we remark that each position gives rise to a string partition, i.e. the partition for one position does not necessarily "carry over" to a subsequent position; in general, we must start from scratch with the new position. Further, we shall not concern ourselves with problems concerning the "size" of the elements of a given partition (e.g. in Fig. 4, where does ⑦ end and ⑧ begin?) since the ensuing

manipulations will be indifferent to such concerns (with respect to the resulting positions); and we shall customarily suppress elements of the (finest possible) partition induced by a position, exhibiting only those relevant to the manipulations involved in passage to a subsequent position.

In the following, it will be useful to designate certain concatenations of (some of) the arcs of a partition arising from a given position as loops. In general, if "F" is a generic body-part (usually, a finger) then by the "F-loop" will be meant the arcs Fn, Fd, and Ff (unordered triple). We shall designate this by

$$\infty \text{ --- loop}$$

For example, referring to Fig. 4, we have

L1∞ --- {①, ②, ③}
L5∞ --- {⑤, ⑥, ⑦}
R5∞ --- {⑨, ⑩, ⑪}
R1∞--- {⑬, ⑭, ⑮} .

And, in general, every string-position will have constituent, identifiable loops. By slight abuse of notation, when no confusion can arise, we may refer to larger sub-constructs as loops as, e.g. in Fig. 4, "the left thumb-little finger loop ":

L15∞ --- {①, ②, ③, ④, ⑤, ⑥, ⑦} ,
R15∞ --- {⑨, ⑩, ⑪, ⑫, ⑬, ⑭, ⑮} .

This is as customary in the string-figure literature. Here is another example:
is

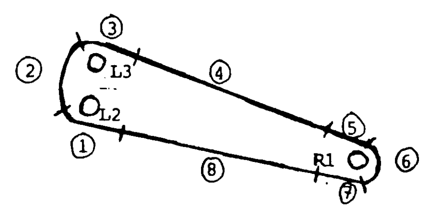


Fig. 5; ∞ -- Example

The partition is

① L2n
② L23d
③ L3f
④ s; L3f-R1f
⑤ R1f
⑥ R1d
⑦ R1n
⑧ s; R1n-L2n

(Note: There is no L2f nor L3n string). We have

L23∞ --- {①, ②, ③}

~~Rl∞~~ --- {⑤, ⑥, ⑦}.

We now initiate the discussion of Problem 1 (Description of string-positions) using the previously developed constructs. We wish to develop a "local" linear description of string positions, from which the associated schema is unambiguously recoverable. To that end, we imagine an ant crawling along the arcs of a given string-position. For example, in Fig. 4, with starting-point on the arc L1n, proceeding in a clockwise direction, our ant's itinerary would be

⊨⇒ L1, L5, R5, R1, L1; L5, R5, ...

to unlimited repetition. Thus the 4 - sequence

⊨⇒ L1, L5, R5, R1 ■

with the understanding that the first symbol in the sequence follows the final symbol -- the string is a closed loop -- is closely associated to the schema of Fig. 4 ... as are the sequences

⊨⇒ R5, R1, L1, L5 ■ (different starting point)

⊨⇒ L1, R1, R5, L5 ■ (counterclockwise orientation)

⊨⇒ L5, L1, R1, R5 ■ (different starting point and orientation),

there being eight such schema-equivalent sequences in all. For definiteness, we pick one of these as canonical of all such sequences arising from a given string-position -- there will be more (or less) than eight such, in general -- according to the following two conventions (concerning "starting point" and "orientation" respectively):

Convention Seq. 1: If LF is the left-hand finger nearest you which is incident with the string (often this will be L1), the starting point for the associated linear sequence will be a point on the LFn-arc.

Convention Seq. 2: The string orientation will be in the direction from the given LFn arc towards LF (which will then be the first entry in the associated linear sequence); i.e. "clockwise".

We remark that several string-figures include positions in which the string is incident with IR only; i.e. L is free of the string. So these conventions will have to be emended subsequently; initially, however, the above will suffice to produce an unique linear sequence associated to a given schema. For example, the linear schema associated to Fig. 5 is

⊨⇒ L23, R1 ■

The first topic of major consequence for the development of linear

sequences associable to string-figures is that of internal string crossings, a matter to which string-manipulation is anything but indifferent; we begin with an extended, illustrative example.

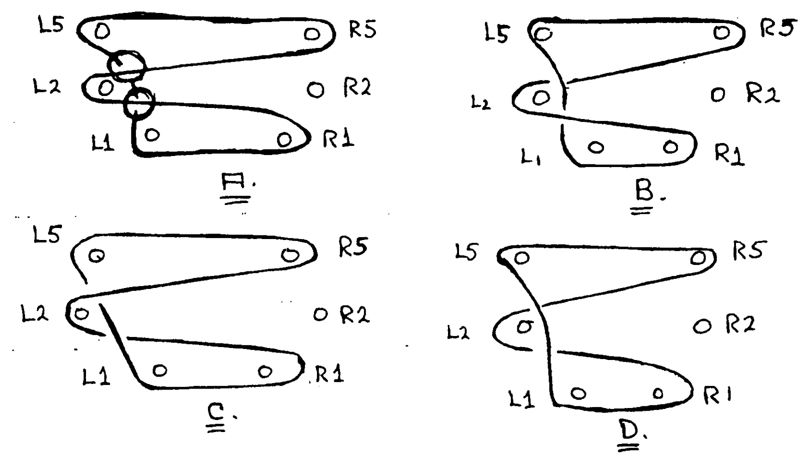


Fig. 6: Crossings

Fig. 6 gives the schemata for four similar string positions, differentiated only by the nature of two (simple) internal string-crossings (O-ed in Fig. 6, A). They may be constructed from Fig. 3 (i.e. O.1) as follows:

A. O.1: Pass L2 to right and pick up, on its back (from below) the Rp-string: # |

B. O.1: L2 pick up Rp from below (cf. A) # pass R12 down thru L2∞ and away to pick up L5f between them; bring this string back towards you and up through L2∞ and, releasing L5, replace this string on L5 # rotate R5∞ 360° away from you* # |

-----0-----

*This may be accomplished, for example, by temporarily grasping R5n and R5f between L1 and L2, then rotating R5 -- together with its loop -- away and down, then back towards you and up to its normal position. (Release L1 and L2 at the completion of the move).

C. 0.1: L2 pick up Rp from below # pass R12 down thru L2ꝏ and towards you to pick up L1n between them, draw this string away from you and back up thru L2ꝏ and, releasing L1, replace this string on L1 # rotate R1ꝏ 360° towards you #|

D. 0.1: R2 pick up Lp from below # pass L2 to right and down into R2ꝏ, then pick up Rp from below # release R2|

On each of these figures, in turn, we perform the exact same sequence of manipulations:

L1 pick up L2n from below (this will give two loops on L1; keep these well-separated, the "new" loop being uppermost) # R1 pick up R5n from below # on both hands, individually (i.e. one at at time) lift the lower 1ꝏ over the upper, and drop it on the palmar side of the (respective) 1 -- i.e. releasing this loop; the teeth may be used to facilitate this manipulation # release L2|

Note that four very dissimilar string-positions result! And, in applying these manipulations to Fig. 6, C, it makes a great difference whether L1 picks up L2n to the left of, or to the right of its crossing by the L1f-string. Of course, the intent of the string-partitioning is to pick out the first alternative by the indicated manipulation; but, had the second alternative been "mistakenly" adopted, a fifth string-position results -- different from each of the previous four! Our conclusions, based on this extremely simple example, are that -- in general -- string-manipulations are strongly dependent on internal crossing-types, and hence these must be carefully differentiated in our future development of the subject. Restatement, in terms of the "Ant crawling along the string" analogy; passing above another string (or below, or around, etc.) is as distinguished an event on his itinerary as is an interaction with the frame, and must be so recorded by the linear sequence associated with his journey.

Motivated by the foregoing discussion, we add nodes to the schema corresponding to a given string-position for each internal crossing; these will be generally designated by an "X" (to differentiate them from the nodes of the frame). Thus the four distinct (manipulation-inequivalent) string-positions

of Fig. 6 may be initially schematicized by

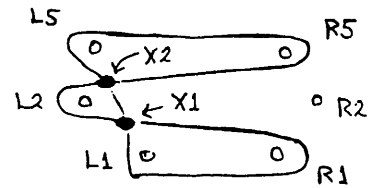


Fig. 7: X-augmented schema

with associated linear sequence

|⟹ L1, x1, x2, L5, R5, x2, L2, x1, R1 ■

The four distinct string-positions of Fig. 6 are then discriminated by local information about the nature of the string self-interaction (crossing-type) at each of x1 and x2. The simplest case, exemplified by the current example(s), is where exactly two strings (arcs)are involved, and one merely passes over (∅) [or under (U)] the other; this will be denoted a simple crossing. The associated linear sequence is, then, uniquely modified by appending the appropriate symbol (∅ or U) to the crossing-entry of that sequence. For example, the four distinct string-positions of Fig. 6 are uniquely specified by the following modifications to the above linear sequence:

Fig. 6. A: |⟹ L1, x1(U), x2(U), L5, R5, x2(∅), L2, x1(∅), R1 ■
B : |⟹ L1, x1(U), x2(∅), L5, R5, x2(U), L2, X1(∅), R1 ■
C : |⟹ L1, x1(∅), x2(U), L5, R5, x2(∅), L2, x1(U), R1 ■
D : |⟹ L1, x1(∅), x2(∅), L5, R5, x2 (U), L2, x1(U), R1■

We postpone, for awhile, a discussion of string-crossings which are not simple, either because more than two strings are involved, or because a simple parity description of the interaction is insufficient. As a simple example of the latter complication, consider the schema

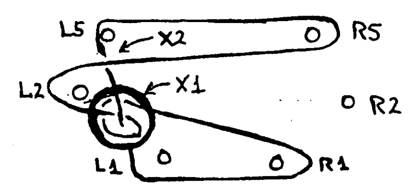


Fig. 8: x1 is not simple

which is obtainable from Fig. 3 by the manipulations

0.1: L2 pick up Rp from below # 1. pass R12 down thru L2ꝏ and towards you to pick up L1n between them, draw this string away from you and back up thru L2∞ and, releasing L1, replace this string **on L1 # 2. Repeat Step 1: 3. Rotate R1 720° towards you #|**

The schema associated to this string position <u>should</u> be

⊨> L1: x1(?): x2(U): L5: R5: x2(Ø): L2: x1(?): R1 #

where the ? — modifiers to the crossing x1 are not matters of simple parity but must, in general, refer to a "dictionary" of crossing-types for well-definedness. We shall develop these matters subsequently, in context, when they become appropriate.

<u>Lemma 1</u>: Let n be a generic natural number. If xn is a simple crossing in a given string-position, then the node xn appears exactly twice in the associated linear sequence. Further, the two appearances are demarked by opposite parities.

<u>Proof</u>: Definitional ■

In certain cases, a given simple crossing will be "cancellable". Consider the following examples:

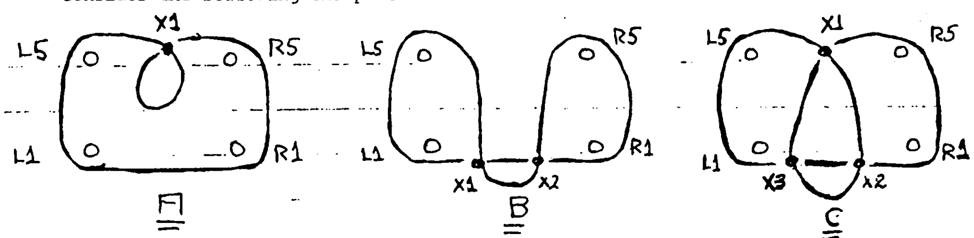


Fig. 9: Extension-cancellable crossings

These three schema, having respective associated linear sequences

A: ⊨> L1: L5: x1(Ø): x1(U): R5: R1 ■

B : ⊨> L1: L5: x1(Ø): x2(Ø): R5: R1: x2(U): x1(U) ■

C : ⊨> L1: L5: x1(Ø): x2(Ø): x3(Ø): x1(U): R5: R1: x2(U): x3(U) ■

all have B #, but <u>not</u> |. In fact, upon applying | to the three illustrated string-positions, we see that they all "collapse" to O.1 (Fig. 3). Such crossings are said to be <u>extension-cancellable</u>. In the following it will be observed that several important classes of string-manipulations depend upon the temporary creation of extension-cancellable string-crossings for their successful completion. These matters are all extremely compatible with the current notation, and will be developed in context. For the nonce we content ourselves with the

following elementary observations: First, we say that two entries in the linear sequence associated to a given string-position are (undirected) <u>adjacent</u> if:

<u>i</u>). ~~one is the first, the other the last, entry in the sequence~~

or <u>ii</u>). one immediately follows the other in the sequence.

Second, after Lemma 1, each simple crossing in a given string-position occurs exactly twice in the associated linear sequence; for definiteness in the following, we shall "tag" these occurrences with a superscript 1 or 2 according to their order of appearance in the canonical sequence.

adj --- adjacent.

For example, the linear sequence associated to Fig. 9, B, would then be

⊨> L1: L5: $x1^{(1)}$(Ø): $x2^{(1)}$(Ø): R5: R1: $x2^{(2)}$(U): $x1^{(2)}$(U) ■

Finally, if n is a generic natural number and xn is a simple crossing in a given string position, we shall write $\pm(xn^{(1)})$ and $\pm(xn^{(2)})$ for the parity of the crossing xn in the associated linear sequence. We remark that Lemma 1 asserts

$$\pm(xn^{(1)}) \neq \pm(xn^{(2)}),$$

one of these being Ø, the other U.

<u>Lemma 2</u>: Let n be a generic natural number.

<u>A</u>. If xn is a simple crossing in a given string-position, and

$$xn^{(1)} \text{ adj } xn^{(2)},$$

then the crossing xn is extension cancellable.

<u>B</u>. If xn and x(n+1) are simple crossings in a given string position which satisfy

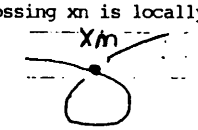
<u>i</u>). $xn^{(1)}$ adj $x(n+1)^{(1)}$

<u>ii</u>). $xn^{(2)}$ adj $x(n+1)^{(2)}$

<u>iii</u>). $\pm(xn^{(1)}) = \pm(x(n+1)^{(1)}) \neq \pm(xn^{(2)}) = \pm(x(n+1)^{(2)})$,

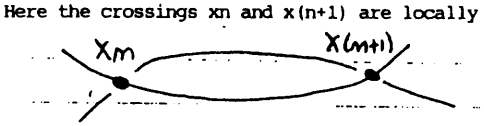
then the crossings xn and x(n+1) are extension-cancellable.

<u>Proof</u>: <u>A</u>. Here the crossing xn is locally homeomorphic to



which is extension-cancellable.

<u>B</u>. Here the crossings xn and x(n+1) are locally homeomorphic to



which are extension-cancellable ■

We remark that Fig. 9, A is an example of Lemma 2, A, while Fig. 9, B is an example of Lemma 2, B. Note that application of Lemma 2, B-type cancellation to the schema of Fig. 9, C essentially reduces that schema to that of Fig. 9, A, which is, itself, extension-cancellable. Compare

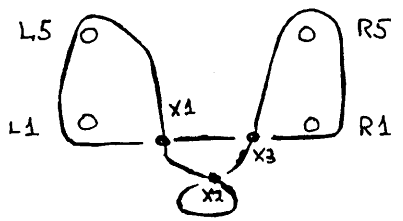


Fig. 10: Two-step extension-cancellation

which Lemma 2, A-type cancellation essentially reduces to Fig. 9, B which, as previously observed, is cancellable. Note that we have restricted ourselves, here, to simple crossings; we must examine the coefficient of friction (μ) for the string under manipulation to decide whether or not the (non-simple) crossing in a string position with schema

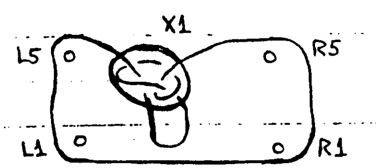


Fig. 11: Conditionally cancellable crossing

is extension-cancellable. For $\mu \approx 0$, the crossing x1 is so cancellable; for $\mu \approx 1$ the crossing x1 gives a knot* upon |. We feel that information as to the modal string-figure string of a specific culture may be gleaned from an analysis of those non-simple crossings which it customarily treats as extension-cancellable or non-cancellable.

----o----

*Knot is one of the most objectionable four-letter words.

It remains to discuss the string-partition induced by a string-position which contains crossings (whether simple or not). To that end we reconsider the schema of Fig. 6, H, with its two simple crossings:

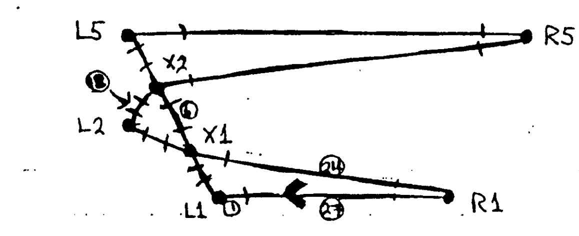


Fig. 12: Stylized reprise of Fig. 6, H.

Complete information, here, is afforded by the associated linear sequence [here "tagged" for future reference]

$\Longmapsto$ L1: $x1^{(1)}$(U): $x2^{(1)}$(U): L5: R5: $x2^{(2)}$(∅): L2: $x1^{(2)}$(∅): R1 ■

The string-partition induced in Fig. 12 is indicated --- every node accounts for two elements of the partition (its "local-neighborhoods"), ordered by "near" and "far" in the case of the fingers, and by "-" and "+" for the crossing nodes, in the direction of orientation (travel along the string), which is indicated --- plus the arcs which connect these node-neighborhoods (e.g. (6), (18), or (27)). As before, the arcs are labelled consecutively in the direction of travel, although -- in the diagram -- most of the labels have been suppressed in an attempt to salvage what little clarity there remains. The respective arc (element) names are, thus, specified; explicitly

(1)	L1n	(10)	L5n	(19)	L2f
(2)	L1f	(11)	L5f	(20)	L2n
(3)	s; L1-x1	(12)	5f	(21)	s; L2-x1
(4)	$x1^{(1)}_-$	(13)	R5f	(22)	$x1^{(2)}_-$
(5)	$x1^{(1)}_+$	(14)	R5n	(23)	$x1^{(2)}_+$
(6)	s; x1-x2	(15)	s; R5-x2	(24)	s; x1-R1
(7)	$x2^{(1)}_-$	(16)	$x2^{(2)}_-$	(25)	R1f
(8)	$x2^{(1)}_+$	(17)	$x2^{(2)}_+$	(26)	R1n
(9)	s; x2-L5	(18)	s; x2-L2	(27)	1n

Note, first, that the arc-names may be directly read-off from the associated linear sequence, and that the crossings require their superscript "tag" for well-definedness of their respective node-neighborhoods (else (4) and (22), for example, would be name-indistinguishable). Also, these "tags" have been suppressed on those straight strings which are node-connectors (e.g. (3), (6), (9), (15), (18), (21), and (24)) as uniqueness, here, obviates further discrimination. They are, however, required for definiteness in a handful of "pathological" string-positions, which will be handled as special cases in the following. Finally, non-simple crossings on three (or more) arcs will receive superscript "tags" 1,2, and 3 (etc.), and an unique naming of partition-elements will, in every case, obtain.

We now begin the development of a functional notation, or calculus, of string-figure manipulation. As with the linear-sequences formally associable to given string-positions, currently in its initial stages of development, the development of the string-figure calculus will take place in consistent stages, whose growing "evolutionary" complexity will mirror the increasing manipulational complexity of the sequence of figures chosen for the sample space. It will be convenient to have a canonical string-position example to illustrate the concepts to be introduced and, for this, we employ Opening A (O.A).

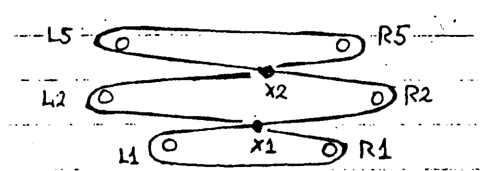


Fig. 13: Opening A (schema)

O.A.: |⟹ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■

This is constructible from O.1, as follows:

O.A. ≡ O.1: R2 pick up Lp from below #
Pass L2 to right and down into R2∞,
then pick up Rp from below #|

The first class of manipulations we shall consider is moving a finger to a particular string and picking up that string, from below, on its back. Note that the finger involved is the "actor" (functor), and the string to be picked up is

the "object" (argument) of this action. For example, consider the following string-manipulation applied to O.A.:

Pass L1 away over all intermediate strings (i.e. L1f, L2n, and L2f) and pick up on its back, from below, the L5n string (arc).

We shall symbolize this in the calculus by $\overrightarrow{L1}(\underline{L5n})$. Note that the arrow over the finger symbolizes both the direction that L1 moves (i.e. away from you) and the path (i.e. over all intermediate strings). The line below the object string (i.e. the argument) symbolizes that this string is to be picked up from below. Similarly,

$\overleftarrow{R5}(\underline{R1n})$ ≡ Pass R5 towards you over all intermediate strings (i.e. R5n, R2f, R2n, and R1f) and pick up on its back, from below, the R1n string. Note that $\overrightarrow{R2}(\underline{R5f})$ is well-defined, but $\overrightarrow{R2}(\underline{R1f})$ is not; as R2 passes away from you, it never encounters the R1f-string: for completeness, we shall consider this to be the "empty-functor", Φ.

Now we consider the notation $\underrightarrow{L1}(\underline{L2f})$:

Pass L1 away from you (over its own far-string, L1f) but under all remaining intermediate strings (i.e. only L2n, here) and pick up on its back, from below, the L2f-string.

The first parenthetical phrase in the above string-manipulation brings up our first convention for the class of manipulations under discussion.

Convention Calc. 1: A functor always passes over strings incident with it (if any).

This means independent of the away/towards direction and the under/over orientation. When we have occasion to effect a manipulation contrary to Convention Calc. 1, we shall employ a special notation, delineated below. Thus,

$\underleftarrow{R5}(\underline{R1f})$ ≡ Pass R5 towards you (over its own near-string, R5n) but under all remaining intermediate strings (i.e. L2f and L2n) and pick up on its back, from below, the R1f string.

In general, for generic finger F, and string, s,

$\overrightarrow{F}(\underline{s})$ ≡ Pass F away over all intermediate strings (if any) and pick up on its back, from below, the s-string.

et cetera.

We now consider functor paths which are more complicated than the simple "over (or under) all intermediate strings." Beginning with the string-position O.A, again, consider the string-manipulation

Pass L1 away over (L1f, convention, and) L2n, but under L2f and pick up L5n from below.

We symbolize this by

$$\overrightarrow{L1}(L2n):\ \underrightarrow{L1}(\underline{L5n}),$$

and "read" it

Pass L1 away over all strings up to, and including, the L2n string, continue passing L1 away under all (remaining) intermediate strings until you come to the L5n-string, which L1 then picks up, from below.

Here, the absence of a "bar" below (and, eventually, above) the argument L2n implies that it is an intermediate string in a (more complex) manipulation; the colon, " : ", may be read "continue"; then L1, having gotten as far as (over) the L2n-string, proceeds under the remaining strings between its present position and the object-string, L5n -- which it then picks up, from below. Similarly, from O.A again, the complex manipulation implied by the notation

$$\overleftarrow{R5}(R2f):\ \underleftarrow{R5}(R2n):\ \overleftarrow{R5}(\underline{R1n})$$

may be readily inferred, if not so easily accomplished, by a R unused to such complicated maneuvers.

The last example, above, brings up an interesting point leading to our second convention: Consider the string of symbols

O.A:: $\overleftarrow{R5}(R2f):\ \underleftarrow{R5}(R2n):\ \overleftarrow{R5}(\underline{R1n})$ #|

Question: How many strings does R5 pick up in the course of this manipulation? That the R1n-string is to be picked up is clearly indicated by the notation; but what about the R2n-string? The more difficult move (in terms of the manual dexterity involved) is what is intended; Only the R1n string is to be picked up by R5, and this is to be drawn back under R2n (which is not, therefore, retained by R5) to #. The contrary situation would have been encoded

O.A:: $\overleftarrow{R5}(R2f):\ \underleftarrow{R5}(R2n):\ \overleftarrow{R5}(\underline{R1n\ \&\ R2n})$ #|

i.e. all strings picked-up in a manipulation appear explicitly as "barred" arguments. Further, in even more complex situations, all confusion is eliminated by the convention

Convention Calc. 2: The symbol " # " at the end of a functor string means that the given functor returns to normal position along its outgoing path.

In particular, it picks up no intermediate strings unless explicitly so indicated.

It remains to discuss the case where a functor picks up its argument (on its back) from above. For example

O.A: $\overrightarrow{L2}(\overline{L5f})$ #| ≡ O.A: pass L2 away over all intermediate strings including L5f, then dipping L2 down and slightly to the right, engage L5f (from the far side and above) on its back, thus picking up this string #|

Functionally, this is equivalent to

O.A: $\overrightarrow{L2}(\underline{L5f})$ # twist upper L2∞ 180° away from you #|

Similarly, the meaning of

O.A: $\overleftarrow{L2}(\overline{L1n})$ #|

may be gleaned from its functional equivalent

O.A: $\overleftarrow{L2}(\underline{L1n})$ # twist upper L2∞ 180° towards you #|

Finally, we define

O.A: $\overrightarrow{R1}(\overline{R5n})$ #|

by its equivalent

O.A: $\overrightarrow{R1}(\underline{R5n})$ # twist upper R1∞ 180° away from you #|

That is, passing a finger away and picking up a given string from above is equivalent to passing the finger away and picking up that string from below, followed by a half-twist away of the resulting upper loop of the manipulating finger. The analogous statement with each (underlined) "away" replaced by the words "towards you" also obtains.

Finally, we may combine the foregoing concepts to produce symbol-strings like

O.A: $\overrightarrow{L1}(L2n):\ \underrightarrow{L1}(\overline{L5n})$ #|

whose meanings, by now, should be apparent, if not so easy to effect, manually. This "function notation" is amazingly versatile, and is as easy to encode as it is to decode ("read"). Further, as will be seen, it lends itself as well to the situational aspect of string-manipulation (i.e. the "effect" of a given movement) as to the descriptive. A simple observation about encoding: Consider the string-position produced by

O.A: release both 1∞'s #|

and suppose we now wish to

Pass L1 away and pick up, from below, the L2n-string.

Question: In the absence of intermediate strings (between L1 and L2n), is this movement to be encoded

$$\overrightarrow{L1}(\underline{L2n}) \quad \text{or} \quad \underset{\rightarrow}{L1}(\underline{L2n})\ ?$$

Either is correct in this situation, as "Pass L1 away ⌈over/under all intermediate strings ⌉ and pick up L2n from below" makes internal reference to the empty set; so the "arrow" in the notation gives directional information only, in this case.

The Calculus so far developed is bilaterally specific when, in fact, the majority of string-figures are made with both hands operating simultaneously, and in concert. We may lift our perspective from the bilaterally specific to the wholistic by employing a "split-level" notation. For example:

$$\underline{O}.A:\ \left\{\begin{matrix}\overrightarrow{L1}(\underline{L5n})\\ \\ \overrightarrow{R1}(\underline{R5n})\end{matrix}\right\}\ \#\ \left\{\begin{matrix}\text{release } L5\infty\\ \\ \text{release } R5\infty\end{matrix}\right\}\quad |$$

This is used to indicate that the manipulations $\overrightarrow{L1}(\underline{L5n})$ and $\overrightarrow{R1}(\underline{R5n})$ are to be effected simultaneously. We shall adopt the convenient shorthand

$$\overrightarrow{1}(\underline{5n}) \equiv \left\{\begin{matrix}\overrightarrow{L1}(\underline{L5n})\\ \overrightarrow{R1}(\underline{R5n})\end{matrix}\right\},$$

emphasizing this wholistic perspective. Similarly

$$\text{release } 5\infty \equiv \left\{\begin{matrix}\text{release } L5\infty\\ \text{release } R5\infty\end{matrix}\right\}.$$

Thus, wholistically, the above example may be more cleanly written, without ambiguity,

$$\underline{O}.A:\ \overrightarrow{1}(\underline{5n})\ \#\ \text{release } 5\infty\ |$$

Of course, the second move in the sequence

$$\underline{O}.A:\ \left\{\begin{matrix}\overrightarrow{L1}(\underline{L2f})\\ \overrightarrow{R1}(\underline{R5n})\end{matrix}\right\}\ \#\ \text{release } 5\infty\ |$$

requires a bilaterally specific perspective for its completion -- in the absence of symmetry -- and the notation admits no obvious shorthand; and should not. But we cannot refrain from mentioning that the symbol-string

$$\underline{O}.A:\ \overrightarrow{L1}(\underline{L2f})\ \#\ \overrightarrow{R1}(\underline{R5n})\ \#\ \text{release } 5\infty|$$

may be thought of as performer's vision of a manipulational sequence in the early learning stages, while*

$$\underline{O}.A:\ \left\{\begin{matrix}\overrightarrow{L1}(\underline{L2f})\\ \overrightarrow{R1}(\underline{R5n})\end{matrix}\right\}\ \#\ \text{release } 5\infty|$$

might represent a subsequent stage of assimilation when it is realized that, in the second step, the thumbs pass away, over, and pick up s; L2f-R5n on either side of the central strings. Thus the notation can be made to mirror a "developmental" aspect of string-figure constructional awareness, to this extent.

It is apparent from the string-figure literature that different collectors consider diverse constructs of the string-positions associable to a given culture as fundamental-whether the native practitioners did or not. Of course, many of these attitudes undoubtedly stem from the pioneering work of Rivers & Haddon on the recording of string-figures; one of these, whose ubiquitousness mandates mention, is the following: Consider the sequence, considered earlier,

$$\underline{O}.A::\ \overrightarrow{1}(2n):\ \underset{\rightarrow}{1}(\underline{5n})\#|$$

This is variously rendered as

$$\underline{O}.A::\ \text{Thumbs away over and down through the } 2\infty:\ \underset{\rightarrow}{1}(\underline{5n})\#|$$

which, perhaps, emphasizes "loops" as a distinguished string-figure sub-construct. Bowing to this viewpoint, we introduce the alternate notation of

$$\underline{O}.A::\ \overrightarrow{1}\downarrow(2\infty):\ \underset{\rightarrow}{1}(\underline{5n})\#|$$

whose second entry is self-explanational in this context. Similarly, we have the alternates

$$\underline{O}.A::\ \underset{\rightarrow}{1}\uparrow(2\infty):\ \overrightarrow{1}(\underline{5n})\#|$$

for

$$\underline{O}.A::\ \underset{\rightarrow}{1}(2n):\ \overrightarrow{1}(\underline{5n})\#|$$

-----o-----

*We shall always put L's manipulation above R's in this split-level notation. By analogy with linear sequences, if we consider an ant crawling along the symbol-string as he reads it, he encounters manipulations for his left "hand" to his left, and those for his right "hand" to his right.

and

O.A:: $\overleftarrow{5}\downarrow$(2oo): $\underleftarrow{5}$ (1f)#|

for O.A:: $\overleftarrow{5}$ (2f): $\underleftarrow{5}$(1f)#|

et cetera. Note that the second entry of

O.A:: $\overrightarrow{1}\uparrow$(2oo): $\overrightarrow{1}$ (5n)#|

is undefined, and will be considered to be Φ, et cetera.

We conclude our discussion of the initial stage of string-figure calculus with a purely wholistic construct; a given hand manipulating strings on the other hand. Let F be the generic finger, s the generic string; we define

$\overrightarrow{\text{LF}}$ (Rs) — pass LF to right over all intermediate strings (if any), and pick up the Rs string from below

$\overrightarrow{\text{LF}}$ ($\overline{\text{Rs}}$) — pass LF to right, over, and pick up Rs from above

$\underrightarrow{\text{LF}}$ (Rs) — pass LF to right, under, and pick up Rs from below

et cetera. And

$\overleftarrow{\text{RF}}$ (Ls) — pass RF to left, over, and pick up Ls from below

$\underleftarrow{\text{RF}}$ ($\overline{\text{Ls}}$) — pass RF to left, under, and pickup Ls from above,

et cetera: Note that "double-arrow right", $\Rightarrow$, means "pass indicated functor to right" -- the under/over orientation being defined positionally, as before -- while "double-arrow left", $\Leftarrow$, means "pass indicated functor to left". Thus

O.A $\equiv$ O.1: $\overleftarrow{\text{R2}}$ (Lp)# $\overrightarrow{\text{L2}}\downarrow$(R2oo): $\underrightarrow{\text{L2}}$ (Rp)#|

Usually, but not always, a right functor will pass to the left; a left functor, to the right.

The foregoing initial development of a string-figure Calculus has produced (in potentio) many string-positions which have no associated schemata (or, therefore, linear sequences) under their current, limited definition. In particular, schemata presently make no provision for multiple loops on a given functor -- a situation which we now remedy. For example, consider the sequence

O.A: $\overrightarrow{\text{L1}}$ (L5n)#|

which produces two loops on L1. We shall always consider multiple loops on a finger to be well-separated -- unless explicitly indicated to the contrary -- and, new loops added to a given finger under manipulation will become uppermost.

Thus, in the position resulting from the symbol-string under consideration, L1 has a lower loop (ℓL1oo) and an upper loop (uL1oo). To schematicize this string-position we split the L1 node into a lower (ℓL1) and an upper (uL1) node, viz.

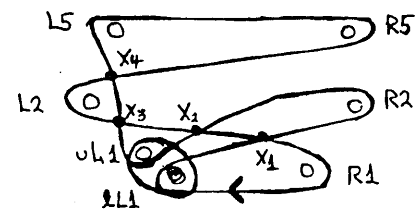


Fig. 14: Multiple L1oo's

all of whose crossings are simple. The associated linear schema is, now, that associated ordinarily to such a (well-defined) schema; in this case

$\models\!\Rightarrow$ ℓL1: x1($\emptyset$): R2: x2($\emptyset$): uL1: x3($\emptyset$): x4($\emptyset$): L5: R5: x4(U): L2: x3(U): x2(U): x1(U): R1 ■

from which a string-partition may be read-off in the usual way. In general if, in a given string position the generic finger, F, has the generic natural number n loops we name these -- beginning at the base of F and proceeding to the tip -- as follows:

n	Foo's
1	Foo
2	ℓFoo, uFoo
3	ℓFoo, mFoo, uFoo
4	ℓFoo, m_1Foo, m_2Foo, uFoo
5	ℓFoo, m_1Foo, m_2Foo, m_3Foo, uFoo
.	
.	
.	
.	
n	ℓFoo, m_1Foo, m_2Foo, ..., m_{n-2}Foo, uFoo

We remark that we know of no string-position arising from a native culture in which n>3 and the loops must be kept distinct; with n=3, the number so known is fewer than 20. The notation above, however, is sufficiently general to allow for that possibility, or to allow for modern invention. The schematization for the case of multiple loops intended here (a convention) will be made clear from a consideration of the following schema-frame for a supposed string-position in which L1 has 2 loops, L2 has 1, L5 has 2, R5 has 1, R2 has 3, and R1 has 2:

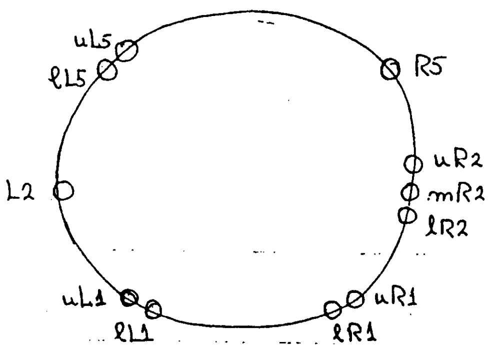


Fig. 15: Schema-frame for multiple loops, I.

The given string-position (here only hypothetically specified) is then imposed on this frame, and the uniquely associable linear sequence (with all crossings) read off. This is the "ordinary" schema which would be associated to a creature having 5 fingers on his L, and 6 on his R -- those fingers being "named" as in the above diagram. Note also that, "for clarity", the nodes resulting from the split of a given (finger-)node are kept more closely grouped -- near the original (unsplit) finger-node's usual frame-position -- compared to the distance between distinct finger-nodes (say, L1 and L2, in the original frame). Finally, in each such grouping, note that the loops lowest on the given finger appear lowest on the page, in the diagram. Finally, let us suppose that the sequence L1→(L5n)#| were applied to whatever string-position is suspended on the above frame. Then the resulting frame would have to be

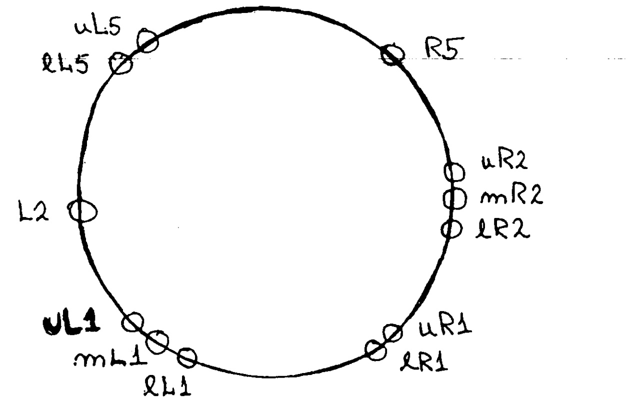


Fig. 16: Schema-frame for multiple loops, II.

The "new" loop on L1 must become the uL1ꝏ (of Fig. 16) while the old uL1ꝏ of (Fig. 15) has been "demoted" to the mL1ꝏ (of Fig. 16); i.e. we adopt the "alias" viewpoint, here: The same loop (the one above the "constant" ℓL1ꝏ) changes names between Fig.s 15 and 16. Again, the resulting schema is "ordinary" (for a six-fingered creature) and has an associated linear sequence: the latter induces an unique string-partition, which provides the well-defined arcs necessary for unambiguous string-manipulation (Calculus). Some manipulations on multiple loops are among the most fundamental maneuvers common to string-figure practitioners the world over, a matter to which we now attend.

The first of the loop-specific operations to be considered is how to get out of one. We shall restrict ourselves to the case of ≤ 3 distinct loops on a given finger, but the generalization to n-loops will be immediate. We employ the notation

☐ --- release.

As usual, let F be the generic finger.

Case 1: F has exactly one loop -- the Fꝏ.

Here

☐ Fꝏ -- lift Fꝏ over the tip of F, and drop it on the palmar side. This will, in general, create a hanging ꝏ -- i.e. "slack" strings in the central design which may (or may not) be absorbed by |.

Case 2: F has exactly two loops -- ℓF∞ and uF∞.

Now

□uF∞ --- lift uF∞ off of F and drop it on the palmar side

exactly as in Case 1, above. However,

□ℓF∞ --- with 1 and 2 of the opposite hand, temporarily remove uF∞ (so that there remains only a single F∞): □F∞: replace former uF∞ (held by 1 and 2 of opposite hand) #

That is, the ℓF∞ is released "below" the uF∞; this is the precise meaning of □ℓF∞ in the split-node schema for F.

Case 3: F has exactly three loops -- ℓF∞, mF∞, and uF∞.

Here

□uF∞ --- same as for Case 2

□mF∞ --- with 1 and 2 of opposite hand, temporarily remove uF∞ (so that there remain two F∞'s, ℓF∞ and uF∞): □uF∞ (as in Case 2): replace former uF∞ (held by 1 and 2 of opposite hand) #

□ℓF∞ --- with 1 and 2 of opposite hand, remove mF∞ and uF∞ (leaving a single F∞): □F∞: replace former mF∞ and uF∞ (held by opposite hand) onto F, maintaining their relative positions to each other #

Again, all above symbols take their definition from the exact meaning of Case 1 (□F∞) applied to the appropriate split-node schema. Two remarks are in order; first, if F has no incident loop, we consider □F∞ to be ϕ. Second, by slight abuse of notation, we shall use

□F ≡ release all loops on F.

In particular, when F has exactly one loop,

□F∞ ≡ □F.

Finally, in Case 2, above, a significant variation of "□ℓF∞" is so common to string-figure manipulation as to warrant its own notation:

N ---- Navaho.

When F has exactly two loops -- ℓF∞ and uF∞, we define

NF ≡ with 1 and 2 of opposite hand, seize the ℓFn-string and lift the ℓF∞ up and over the uF∞, and drop it on the palmar side #.

Note that, in every case of releasing a loop from a finger, the loops which

remain loops on that finger (if any) are subject to renaming via the convention (page 21) before subsequent manipulation.

The second of the loop-specific operations is the twist:

> ---- twist 180° away from you

< ---- twist 180° towards you.

For example, consider the manipulation string

O.A: >L2∞ #|

The second movement is accomplished as follows:

>L2∞ ≡ Seize L2∞ between R1 and R2 and lift it off L2: (looking directly at Lp) turn this loop over 180° clockwise, and replace it on L2 #

Similarly,

<L2∞ ≡ Seize L2∞ between R1 and R2: □L2: turn this loop over 180° counterclockwise: replace loop on L2 #

By analogy -- remark this difference carefully --

>R2∞ ≡ Seize R2∞ between L1 and L2 : □R2: (looking directly at Rp) turn this loop over 180° counterclockwise: replace on R2 #

Note: looking at Lp, clockwise is "away from you" when B are in #; looking at Rp, counterclockwise is "away from you when B are in #. Thus, finally, <R2∞ has the same definition as that for >R2∞, with the word "counterclockwise" replaced by "clockwise". To easily recall which of the symbols -- > or < -- means "away from you", and which "towards you", return to the analogy of the ant crawling along the symbol-string for a given manipulation as he reads it; when he encounters the symbol ">", he sees it as an arrowhead, pointing "away" from him -- the symbol "<", as an arrowhead pointing "towards" him.

In like manner, we may easily effect the twist of a given loop on a finger incident with multiple loops: using the other hand, carefully remove all loops (if any) on the finger above the loop to be manipulated, perform the indicated twist on that loop, and carefully replace the original loops on the given finger. This is the precise meaning of the indicated twist on the split-node schema for the finger in question.

Some notational shorthands and conveniences:

If F is a generic finger,

$$>F\infty \equiv \left\{\begin{matrix} > LF\infty \\ > RF\infty \end{matrix}\right\}$$

as is usual for split-level notation, while

$$>> F\infty \equiv\ > F\infty : >F\infty$$

Note that >> RFꝏ, for example, is easily accomplished by seizing RFn and RFf between L1 and L2, and then rotating RF away, down, back towards you, and up to # (□L1 and L2). When F has only one loop -- or, later, when F has multiple loops that need not be maintained as distinct -- we shall abuse notation slightly, in the usual way, by writing

>>RF ≡ seize all RF strings, close to RF, and rotate RF away, down, towards you, and back up to # (releasing all seized RF strings).

et cetera. As a final remark, if F has no loop, we consider the instruction <Fꝏ to be Φ (etc.).

The last of the loop-specific operations to be considered in this initial stage of the subject is the translation of loops. The development will parallel that of the functor calculus but, strictly speaking, loops are not functors, but constructs (arc concatenations). Consider, by way of example, the manipulational symbol string

$\underline{O}.A$: $\overleftarrow{2}$ (1f) #□1|.

This has the effect of translating the 1ꝏ to 2 where, since 2 already has a loop, it becomes the u2ꝏ. We shall occasionally substitute the symbol string

$\underline{O}.A$: $\overrightarrow{1\infty}$ → 2 #|

for the previous notation. Here

$\overrightarrow{1\infty}$ → 2 ≡ pass the 1ꝏ away over all intervening strings (if any) and place it, as a loop, directly upon 2.

We shall extend the convention that all new strings picked up by a given functor become uppermost on that functor (unless otherwise noted) to the present situation, whence remarking that the new 2ꝏ is to be the u2ꝏ will be redundant. Similarly, with the starting-position

$\dot{\underline{O}}.A$: $\overrightarrow{2\infty}$ →3: $\overrightarrow{1\infty}$ →2# $\overrightarrow{RI}$ ($\overline{R2n}$)# L1↑(R1ꝏ)#|

we shall define the 5ꝏ translation

$\overleftarrow{5\infty}$(3ꝏ): 5ꝏ →1 ≡ pass the 5ꝏ towards you over the 3ꝏ, continue, pass 5ꝏ towards you under all remaining strings (here, the 2ꝏ) and place it, as a loop, on 1,

by direct analogy with the previous functor-notation. Similarly, the unique meanings of the manipulations

5ꝏ↑(2ꝏ): $\overleftarrow{5\infty}$→ 1#|

>$\overrightarrow{1\infty}$ →3#|

and the composite

>$\overrightarrow{1\infty}$↓(2ꝏ): 1ꝏ →4#|

should now be clear.

We shall also use loop-translation for intra-functor loop manipulation: Suppose that the generic finger, F, has exactly two incident loops, ℓFꝏ and uFꝏ. We write

ℓFꝏ → uFꝏ ≡ pass ℓFꝏ over uFꝏ to the tip of F.

Thus, in effect, ℓFꝏ and uFꝏ change places -- the lower loop passing over the upper. For the contrary situation, we write "uFꝏ → ℓFꝏ" where, again, the ℓFꝏ and uFꝏ change places -- this time the upper loop passing over the lower. The convention is this: The loop to be moved always passes over the unmoved loop (as well as all intermediate loops, if any). Hence

N F ≡ ℓFꝏ→ uFꝏ:□uFꝏ.

□ℓFꝏ ≡ uFꝏ→ ℓFꝏ:□uFꝏ.

This, essentially, concludes the bilaterally specific translation of loops.

There is a fundamental wholistic loop transference known as the "exchange" Consider a string-position in which both RF and LF have a single loop each. By

X F (R) ≡ exchange F loops, passing right over left

we shall mean the manipulation

Bend RF and LF towards each other slightly; $\overleftarrow{RF}$(LFd) on tip of RF (i.e. above the RFꝏ already retained there):□LFꝏ#
$\overrightarrow{LF}$(uRd): LF(ℓRd)# (releasing ℓRFꝏ -- over uRFꝏ)|

This exchanges the Fꝏ's, the left Fꝏ having passed through the right.
In like manner,

X F (L) ≡ Bend RF and LF towards each other: $\overrightarrow{LF}$(RFd) on tip of LF: □RFꝏ # $\overleftarrow{RF}$(uLd): RF(ℓFd) # (□ℓLFꝏ- over uLFꝏ)|

As a check on our understanding of the exchange moves, we perform the sequences

O.A: X2(L) #|,

which produces two non-simple crossings in the resulting central design, versus

O.A: X2(R) #|,

which falls apart.

The last topic to be considered in this initial development of loop specific manipulations is that of the direct cross-hand transfer. To that end, let F be the generic finger, and suppose that LF has a single loop, while RF has no loops. Then

$$LF\infty \Longrightarrow RF \equiv \overleftarrow{\overleftarrow{RF}}\downarrow(LF\infty):\Box LF \text{ #|},$$

thus transferring the $LF\infty$ to RF. When LF and RF have multiple distinct loops, the above symbol remains defined under the conventions that the loop which moves always passes over all loops above it (if any) on the same finger, and that any new loop picked up by a finger becomes the uppermost loop on that finger (if, indeed, there are any such loops). Similarly, if RF has a single loop, while LF has none then, by symmetry, we define

$$RF\infty \Longrightarrow LF \equiv \overrightarrow{\overrightarrow{LF}}\downarrow(RF\infty):\Box RF \text{ #|},$$

Thus transferring the $RF\infty$ to LF.

End Stage 1 Development

BIBLIOGRAPHY: FORMAL SYSTEMS AND NOTATIONAL SHORTHANDS

Amir-Moéz, A.R.: MATHEMATICS AND STRING FIGURES.

Amir-Moéz, A.R. & J.D. Hamilton: ART AND MATHEMATICS OF STRING FIGURES.

Firth, R. & H.C. Maude: TIKOPIA STRING FIGURES.

Holden, A.: ORDERLY TANGLES.

Kamiya, K.: VARIATIONS OF JACOB'S LADDER.

Maddison, R.: STRING FIGURES.

Michelson, T.: FOX PHYSICAL ANTHROPOLOGY.

Natubori, K.: CATS CRADLE FOR ONE PERSON.

Noble, P.D.: STRING FIGURES: Reply to R. Maddison: STRING FIGURES.
A CONTRIBUTION TOWARDS A SHORTHAND NOTATION FOR FIELD-WORKERS RECORDING STRING FIGURES.
STRING GAMES AND PAPER EQUIVALENT MODELS.

Read, R.: AN ABBREVIATED NOTATION FOR RECORDING STRING FIGURES.

Rivers, W.H.R. & A.C. Haddon: A METHOD OF RECORDING STRING FIGURES AND TRICKS

Roth, W.E.: STRING FIGURES, TRICKS, AND PUZZLES OF THE GUIANA INDIANS.

Walker, J.: CAT'S CRADLES AND OTHER TOPOLOGIES FORMED WITH A TWO-METER LOOP OF FLEXIBLE STRING.

The oldest known string-figure in Western literature is attributed to the Greek Heraklas, about the first century A.D., in a manuscript entitled "Brokhos". And, although the original work did not survive, it is extensively cited -- perhaps reproduced in translation -- in the medical treatise Iatrikon Synagogos by one Oribasius of Pergamum.(1)

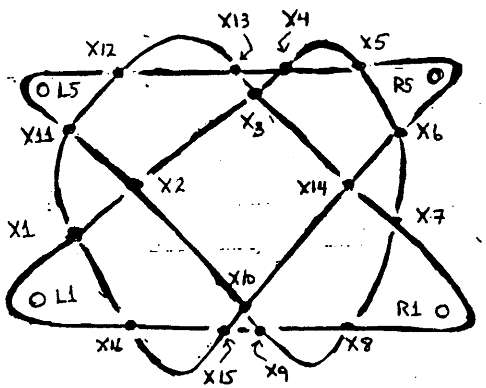


Fig. 17: Brochos (sling)

⟾ L1: x1(∅): x2(U): x3(U): x4(U): x5(∅): x6(∅): x7(U): x8(U): x9(∅): x10(U): x2(∅): x11(U): L5: x12(U): x13(∅): x4(∅): x5(U): R5: x6(U): x14(∅): x10(∅): x15(∅): x16(U): x1(U): x11(∅): x12(∅): x13(U): x3(∅): x14(U): x7(∅): R1: x8(∅): x9(U): x15(U): x16(∅) ■

Although we cannot know Heraklas' method of construction, the figure may be simply made as follows:

O.A: $\vec{1}$(2f)# ⮌2(ℓ1f)#□1| $\vec{1}$↓(ℓ2∞): 1→(5f)#□5| > $\overrightarrow{u2\infty}$ —> 5:□2#| (gently).

We propose to analyze this figure constructionally, by sequentially illuminating each of its intermediary string positions;

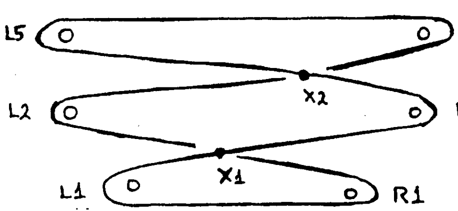


Fig. 18.I: Brochos*, O.A.

* cf. Fig. 13

⇒ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■

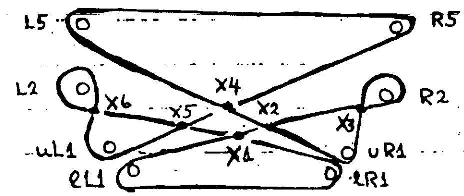


Fig. 18.II: Brochos, O.A: $\vec{1}$(2f)#

⇒ ℓL1: x1(∅): x2(U): x3(U): R2: x3(∅): uR1: x2(∅): x4(∅): L5: R5: x4(U): x5(∅): uL1: x6(∅): L2: x6(U): x5(U): x1(U): ℓR1 ■

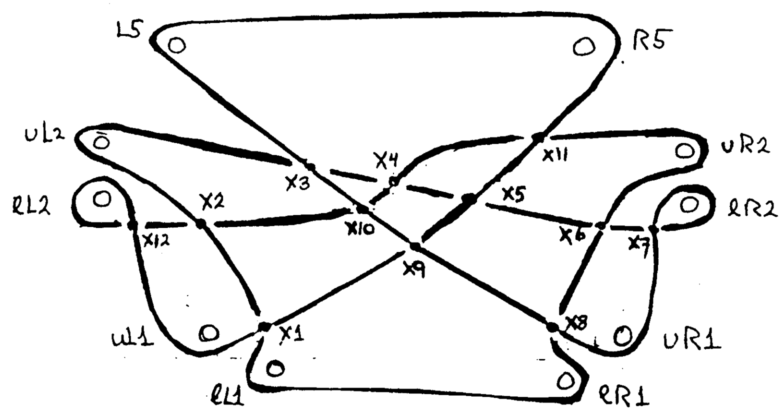
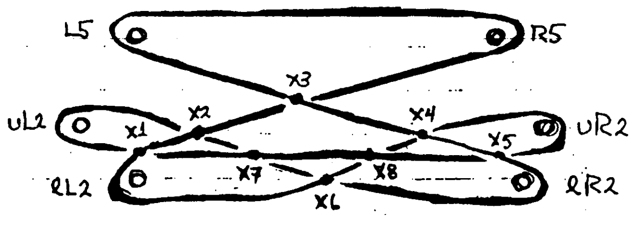


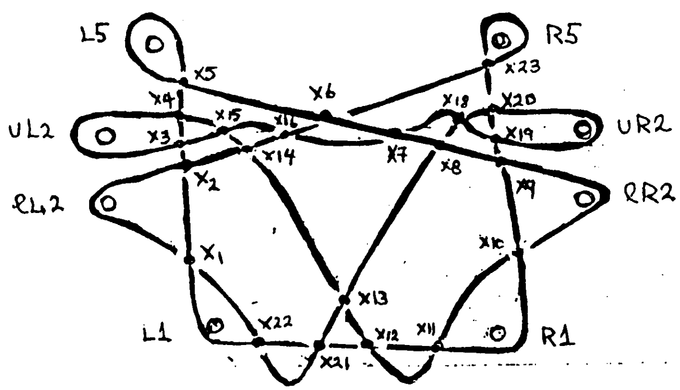
Fig. 18.III: Brochos, O.A: $\vec{1}$(2f)# ⮌2(ℓ1f)#

⇒ ℓL1: x1(U): x2(∅): uL2: x3(U): x4(U): x5(U): x6(U): x7(U): ℓR2: x7(∅): uR1: x8(∅): x9(∅): x10(∅): x3(∅): L5: R5: x11(∅): x5(∅): x9(U): x1(∅): uL1: x12(∅): ℓL2: x12(U): x2(U): x10(U): x4(∅): x11(U): uR2: x6(∅): x8(U): ℓR1 ■



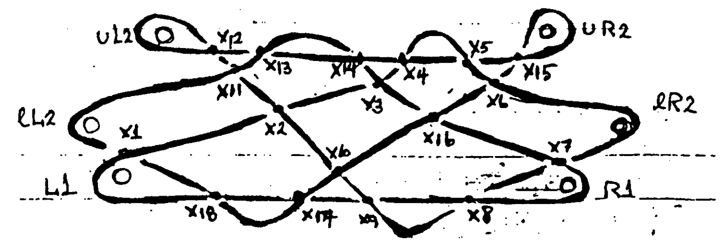
<u>Fig. 18.IV</u>: Brochos, <u>O</u>.A: $\overrightarrow{1}$ (<u>2f</u>)# $\overleftarrow{2}$ (<u>ℓ1f</u>)# □1|

⊨ ℓL2: x1(Ø): x2(Ø): x3(U): R5: L5: x3(Ø): x4(Ø): x5(Ø): ℓR2: x6(U): x7(U): x2(U): uL2: x1(U): x7(Ø): x8(Ø): x5(U): uR2: x4(U): x8(U): x6(Ø) ■



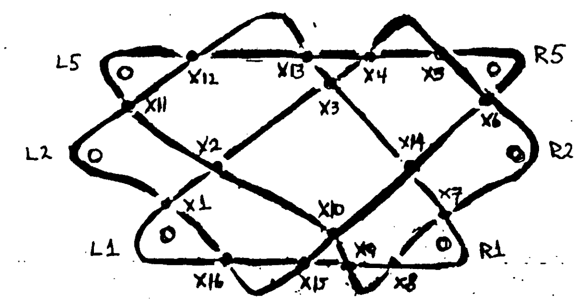
<u>Fig. 18.V</u>: Brochos, <u>O</u>.A: $\overrightarrow{1}$ (<u>2f</u>)# $\overleftarrow{2}$ (<u>ℓ1f</u>)#□1| $\overrightarrow{1}$↓(ℓ2∞): $1_{\rightarrow}$(<u>5f</u>)#

⊨ L1: x1(Ø): x2(U): x3(U): x4(U): x5(U): L5: x5(Ø): x6(Ø): x7(Ø): x8(Ø): x9(Ø): ℓR2: x10(U): x11(U): x12(Ø): x13(U): x14(U): x15(U): x4(Ø) uL2: x3(Ø): x15(Ø): x16(U): x7(U): x18(Ø): x19(Ø): uR2: x20(Ø): x18(U): x8(U): x13(Ø): x21(Ø): x22(U): x1(U): ℓL2: x2(Ø): x14(Ø): x16(Ø): x6(U): x23(Ø): R5: x23(U): x20(U): x19(U): x9(U): x10(Ø): R1: x11(Ø): x12(U): x21(U): x22(Ø) ■



<u>Fig. 18.VI</u>: Brochos, <u>O</u>.A: $\overrightarrow{1}$ (<u>2f</u>)# $\overleftarrow{2}$ (<u>ℓ1f</u>)#□1| $\overrightarrow{1}$↓(ℓ2∞): $1_{\rightarrow}$(<u>5f</u>)#□5|

⊨ L1: x1(Ø): x2(U): x3(U): x4(U): x5(Ø): x6(Ø): ℓR2: x7(U): x8(U): x9(Ø): x10(U): X2(Ø): x11(U): x12(U): uL2: x12(Ø): x13(U): x14(Ø): x4(Ø): x5(U): x15(Ø): uR2: x15(U): x6(U): x16(Ø): x10(Ø): x17(Ø): x18(U); x1(U): ℓL2: x11(Ø): x13(Ø): x14(U): x3(Ø): x16(U): x7(Ø): R1: x8(Ø): x9(U): x17(U): x18(Ø) ■



<u>Fig. 18. VII</u>: Brochos, <u>O</u>.A: $\overrightarrow{1}$ (<u>2f</u>)# $\overleftarrow{2}$ (<u>ℓ1f</u>)# □1| $\overrightarrow{1}$↓(ℓ2∞): $1_{\rightarrow}$(<u>5f</u>)#□5|
> $\overrightarrow{u2\infty}$ ⟶ 5

⊨ L1: x1(Ø): x2(U): x3(U): x4(U): x5(Ø): x6(Ø): R2: x7(U): x8(U): x9(Ø): x10(U): x2(Ø): x11(U): L5: x12(U): x13(Ø): x4(Ø): x5(U): R5: x6(U): X14(Ø): x10(Ø): x15(Ø): x16(U): x1(U): L2: x11(Ø): x12(Ø): x13(U): x3(Ø): x14(U): x7(Ø): R1 x8(Ø): x9(U): x15(U): x16(Ø) ■

The final manipulation for the figure, □2#| (gently), produces the schema of Fig. 17, and the associated linear sequence there displayed.

Let us examine the effect that the individual manipulations of the construction have on the constituent loops: There are three (pairs -- R & L) of these to be considered, the 1∞, the 2∞, and the 5∞ (cf. Fig. 18.I: O.A). First, on O.A,

Brochos, Step 1 } $\vec{1}$(2f)# $\overleftarrow{2}$(ℓ1f)# □1| ≡ $\overrightarrow{1\infty}$↓(2∞): 1∞ (2f): $\overleftarrow{1\infty}$ → 2#|

That is, the sequence of moves on the left, above -- occurring as the "first step" of the construction of Brochos -- has the loop-specific effect of passing the 1∞ away, down through the 2∞, away under its far string (2f), then back up towards you and over that string (2f) directly back to 2, where -- since 2 already has a loop -- it becomes the u2∞. Secondly, on this figure (and on O.A),

Brochos, Step 2 } $\vec{1}$↓(ℓ2∞): 1 (5f)# □5| ≡ >5∞↑(ℓ2∞): $\overleftarrow{5\infty}$ → 1#|

That is, the left-hand sequence effects the loop-specific translation of twist the 5∞ 180° away from you, under all intervening strings and up through the ℓ2∞; continue passing the 5∞ towards you over ℓ2n and directly onto 1. Finally, the penultimate move is

Brochos, Step 3 } >$\overrightarrow{u2\infty}$ → 5#|,

which is already loop-specific. We remark that Steps 1 and 3 combine to give a motion of the 1∞, namely

>$\overrightarrow{1\infty}$↓(2∞): 1∞ → 5#|

which is exactly analogous to the motion of the 5∞,

>5∞ ↑(2∞): $\overleftarrow{5\infty}$ → 1#|

Thus, if we didn't have to worry about how the functors (here, fingers) were to effect such a manipulation, we could directly view the "heart" of (this construction of) Brochos as the beautiful, simultaneous, sequence (2)

Brochos, "Heart" } O.A: { > $\overrightarrow{1\infty}$↓(2∞): 1∞ → 5 ; > 5∞ ↑(2∞): $\overleftarrow{5\infty}$ → 1 } : □2|

In actuality, of course, this simultaneous sequence is impossible to realize; that's why, in the construction, the 1∞ has to "get out of the way, and rest (on 2) for a moment" while the 5∞ completes its movement. The 1∞ then continues (with a twist) to its appointed destination (on 5). But we view the concept of the "Heart"-sequence as a gedanken-experiment of fundamental importance in the deeper understanding of the string-figures of the world; one which would, perhaps, lend itself well to a computer-graphics realization.

Now let us make precise the sense in which the two manipulation sequences

(1) >$\overrightarrow{1\infty}$↓(2∞): 1∞ → 5

and

(2) >5∞↑(2∞): $\overleftarrow{5\infty}$ → 1

are "analogous" in the context of O.A. First note that an imaginary line ℓ connecting L2d to R2d is a line of reflection(3) for the 2-dimensional schema of the string-position O.A:

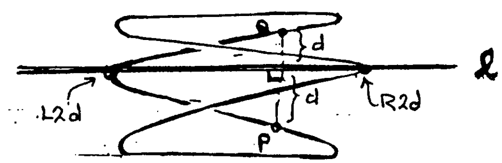


Fig. 19: A reflection-line symmetry for O. A.

What this means is that, essentially,* from each point P on the figure below ℓ, if a perpendicular be dropped to ℓ and extended an equal distance beyond, the terminus of this perpendicular will again be a point, Q, of the figure; and conversely. Thus viewed, the 5∞ is the ℓ-reflection of the 1∞, and the motion >5∞ is the ℓ-reflection of the motion >1∞, et cetera. The nature of the analogy between the manipulation sequences (1) and (2), above, is now clear:

(2). >5∞↑(2∞): $\overleftarrow{5\infty}$ → 1

is the ℓ-reflection of

(1). >$\overrightarrow{1\infty}$↓(2∞): 1∞ → 5,

on O.A.

The concept of "symmetry" is a useful one for string-figures in general.

----o----

* i.e. with ideal hand-positions.

For example, perform O.A on the hands and imagine yourself viewing this string-position from the far (little-finger) side of the hands. From this perspective, the "Heart"-sequence for Brochos appears to be

$$\underline{O}.A: \left\{ \begin{array}{l} < \underrightarrow{1\infty}\uparrow(2\infty) : \overrightarrow{1\infty} \rightarrow 5 \\ < \overleftarrow{5}\infty\downarrow(2\infty) : \underleftarrow{5\infty} \rightarrow 1 \end{array} \right\} : \square 2|$$

Thus, if we readopt our more usual "near-side" viewpoint and perform the above manipulation-sequence, we strongly conjecture that a string-position "very like" Brochos will result. In fact, we obtain

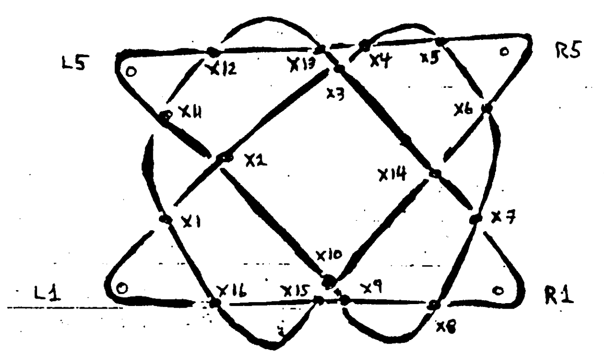


Fig. 20: A pseudo-Brochos

with associated linear schema

⊨ L1: x1(U): x2(∅): x3(U): x4(∅): x5(U): x6(U): x7(∅): x8(∅): x9(U): x10(U): x2(U): x11(∅) L5: x12(∅): x13(U): x4(U): x5(∅): R5: x6(∅): x14(U): x10(∅): x15(U): x16(∅): x1(∅): x11(U): x12(U): x13(∅): x3(∅): x14(∅): x7(U): R1: x8(U): x9(∅): x15(∅): x16(U) ■

And, of course, this figure "is" Brochos -- if we elide the fine points of internal crossing-types. Considering these "fine points", however, we see that the figure isn't the Brochos, nor can it be obtained from that figure by a rigid motion; the parities of the crossings just can't be made to correspond exactly. We remark that if, in the "Heart"-sequence for pseudo-Brochos (Fig. 20), we replace O.A by

$$\underline{O}.A' \equiv \underline{O}.1: \overrightarrow{L2}(\underline{Rp})\# \ \overleftarrow{R2}\downarrow(L2\infty) : \underleftarrow{R2}(\underline{Lp})\#|$$

(i.e. just like O.A, except the Rp-string is taken up first) the resulting

figure is related to the Brochos by a rigid motion [rotate the figure 180° in its plane]. Since the example accompanying Fig. 6 has shown that, in general, string manipulations are anything but indifferent to the matter of internal crossing-types, we shall have to attend carefully to such details, in the future. For the present, however, we content ourselves with an alternate construction of pseudo-Brochos, popularly known as "The Eclipse" (see, e.g. W.W. Rouse Ball, Figure No. 11); the mouth (M) is required for the construction:

Eclipse: $\underline{O}.A: \overrightarrow{M}(\underline{5f})\# \square 5| \ \overleftarrow{5}(\underline{1n})\# \square 1| \ \underleftarrow{2}\uparrow(M\infty)$ (near M): $\square M\#|$ $N2|$ (gently).

Before continuing with our look at the Brochos, we pause to introduce an alternate extension of this, and a great many other, important, beautiful string-figures. It is variously known as "Pindiki", "Caroline Extension", etc. and, although it requires not a little practice to gain dexterity with it, this is well worth the time and effort involved, as it affords a dramatic unfolding of the central design for final display. The description below is from Kathleen Haddon (Artists in String, p. 156):

'Pindiki' is a native name for the final extension of many figures. It consists of passing 2 proximal to 1f and bringing them up through the 1∞, so that this string makes a half-turn around their tips, at the same time keeping 1 closely pressed against 2 to hold 1f firm. Then extend the figure by turning B palms away from you.

We shall adopt the notation

$\mathbb{P}$ --- Pindiki.

To display the Brochos via the Pindiki, we proceed as follows:

Make the Brochos: $> \overrightarrow{1\infty} \rightarrow 2\# \ \underrightarrow{1}(\underline{5n \ \& \ 2n})\#\square 2 \ \mathbb{P}$

Note: The above symbol sequence gives the exact construction of the Nauru figure "Sun" (Ekwan Ib.; Honor Maude, The String Figures of Nauru Island, No. 52):

$\underline{O}.A.: \overrightarrow{1}(\underline{2f})\# \ \overleftarrow{2}(\underline{\ell 1f})\#\square 1| \ \overrightarrow{1}\downarrow(\ell 2\infty) : \underrightarrow{1}(\underline{5f})\#\square 5| \ > \overrightarrow{u2\infty} \rightarrow 5 : \square 2\#|$ (gently):
$> \overrightarrow{1\infty} \rightarrow 2\# \ \underrightarrow{1}(\underline{5n \ \& \ 2n})\#\square 2 \ \mathbb{P}$

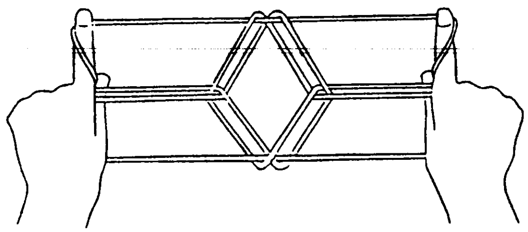


Fig. 21: Ekwan Ib.

[The "plate" is from C.F. Jayne, String Figures, Fig. 864.] Here we at once gain new insight into the Brochos and, at the same time, illustrate the string/frame interaction of the classic Pindiki position. Strings are tighter, neater; the central figure crisper, more geometrical, a "double diamond" -- and the 16 original simple-crossings have more apparently grouped themselves into four (complex) crossings, consisting of two obviously related pairs [left-right and top-bottom]. We shall pursue these matters later; let us now return to a more construction-oriented approach to the Brochos.

Let n>1 be the generic natural number; let us agree that, if a sequence from the string-figure Calculus is enclosed in square brackets -- raised to the n^th power -- then the sequence of moves within the brackets is to be performed n times, consecutively. For example, suppose we insert such brackets into the Brochos construction as follows:

O.A: [$\vec{1}$ (2f) # $\overleftarrow{2}$ (ℓ1f) # □1| $\vec{1}$↓(ℓ2ꝏ): 1→(5f) # □5| > $\overrightarrow{u2ꝏ}$ ⟶ 5]^2: □2#| (gently).

This will be taken as a shorthand for the lengthy symbol-string

O.A: $\vec{1}$ (2f) # $\overleftarrow{2}$ (ℓ1f) # □1| $\vec{1}$↓(ℓ2ꝏ): 1→(5f) # □5| > $\overrightarrow{u2ꝏ}$ ⟶ 5: $\vec{1}$ (2f) # $\overleftarrow{2}$ (ℓ1f) #
□1| $\vec{1}$↓(ℓ2ꝏ): 1→(5f) # □5| > $\overrightarrow{u2ꝏ}$ ⟶ 5: □2#| (gently)

et cetera. Similarly, we shall obtain -- for the corresponding "Heart"-sequence --

O.A: [> $\overrightarrow{1ꝏ}$↓ (2ꝏ): 1ꝏ→ ⟶ 5 / > 5ꝏ→↑ (2ꝏ): $\overleftarrow{5ꝏ}$ ⟶ 1]^2 : □2|

Now, if this sequence of manipulations is followed with string on hands, the resulting figure is

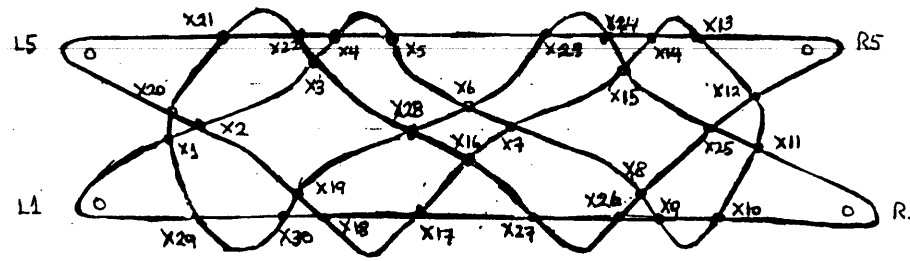


Fig. 22: Double-Brochoi Construction

The crossing-parities are elided in the above schema, but are explicitly given by the associated linear sequence:

|⟹ L1: x1(∅): x2(U): x3(∅): x4(U): x5(∅): x6(∅): x7(∅): x8(∅): x9(∅): x10(U): x11(U): x12(∅): x13(∅): x14(U): x15(U): x7(U): x16(∅): x17(U): x18(∅): x19(U): x2(∅): x20(U): L5: x21(U): x22(∅): x4(∅): x5(U): x23(U): x24(∅): x14(∅): x13(U): R5: x12(U): x25(∅): x8(U): x26(∅): x27(U): x16(U): x28(U): x3(U): x22(U); x21(∅): x20(∅): x1(U): x29(U): x30(∅): x19(∅): x28(∅): x6(U): x23(∅): x24(U): x15(∅): x25(U): x11(∅): R1: x10(∅): x9(U): x26(U): x27(∅): x17(∅): x18(U): x30(U): x29(∅) ■

And, when this double-Brochoi figure is extended via the Pindiki technique, i.e.

Make the Double-Brochoi: > $\overrightarrow{1ꝏ}$ ⟶ 2# 1→(5n & 2n) # □2 IP

we find, after some minimal rearrangement of the central design,

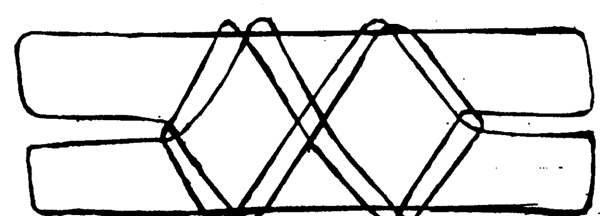


Fig. 23: Two Double-diamonds

Thus, Fig. 21 represents one double-diamond, Fig. 23 represents two double-diamonds and, in general, the manipulation-sequence

O.A: [$\overrightarrow{1}$ (2f)# $\overleftarrow{2}$ (ℓ1f)#□1| $\overrightarrow{1}$↓(ℓ2ꝏ): 1↘(5f)#□5| > $\overrightarrow{u2ꝏ}$ → 5]^n: □2#| (gently):
> $\overrightarrow{1ꝏ}$ → 2# 1↘(5n & 2n)#□2 P

will produce any number, n, of double-diamonds, length of string permitting.

As in the case n=1, the symmetric "Heart"-sequence for n=2,

O.A: $\left[\begin{matrix} \text{<1ꝏ↑(2ꝏ): } \overrightarrow{1ꝏ} \to 5 \\ \text{<}\overleftarrow{5ꝏ}\text{ ↓(2ꝏ): 5ꝏ} \to 1 \end{matrix}\right]^2$: □2|

produces a pseudo-double-Brochoi; the difference in this case (and for n even, in general) is that this figure is related to the double-Brochoi by a rigid motion [rotate the figure 180° in its plane]. The pseudo-double-Brochoi is known to the Maori as "Koura" [Andersen, Maori String Figures, No. 13], where it is constructed thus:

Koura: O.A: <<5: $\overrightarrow{1}$ (5n & 5f)# $\overleftarrow{2}$ (ℓ1f)#□1| N2| (gently)

As an interesting final remark in this direction, if we construct the "Heart"-sequence for Koura directly, we find

O.A: <<5ꝏ:: $\overrightarrow{1ꝏ}$ (2ꝏ): 1ꝏ↘(5ꝏ): $\overleftarrow{1ꝏ}$↓(2ꝏ): 1ꝏ → 1::□2| (gently),

an interesting alternative to that announced above. [Note: The use of the double-colon, "::", above, indicates that the manipulational symbol-string so encapsulated is a single, complex movement (here, of the 1ꝏ), whose description requires a multiple, extended phrase.]

It remains to give a detailed "crossing-analysis" of the construction of the string-figure the Brochos (Figs. 18, I-VII, and 17). And, in this context, the information gained from our previous observations must be considered somewhat "superficial"; the insights gained therefrom will, nevertheless, be very useful to the present, more fundamental, investigations.

We begin by investigating the passage from Fig. 18.I [O.A] to Fig. 18.II [O.A: $\overrightarrow{1}$ (2f)#]. Here 1 has picked up 2f — to become an u1ꝏ -- as a preparational step for the passage of $\overrightarrow{1ꝏ}$↓(2ꝏ): 1ꝏ↘(2f): $\overleftarrow{1ꝏ}$ → 2. Clearly,

Fig. 18.I		Fig. 18.II
x1	⟶	x1
x2	⟶	x4

and the crossings x2, x3, x5, x6 of Fig. 18.II created by $\overrightarrow{1}$ (2f)# are temporary; in the sense that they will "disappear" when 1 ultimately relinquishes its loops in Fig. 18.IV. We shall refer to such crossings as "constructional" -- C-crossings -- and think in terms of the analogy of the scaffolding of a building under construction. Crossings which appear in an intermediate string-position that maintain themselves to the final string-position in the sequence, will be referred to as "structural" -- S-crossings. It is too early to decide whether crossings x1 and x2 (Fig. 18.I) are C-crossings or S-crossings for the Brochos.

Consider, next, the passage from Fig. 18.II [O.A: $\overrightarrow{1}$ (2f)#] to Fig. 18.III [O.A: $\overrightarrow{1}$ (2f)# $\overleftarrow{2}$ (ℓ1f)#]. Here the original 1f-string has completed the journey, relative to the 2ꝏ, that the entire 1ꝏ is destined to take (upon completion of the move, i.e. □1|). The fate of the crossings of Fig. 18.II under the indicated string-manipulation is easy to determine:

Fig. 18.II		Fig. 18.III	Fig. 18.II		Fig. 18.III
x1	⟶	x4	x4	⟶	x9
x2	⟶	x3	x5	⟶	x11
x3	⟶	x7	x6	⟶	x12

and the crossings x1, x2, x5, x6, x8, x10 of Fig. 18.III are created by the move $\overleftarrow{2}$ (ℓ1f)# applied to Fig. 18.II.

The passage from Fig. 18.III to Fig. 18. IV, under □1|, is an interesting exercise in "schema-chasing", whose crossing-specific results are

	Fig. 18.III		Fig. 18.IV		Fig. 18.III		Fig. 18.IV
	x1	⟶	x1	C.	x7	⟶	ϕ
	x2	⟶	x8		x8	⟶	x5
C.	x3	⟶	ϕ		x9	⟶	x3
	x4	⟶	x6		x10	⟶	x4
	x5	⟶	x2	C.	x11	⟶	ϕ
	x6	⟶	x7	C.	x12	⟶	ϕ

Here, for the generic natural number, n, we have signified that the crossing xn disappears under the given string-manipulation (here, □1|) by the notation xn ⟶ ϕ; thus xn was a C-crossing in the previous figure. We remark that {x3, x7} and {x11, x12} are pairwise extension-cancellable by (several applications of) Lemma 2.B. Thus, the observed temporary nature of the crossings x2, x3, x5, x6 of Fig. 18.II has been validated:

Fig. 18.II		Fig. 18.III		Fig. 18.IV	
x2	⟶	x3	⟶	∅	Lemma 2.B
x3	⟶	x7	⟶	∅	
x5	⟶	x11	⟶	∅	Lemma 2.B
x6	⟶	x12	⟶	∅	

The passage from Fig. 18.IV to Fig. 18.V, under $\vec{1}\downarrow(\ell 2\infty)$: $1_{\rightarrow}$(5f)# effects the alias transformation

Fig. 18.IV		Fig. 18.V	Fig. 18.IV		Fig. 18.V
x1	⟶	x16	x5	⟶	x8
x2	⟶	x14	x6	⟶	x13
x3	⟶	x6	x7	⟶	x15
x4	⟶	x7	x8	⟶	x18

while creating the crossings x1, x2, x3, x4, x5, x9, x10, x11, x12, x19, x20, x21, x22, x23 [Note: There is no crossing numbered x17 for reasons which are none of your business.]. Now, clearly, crossings x11, x12, x13, x21, x22 are S-crossings -- comprising, as they do, the double-diamond's involvement with the ln-string in the final figure (Fig. 17) -- while, equally clearly, x5 and x23 (of Fig. 18.V) are C-crossings, destined for cancellation under the inevitable □5|. Equally apparent is the fact that, upon the move □5, the crossings x2, x6, x9 -- being three separate crossings of 5n with 5f -- will collapse to a single crossing of the ℓL2f and the ℓR2f strings. The exact alias crossing-transformation is given below:

	Fig. 18.V		Fig. 18.VI		Fig. 18.V		Fig. 18.VI
	x1	⟶	x1		x12	⟶	x9
	x2	⟶	x3		x13	⟶	x10
	x3	⟶	x4		x14	⟶	x11
	x4	⟶	x2		x15	⟶	x12
C.	x5	⟶	∅		x16	⟶	x13
C.	x6	⟶	∅		x18	⟶	x15
	x7	⟶	x5		x19	⟶	x14
	x8	⟶	x6		x20	⟶	x16
C.	x9	⟶	∅		x21	⟶	x17
	x10	⟶	x7		x22	⟶	x18
	x11	⟶	x8	C.	x23	⟶	∅

Here x5 and x23 are each extension-cancellable by Lemma 2.A, while {x6,x9}

are so cancellable by (several applications) of Lemma 2.B. In Fig. 18. VI, all crossings except x12, x15 are S-crossings.

The penultimate step of the figure, from Fig. 18.VI to Fig. 18. VII via > $\overrightarrow{u2\infty}$ ⟶ 5 effects the transform

	Fig. 18.VI		Fig. 18.VII		Fig. 18.VI		Fig. 18.VII
	x1	⟶	x1		x10	⟶	x10
	x2	⟶	x2		x11	⟶	x11
	x3	⟶	x3	C.	x12	⟶	∅
	x4	⟶	x4		x13	⟶	x12
	x5	⟶	x5		x14	⟶	x13
	x6	⟶	x6	C.	x15	⟶	∅
	x7	⟶	x7		x16	⟶	x14
	x8	⟶	x8		x17	⟶	x15
	x9	⟶	x9		x18	⟶	x16

and every crossing which remains in Fig. 18.VII is structural.

The final step, from Fig. 18.VII to Fig. 17 (the Brochos) via □2 is trivial -- delete L2 and R2 from the linear sequence associated to Fig. 18.VII to get that associated to Fig. 17 -- and the alias crossing-transformation is the identity.

The above "crossing-analysis" of the step-by-step evolution of the string-position the Brochos suffers from the "defect" of being strongly schema-dependent in its derivation of the alias crossing-transformations associated to each of its steps; when -- from the present perspective -- the fundamental constructs involved are the linear sequences (uniquely identifying the static string-positions) and the symbol Calculus (uniquely specifying transitions between string-positions). Of course it is realized that the schemata are extremely handy "visual adjuncts" to these theoretical constructs -- which, for example, are easily apprehendable by all, provide geometric insight, and make it relatively simple to learn "new" figures -- and so they will never disappear from this development of the subject. But our reliance and dependence on them should (4) (and will!) abate -- and finally disappear -- as knowledge and fluency with linear sequences and the Calculus increases. While we feel it is premature to launch into a full-blown discussion of such matters at the present time, a few elementary remarks now will motivate and add insight to the subsequent development, concept evolution, and notation introduced.

A big clue as to how this independence of schemata is to be achieved comes from a closer examination of the transition from Fig. 18.III to Fig. 18.IV via □1|, a "release" move. What must occur in this dynamic is that -- in the linear sequence associated to Fig. 18.III — we

(1) Delete 1 -- i.e. erase ℓL1, uL1, ℓR1, uR1 from the linear sequence associated to Fig. 18.III;

(2) Delete all extension-cancellable crossings;

(3) Rearrange the resulting sequence according to the "canonical" conventions, if necessary.

The result must be the linear sequence associated to Fig. 18.IV -- or a sequence "equivalent" to it, in a sense yet to be defined. Similarly, for the passage -- say -- from Fig. 18.I (O.A) to Fig. 18.II, a "pickup" move, we gain insight by considering the "inverse" transformation -- from Fig. 18.II to Fig. 18.I via □ul∞ -- a "release" move again. In the linear sequence associable to Fig. 18.II; i.e.:

(1) ⊨> ℓL1: x1(∅): x2(U): x3(U): R2: x3(∅): ~~uR1~~: x2(∅): x4(∅): L5: R5: x4(U): x5(∅): ~~uL1~~: x6(∅): L2: x6(U): x5(U): x1(U): ℓR1 ■

we first delete the ul∞ [and drop the premodifiers from the expressions for the remaining l∞ in the sequence] to obtain

(2) ⊨> L1: x1(∅): x2(U): x3(U): R2: x3(∅): x2(∅): x4(∅): L5: R5: x4(U): x5(∅): x6(∅): L2: x6(U): x5(U): x1(U): R1 ■

Next, we use Lemma 2 to delete all resulting extension-cancellable crossing [Here {x2,x3} and {x5, x6}] to obtain

(3) ⊨> L1: x1(∅): R2: x4(∅): L5: R5: x4(U): L2: x1(U): R1 ■

which, apart from the "name" for the crossing x4, is Fig. 18.I: O.A. The "trick" here is to proceed back up this chain of linear sequences — i.e. in the other direction, from Fig. 18.I (O.A) to Fig. 18.II -- via insertion of the nodes uL1 and uR1 at the appropriate places in some linear sequence "equivalent" to the one for Fig. 18.I. Note: Here, we may replace "equivalent" by "cancellation-equivalent" because the situation is so simple; a more general type of equivalence between linear sequences is required, in future. The sole clue

as to which of the many sequences ultimately equivalent to that for Fig. 18.I is to be chosen, and to where in that sequence the nodes uL1 and uR1 are to be inserted, is the Calculus transform-manipulation uniquely specifying passage from Fig. 18.I to Fig. 18.II; i.e. $\vec{1}$(2f)#. This uniquely specifies the partition-element 2f — itself uniquely specified by the linear-sequence -- and so establishes "pointers" in the sequence associated to Fig. 18.I:

⊨> L1: x1(∅): R2↓: x2(∅): L5: R5: x2(U):↓ L2: x1(U): R1 ■

Further, since 1 has a single loop, and 1 is nearer you than is 2 (i.e. 1<2) -- and, in fact, 2-1=1 is also relevant — a single Lemma 2.B-type extension-cancellable crossing is to be inserted just before -- and just after -- each of R2 and L2 in the above sequence. Irrespective of the "names" for these (four) inserted crossings, we obtain the linear sequence (2) (page 44). The new nodes, uR1 and uL1 are to be inserted between the nodes of these cancellable crossings in the obvious way; this gives the desired linear sequence of Fig. 18.II ((1), page 44). We shall content ourselves, for the nonce, with these two examples of linear-sequence modification by the manipulational Calculus; we shall return to it in developmental stages as more and more appropriate groundwork for it has been laid.

As a final topic to be introduced in the context of this figure, the Brochos, we return to its Pindiki extension Fig. 21, and the comments following that figure.

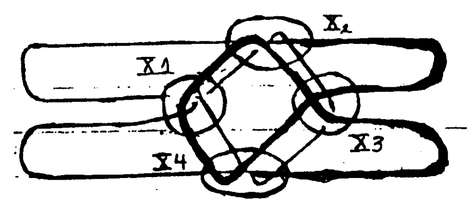


Fig. 24: Two-colored Brochos (+ Pindiki)

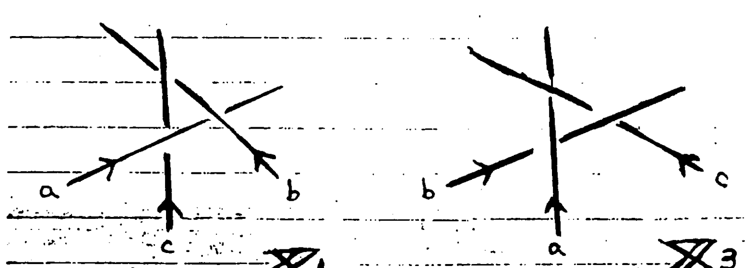
Here there is a (black) central loop — from the left -- and a (red) central loop — from the right; and these have four points of involvement with each other, and the near and far straight-strings of the figure. In terms of the ongoing "Ant crawling along the string" analogy, he encounters four "freeway interchanges" during his round trip -- designated here as X1, X2, X3, X4, respectively, -- through each of which he passes a total of three times. We shall refer to these as complex crossings, and note that each of these is composed of a number (3 or 5) of simple crossings. Further, the complex crossings X1 and X3 are obviously related to each other (by a symmetry), as are X2 and X4. In fact -- referring to Fig. 17 again --

Fig. 24		Fig. 17
X1	----	x1, x2, x11
X2	----	x3, x4, x5, x12, x13
X3	----	x6, x7, x14
X4	----	x8, x9, x10, x15, x16

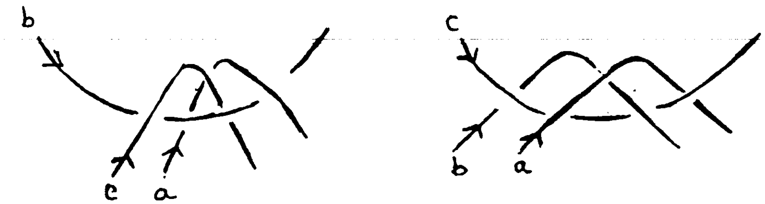
and

1. X1 and X3 are reflections in the plane perpendicular to the plane of the figure, passing through x3 and x10.
2. X2 and X4 lack one crossing parity (e.g. x10) of being reflections in the line through x2 and x14 (an asymmetry inherited from O.A).

The "complex-crossing" viewpoint has its good news/bad news aspect: On the one hand, the total number of crossings is reduced from 16 (Fig. 17) to 4 (Fig. 24); on the other, the notion of what now constitutes a "crossing" has become much more complicated. But, just as the formidable seven-level freeway interchange in downtown Los Angeles is functionally masterable, so are the complex crossing types. What is needed is a Dictionary of Complex Crossings: By way of example, four of the entries in such a Dictionary would surely look like



X1 **X**3



X2 **X**4

Fig. 25: Entries in **X** - Dictionary (examples)

Here we use the bold-face **X** for a complex crossing-type; the four examples above are exactly those that appear in the Brochos. Note that each entry of the **X**-Dictionary consists of several labeled strings -- each with an (arbitrarily) assigned orientation -- in a schema which delineates their precise interactions with one another (and, ultimately, themselves). The positioning of these schema (as to size or rotation in their own plane) is unimportant -- as these are "local" constructs; what is important is that they not be "turned over", but are to be presented exactly as they appear when viewed from (directly) above. In terms of these complex crossings, the linear sequence associated to the Brochos would be

|⟹ L1: x1(1,a+): x2(2,a+):x3(3,a+): x4(4,a+): x1(1,b+): L5: x2(2,b+): R5: x3(3,b+): x4(4,b+): x1(1,c+): x2(2,c+): x3(3,c+): R1: x4(4,c+) ■

exhibiting exactly four distinct (complex) crossings. Here, each crossing is numbered, as before, and modified by an **X**-Dictionary number, a letter (designating which string in the corresponding **X**-Dictionary schema is relevant) and a "+" or "-" (designating whether travel along the given string is in the directions of its orientation, or opposite thereto). Note that the **X**-Dictionary examples, above, were chosen with the Brochos in mind; hence, in this case the crossing-numbers equal the **X**-Dictionary numbers, and the direction of travel, in every case, is that of the relevant string's given orientation. Of course this will not happen, in general, for future string-figures with these referents; their present occurrence merely reflects the ad hoc nature of the current example.

Thus, to read and interpret the above linear sequence, we proceed from L1 to the first crossing, x1, which is complex (since its modifier is not a simple Ø or U). The modifier, "1,a+", of crossing x1 points to the **X**-Dictionary entry

$\mathbb{X}_1$ -- a three-line schema -- and to the line labeled "a" therein. The "+" on the string-label "a" in the modifier indicates that we are to proceed through the interchange $_1$, along the line labeled a, in the indicated direction. Et cetera. We remark that, upon regrouping the above linear sequence, viz:

$\models\!\Rightarrow$ L1:

Black Central Loop $\}\rightarrow$ x1($\underline{\underline{1}}$,a+): x2($\underline{\underline{2}}$,a+): x3($\underline{\underline{3}}$,a+): x4($\underline{\underline{4}}$,a+): x1($\underline{\underline{1}}$,b+):

L5: x2($\underline{\underline{2}}$,b+): R5:

Red Central Loop $\}\rightarrow$ x3($\underline{\underline{3}}$,b+): x4,($\underline{\underline{4}}$,b+): x1($\underline{\underline{1}}$,c+): x2($\underline{\underline{2}}$,c+): x3($\underline{\underline{3}}$,c+):

R1: x4($\underline{\underline{4}}$,c+) ∎

the paired central loop-structure displayed in the two-colored Brochos (Fig. 24) becomes apparent directly from the linear sequence. This is to be compared with the (simple-crossing) linear sequence of Fig. 17, where the existence of these distinguished substructures is far from obvious. Finally, it is to be observed that, of course, Lemma 1 will not be a valid assertion for complex crossings, in general. The extension of this lemma to complex crossings, generally, requires the introduction of the parameters of such a crossing; specifically, let n be the generic natural number, and let $\mathbb{X}_n$ be the generic $\mathbb{X}$-Dictionary entry. We define

$\alpha(\mathbb{X}_n) \equiv$ the number of distinct strings (arcs) which comprise the schema for $\mathbb{X}_n$,

$\chi(\mathbb{X}_n) \equiv$ the number of distinct simple crossings among the $\alpha(\mathbb{X}_n)$ strings in the schema for $\mathbb{X}_n$.

Thus, for example, in the case of the Brochos, we have

$$\alpha(\mathbb{X}_1) = \alpha(\mathbb{X}_2) = \alpha(\mathbb{X}_3) = \alpha(\mathbb{X}_4) = 3$$

while

$$\chi(\mathbb{X}_1) = \chi(\mathbb{X}_3) = 3, \text{ and } \chi(\mathbb{X}_2) = \chi(\mathbb{X}_4) = 5$$

respectively. When xn is a simple crossing, we define $\alpha(xn) = 2$ and $\chi(xn) = 1$, for completeness. The complex-crossing generalization of Lemma 1 is framed in terms of the first of the above parameters:

Lemma 3: Let n be the generic natural number, and let xn be a crossing in a given string-position. Then the node xn appears exactly $\alpha(xn)$ times in the associated linear sequence. Further, these $\alpha(xn)$ appearances all have distinct second entries.

Proof: A complete circuit of the closed loop of string passes through each

crossing exactly once on each of its constituent strings. ∎

Assertions like (1) and (2) (page 46) contrast and compare geometric information about $\mathbb{X}$-Dictionary entries. The various crossing-types encountered in the Brochos and future string-figures will generate the $\mathbb{X}$-Dictionary, the interrelation of whose entries will generate many useful geometric theorems. These may reasonably be expected to enrich and deepen our insight into the subject as a whole.

We conclude our discussion of the complex of figures surrounding the Brochos with another constructional variant (Honor Maude, String Figures From New Caledonia and the Loyalty Islands; No. 8, The Sun); the wrist (W) is required in the construction:

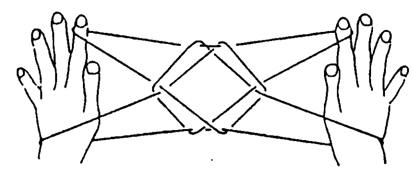


Fig. 26: The Sun

O.A: $\overleftarrow{2345}$(1n)#□1| $\overrightarrow{2345}$∞ ⟶ W:: $\overleftarrow{1}$(Wn): $\underset{\rightarrow}{1}$(Wf): $\overleftarrow{1}$($\overline{5f}$)#□5: $\overrightarrow{2}$∞ ⟶ 5|

$\overrightarrow{1}$(5n)# $\overleftarrow{2}$(ℓ1f)#□1 & 5| (gently).

Note: The extended 1 move (following the double-colon, above) is the most complex manipulation yet met with. Here the thumbs rotate towards you over Wn, and continue their rotation away, under this string, all the way to the far side of the figure and under Wf; then back towards you over this string (only!) and up into the 5∞. Now hook 1 away over 5f from above, and retrace 1's rotation back to #.

---- End Brochos Discussion ----

THE BROCHOS NOTES

(1) (p. 30) cf. C.L.Day, Quipus and Witches' Knots, p. 124, where this figure appears as No. 13, the "4-loop Plinthios Brokhos" -- or "4-loop bandage noose" [Bandage Noose = Sling]. We shall refer to it as the "Brochos" (plural, Brochoi") in the following. The above author gives

Brochos: O.A: $>\overrightarrow{1\infty} \rightarrow$ 4: $<\overleftarrow{5\infty} \rightarrow$ 2: N2| (gently).

for Heraklas' probable construction [cf. p. 34, Note (2), below]. We shall pursue another approach to the construction, pursuant to our developmental purposes, illuminating the above "probable" construction as the "pseudo-Brochos" (Fig. 20).

(2) (p. 34) cf. "The Well" (K. Haddon and H. Treleaven: "Some Nigerian String Figures", No.45) for another genuinely instructive example of a "source" string-figure whose construction is, essentially, a "Heart"-sequence leading to the Brochos-complex of figures:

Well: O.A: $>\overrightarrow{1\infty} \rightarrow$ 4:: $<\overleftarrow{5\infty}\downarrow(2\infty)$: $\underleftarrow{5\infty} \rightarrow$ 1::□2| (gently)

(3) (p. 35) But note that ℓ is not a line of reflection for the (3-dimensional) string-position, O.A, itself; as, for example, the image of the crossing x1 is not x2, but a crossing of parity opposite that of x2. Compare

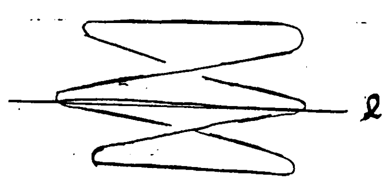


Fig. 19.A: O.A*, with line of reflection, ℓ

O.A* ≡ O.1:□5| $\overleftarrow{R2}$ (L1f)# $\overrightarrow{L2}\downarrow(R2\infty)$: $\underrightarrow{L2}$ (R1f)# $\overleftarrow{R5}$ (L2f)# $\overrightarrow{L5}\downarrow(R5\infty)$:
$\underrightarrow{L5}$ (R2f)#|

To appreciate the difference between O.A and O.A*, compare the string-positions which result from the two manipulational sequences

O.A:□2| and O.A*:□2|

(4) ~~(p. 43) To a Mathematician, a "proof by picture" is not a proof at all.~~ For while it is true that "a picture is worth a thousand words", often many of those words will be outright lies -- or, at best, misrepresentations of the truth.

BIBLIOGRAPHY: BROCHOS

Andersen, J.C.: MAORI STRING FIGURES.
Wahine #3

Ball, W.W. Rouse: FUN WITH STRING FIGURES.
Veiled Sun or Eclipse #11

Beaglehole, P.: STRING FIGURES FROM PUKAPUKA.
Paiokiole (Lozenge) Fig. 36

Centner, T.H.: L'ENFANT AFRICAIN ET SES JEUX.
Inamfumu wa Lutanda la Reine des Etoiles Fig. 195, #C

Compton, R.H.: STRING FIGURES FROM NEW CALEDONIA AND THE LOYALTY ISLANDS.
Butterfly #7
Nameless #13A

Cunnington, W.A.: STRING FIGURES AND TRICKS FROM CENTRAL AFRICA.
Mwezi (Moon) #15

Dayrell, E.: NOTES ON "NYAM TUNERRA", OR CAT'S CRADLE.
Mon mukai (Young Girl) Fig. 17

de Palavecino, M.D.M.: ALGUNOS JUEGOS INFANTILES CRIOLLOS E INDIGENAS DEL NORTE ARGENTINO.
No name Fig. 4

Devonshire, C.W.: RAROTONGAN STRING FIGURES.
Titia a Paraone P. 122

Edge-Partington, J. & C. Heape: AN ALBUM OF THE WEAPONS, TOOLS, ORNAMENTS, ARTICLES OF DRESS, ETC., OF THE NATIVES OF THE PACIFIC ISLANDS.
Series 1, Plate 341 #1

Emory, K. & H.C. Maude: STRING FIGURES OF THE TUAMOTUS.
Tifai Manania (Female Turtle) #1A

Eschlimann, P.H.: L'ENFANT CHEZ LES KUNI.
Ukukobokobo Fig. 1

Evans-Pritchard, E.E.: ZANDE STRING FIGURES.
Woman (Extended from "Man") p. 231

Firth, R. & H.C. Maude: TIKOPIA STRING FIGURES.
Foi nupu Pu Taumako (a former chief of Taumako) #2

Grimble, A.: GILBERTESE ASTRONOMY AND ASTRONOMICAL OBSERVANCES.
Sun p. 215

Gryski, C.: MANY STARS AND MORE STRING GAMES.
Giant Clam pp. 28-29

Haddon, K.: CAT'S CRADLES FROM MANY LANDS.
Mwezi (Moon) #20

Haddon, K. & H.A. Treleaven: SOME NIGERIAN STRING FIGURES.
A Well #15

Handy, W.C.: STRING FIGURES FROM THE MARQUESAS AND SOCIETY ISLANDS.
Honu Vahine p. 72

Hornell, J.: STRING FIGURES FROM FIJI AND WESTERN POLYNESIA.
Tavola Tree Fig. 33
Humu (Coal Hole) Fig. 61
Paa (Crab) Fig. 65

STRING FIGURES FROM GUJARAT AND KATHIAWAR.
Bagli-No-Kotho (Nest of the Crane) #14

Jayne, C.F.: STRING FIGURES.
Carrying Money (made from 10 Men) Fig. 359

Kaula, E.M.: THE FIRST BOOK OF AUSTRALIA.
No name (drawing) p. 25

Keeler, C.E.: CUNA INDIAN ART.
No name (photo) p. 145

Koch-Grünberg, T.: ZWEI JAHRE UNTER DEN INDIAREN REISEN IN NORD -WEST BRASILIEN.
Hausspinne (photo) Vol. II, p. 130,131.

La Fay, H.: EASTER ISLAND AND ITS MYSTERIOUS MONUMENTS.
Nelo-like figure (photo) p. 101

Lehmann-Nitsche, R.: LA ASTRONOMIA DE LOS TOBAS, I.
La Estrella (photo) Fig. 1,p. 27

Lutz, F.E.: STRING FIGURES FROM THE PATOMANA INDIANS OF BRITISH GUIANA.
Star Fig. 11

Maude, H.C.: THE STRING FIGURES OF NAURU ISLAND
Ekwan Series #52,53,54
Eongatubabo #68

SOLOMON ISLANDS STRING FIGURES.
Nepe (Moon) #1
Niu (Star) #2

STRING FIGURES FROM NEW CALEDONIA AND THE LOYALTY ISLANDS.
The Sun & The Butterfly #8

Maude, H.C. & H.E. Maude: STRING-FIGURES FROM THE GILBERT ISLANDS.
Taai I. (Sun) #9
Boua-n Nei Taai (Pillars of the Sun) #32

Martinez-Crovetto, R.: JUEGOS DE HILO DE LOS ABORIGENES DEL NORTE DE PATAGONIA.
Culodeavestruz #1

McCarthy, F.D.: THE STRING FIGURES OF YIRRKALLA.
Sun #22

Métraux, A.: ETUDES SUR LA CIVILISATION DES INDIENS CHIRIGUANO.
Etoille (Star) Fig. 89.1

QUELQUES JEUX DE FICELLE L'AMERIQUE DU SUD.
Coeur (Heart) Fig. 32

Noble, P.D.: STRING FIGURES OF PAPUA NEW GUINEA.
Sasa #16

ECHO (IMITATION) FROM SOUTHERN PAPUA.
Echo pp. 2-5

Noguchi, H.: ZUKEI-ASOBI NO SEKAI.
Wahine pp. 132-133

Nordenskiöld, E.: DE SYDAMERIKANSKA INDIANERNAS' KULTURHISTORIA.
No name Fig. 110

Öhrvall, H.: NÅGOT OM KNUTAR I ANTIKEN, SÄRSKILDT HOS OREIBASIOS.
Plinthios o Tetra Plate 2, #27

Read, R.: RECONSTRUCTIONS OF SOME SOUTH AMERICAN STRING FIGURES.
A Star #4

Reed, A.W.: GAMES THE MAORIS PLAYED.
(drawing) Cover, p.21

Roth, W.E. GAMES, SPORTS, AND AMUSEMENTS.
No name Plate 10, #1

STRING FIGURES, TRICKS, AND PUZZLES OF THE GUIANA INDIANS.
Star #649

Rydén, S.: SOUTH AMERICAN STRING FIGURES.
Star Fig. 3, #3

Schebesta, P.: DIE BAMBUTI-PYGMAEN VON ITURI, Vol. II.
Ibúpe ,(photo) Bildtafel 22, Fig.71
Otse Agbe (photo) Bildtafel 22, Fig.73

Senft, B. & G.: NINIKULA FADENSPIELE AUF DEN TROBRIAND-INSELN, PAPUA-NEUGUINEA.
Kapiva Fig. 4

Stachewski, F.: DIE BANJANGI.
De Kott p. 53, #1

Starr,F.: ETHNOGRAPHIC NOTES FROM THE CONGO FREE STATE.
Naintu (Woman) Fig. 24

Tessmann, G.: DIE KINDERSPIELE DER PANGWE.
Girl Fig. 38G

Torday, E. & T.A. Joyce: NOTES ETHNOGRAPHIQUES SUR LES PEUPLES COMMUNÉMENT APPELES BAKUBA.
No name (drawing) Fig. 68, B

Trapp,K.: BANTU-SPIELE.
Sun Fig. 2

III. OSAGE DIAMONDS

Perhaps the two most widely-known -- or, at least, recognized* string-figures among modern urbanites today are the Jacob's Ladder and Crows Feet. We shall investigate the former of these, calling it Osage Diamonds, after C.F. Jayne: String Figures, Fig. 50 [The "plate" is from R.M. Abraham: Easy-to-do Entertainments and Diversions, No. 147, where it is called "The Fence"]:

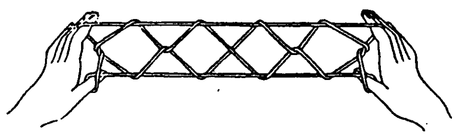


Fig. 27: Osage Diamonds (Ø ◇s)

The construction is very easy but, at one point, requires a word of explanation.

Osage Diamonds: O.A: □1| 1↳(5f)# 1⃗(2f)#□5| 5⃖(1f)#□1| 1⃗(5n)#| 1⃗(2n)# N1|:: 2⃖↓(1-△s): <2(#):□5| (palms away).

Notes about the construction: (1) the move 1⃗(2n) requires quite some dexterity with 1 -- as, of course, the 2n-string is to be picked up near the base of 2; i.e. before it is crossed by the s; 1f-5n string -- and, at least in the learning stages, is best accomplished with the aid of the opposite hand. (2) the move 2⃖↓(1-△s) is a shorthand for...

2⃖↓(1-△s) ≡ pass 2 towards you, over, and down into the small (inverted) triangles close to the base of 1.

Specifically, the △ involved is composed entirely of continuations of the 5n-string -- where it crosses 1n, s;1f-2n, and its own continuation -- near the base of 1. These matters will be further explicated in the subsequent constructional analysis of the figure.

* "Oh, i used to be able to make that one."

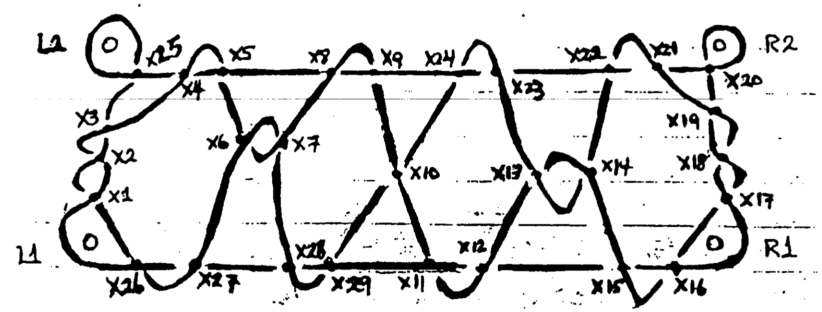


Fig. 28: Schema for Osage Diamonds

The associated linear sequence is

|⇒ L1: x1(Ø); x2(U): x3(Ø): x4(Ø): x5(U): x6(U): x7(Ø): x8(Ø): x9(U): x10(Ø): x11(U):x12(Ø): x13(U): x14(Ø): x15(Ø): x16(U): x17(U): x18(Ø): x19(U): x20(U): R2: x20(Ø): x21(U): x22(Ø): x23(U): x24(Ø): x9(Ø): x8(U): x5(Ø): x4(U): x25(Ø): L2: x25(U): x3(U): x2(Ø): x1(U): x26(U): x27(Ø): x6(Ø): x7(U): x28(Ø): x29(U): x10(U):x24(U): x23(Ø): x13(Ø): x14(U): x22(U): x21(Ø): x19(Ø): x18(U): x17(Ø): R1: x16(Ø): x15(U): x12(U): x11(Ø): x29(Ø): x28(U): x27(U): x26(Ø) ■

As with the Brochos earlier, we proceed to a constructional analysis of Osage Diamonds through sequential illumination of its intermediary string-positions; only this time we record the crossing alias transformations as we go:

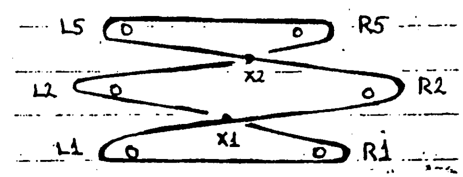


Fig. 29.I: Osage Diamonds: O.A

|⇒ L1: x1(Ø): R2: x2(Ø): L5: R5: x2(U): L2: x1(U): R1 ■

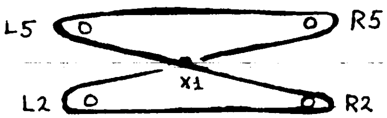


Fig. 29.II: Osage Diamonds: O.A: □1|

|⇒ L2: x1(U): R5: L5: x1(∅): R2 ■

	Fig. 29.I		Fig. 29.II
C	x1	⟶	∅
	x2	⟶	x1

Note: The crossing x1 of Fig. 29.I becomes extension-cancellable -- by Lemma 2.A -- upon □1.

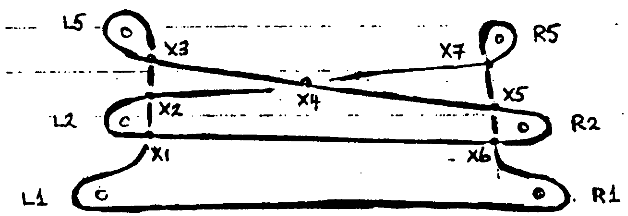


Fig. 29.III: Osage Diamonds: O.A: □1| $\underrightarrow{1}$(5f)#

|⇒ L1: x1(U): x2(U): x3(U): L5: x3(∅): x4(∅): x5(∅): R2: x6(∅): x1(∅): L2: x2(∅): x4(U): x7(∅): R5: x7(U): x5(U): x6(U): R1 ■

Fig. 29.II		Fig. 29.III
x1	⟶	x4

The crossings x1, x2, x3; x5, x6, x7 of Fig. 29.III are created by the movement $\underrightarrow{1}$(5f)# applied to Fig. 29.II.

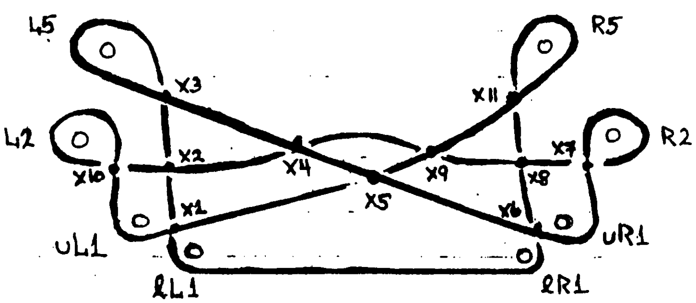


Fig. 29.IV: Osage Diamonds: O.A: □1| $\underrightarrow{1}$(5f)# $\vec{1}$(2f)#

|⇒ ℓL1: x1(U): x2(U): x3(U): L5: x3(∅): x4(∅): x5(∅): x6(∅): uR1: x7(∅): R2: x7(U): x8(∅): x9(U): x4(U): x2(∅): x10(U): L2: x10(∅): uL1: x1(∅): x5(U): x9(∅): x11(∅): R5: x11(U): x8(U): x6(U): ℓR1 ■

Fig. 29.III		Fig. 29.IV
x1	⟶	x2
x2	⟶	x1
x3	⟶	x3
x4	⟶	x5
x5	⟶	x6
x6	⟶	x8
x7	⟶	x11

The crossings x4, x7, x9, x10 of Fig. 29.IV are created by the movement $\vec{1}$(2f)# applied to Fig. 29.III.

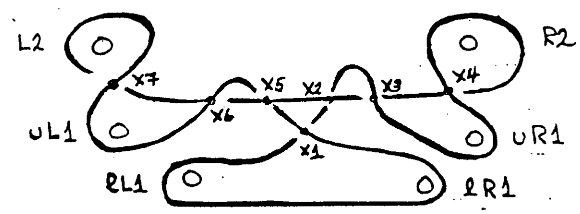


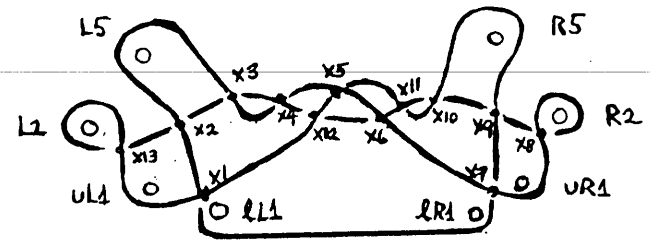
Fig. 29.V: Osage Diamonds: O.A: □1| $\underrightarrow{1}$(5f)# $\vec{1}$(2f)#□5|

|⇒ ℓL1: x1(U): x2(U): x3(∅): uR1: x4(∅): R2: x4(U): x3(U): x2(∅): x5(∅): x6(U): x7(U): L2: x7(∅): uL1: x6(∅): x5(U): x1(∅): ℓR1 ■

	Fig. 29.IV		Fig. 29.V
	x1	⟶	x1
	x2	⟶	x2
C.	x3	⟶	∅
	x4	⟶	x3
C.	x5	⟶	∅

	Fig. 29.IV		Fig. 29.V
C.	x6	⟶	∅
	x7	⟶	x4
	x8	⟶	x5
	x9	⟶	x6
	x10	⟶	x7
C.	x11	⟶	∅

The crossings x3 and x11 of Fig. 29.IV become extension-cancellable -- by Lemma 2.A -- upon □5; while, clearly, x1, x5, x6 of Fig. 29.IV -- being three separate crossings of 5n with 5f -- all collapse to a single crossing of the ℓL1f and ℓR1f-strings. In fact, we eliminate the pair {x5,x6} of Fig. 29.IV via (several applications of) Lemma 2.B. We remark, at this point, the creation of the complex S-crossings formed by {x2,x3} and {x5,x6} of Fig. 29.V.

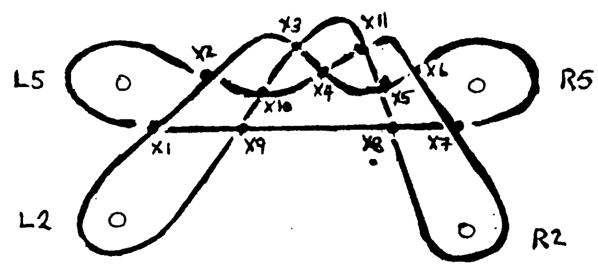


<u>Fig. 29.VI</u>: Osage Diamonds: <u>O</u>.A: □1| $\underset{\rightarrow}{1}$(<u>5f</u>)# $\overrightarrow{1}$(<u>2f</u>)#□.5| $\overleftarrow{5}$(<u>1f</u>)#

|⇒ ℓL1: x1(U): x2(∅): L5: x3(∅): x4(U): x5(∅): x6(∅): x7(∅): uR1: x8(∅): R2: x8(U): x9(U): x10(U): x11(∅): x6(U): x12(U): x4(∅): x3(U): x2(U): x13(U): L2: x13(∅): uL1: x1(∅): x12(∅): x5(U): x11(U): x10(∅): R5: x9(∅): x7(U): ℓR1 ■

<u>Fig. 29.V</u>		<u>Fig. 29.VI</u>
x1	⟶	x5
x2	⟶	x4
x3	⟶	x6
x4	⟶	x8
x5	⟶	x11
x6	⟶	x12
x7	⟶	x13

The crossings x1, x2, x3, x7, x9, x10 of Fig. 29.VI are created by the movement $\overleftarrow{5}$(<u>1f</u>)# applied to the string-position of Fig. 29.V.

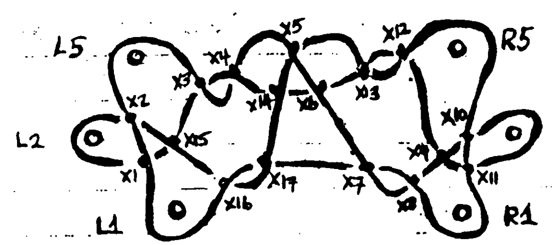


<u>Fig. 29.VII</u>: Osage Diamonds: <u>O</u>.A:□1| $\underset{\rightarrow}{1}$(<u>5f</u>)# $\overrightarrow{1}$(<u>2f</u>)#□5| $\overleftarrow{5}$(<u>1f</u>)#□1|

|⇒ L2: x1(∅): x2(∅): x3(U): x4(∅): x5(∅): x6(U): R5: x7(U): x8(∅): x9(∅): x1(U): L5: x2(U): x10(∅): x4(U): x11(U): x6(∅): x7(∅): R2: x8(U): x5(U): x11(∅): x3(∅): x10(U): x9(U) ■

<u>Fig. 29.VI</u>		<u>Fig. 29.VII</u>	<u>Fig. 29.VI</u>		<u>Fig. 29.VII</u>
x1	⟶	x1	<u>C</u>. x8	⟶	∅
x2	⟶	x9	x9	⟶	x8
x3	⟶	x10	x10	⟶	x5
x4	⟶	x3	x11	⟶	x11
x5	⟶	x4	x12	⟶	x2
x6	⟶	x6	<u>C</u>. x13	⟶	∅
x7	⟶	x7			

Here, for example, as x6 and x7 of Fig. 29.VI "slide" over intermediate strings (and crossings) to x6 and x7 of Fig. 29.VII, respectively, a (temporary) crossing of the (former) uR1n string over the R2n string is created between x8 and x9 on the R2n-string of Fig. 29.VI. This crossing and x8 are extension-cancellable by Lemma 2.B. In exactly analogous manner, x13 of Fig. 29.VI is extension cancellable. [We remark that in a case like this, □1, where there are <u>two</u> 1∞'s to be dropped, it is often helpful in computing the <u>alias</u> crossing transformation between the two figures involved to insert an intermediate position, corresponding to either □u1∞ <u>or</u> □ℓ1∞ -- whichever is the more helpful for the computation (here, in fact, we have used the intermediate diagram for □u1∞) -- then computing the transformation from this diagram to the one desired (<u>i.e</u>. releasing the remaining 1∞). The composition of these two <u>alias</u> transformations gives the desired result. Here, we have suppressed the "intermediate step", presenting only the conclusions.]



<u>Fig. 29.VIII</u>: Osage Diamonds: <u>O</u>.A:□1| $\underset{\rightarrow}{1}$(<u>5f</u>)# $\overrightarrow{1}$(<u>2f</u>)#□5| $\overleftarrow{5}$(<u>1f</u>)#□1| $\overrightarrow{1}$(<u>5n</u>)#|

|⇒ L1: x1(∅): x2(∅): L5: x3(∅): x4(U): x5(∅): x6(∅): x7(∅): x8(U): x9(∅): x10(U): R2: x11(U): x9(U): x12(U): x13(∅): x6(U): x14(U): x4(∅): x3(U): x15(U): x1(U): L2: x2(U): x15(∅): x16(U): x17(∅): x14(∅): x5(U): x13(U): x12(∅): R5: x10(∅): x11(∅): R1: x8(∅): x7(U): x17(U): x16(∅) ■

Fig. 29.VII		Fig. 29.VIII	Fig. 29.VII		Fig. 29.VIII
x1	→	x17	x7	→	x7
x2	→	x14	x8	→	x11
x3	→	x13	x9	→	x1
x4	→	x5	x10	→	x3
x5	→	x12	x11	→	x4
x6	→	x6			

The crossings x2, x8, x9, x10, x15, x16 are created by the move $\overset{\rightarrow}{1}$(5n)#| applied to the Fig. 29.VII.

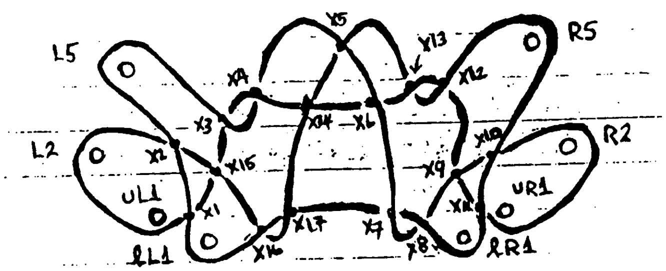


Fig. 29.IX: Osage Diamonds: O.A:□1| $\underset{\rightarrow}{1}$(5f)# $\overset{\rightarrow}{1}$(2f)#□5| $\overset{\leftarrow}{5}$(1f)#□1| $\overset{\rightarrow}{1}$(5n)#| $\overset{\rightarrow}{1}$(2n)#

The linear sequence associated to Fig. 29.IX is derived from that for Fig. 29.VIII via the following replacements in that former sequence

L1 → ℓL1		R1 → ℓR1	
L2 → uL1:L2		R2 → uR1:R2	

The alias crossing-transformation is the identity.

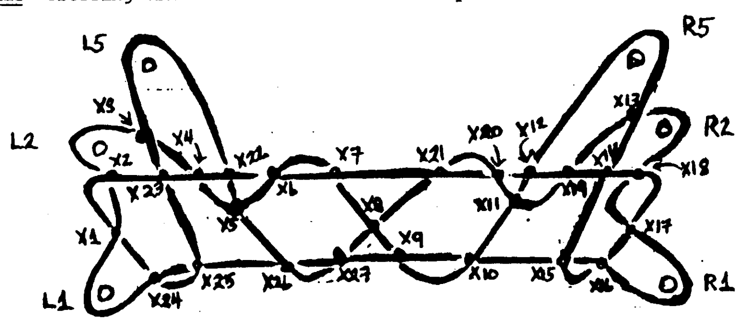


Fig. 29.X: Osage Diamonds: O.A:□1| $\underset{\rightarrow}{1}$(5n)# $\overset{\rightarrow}{1}$(2f)#□5| $\overset{\leftarrow}{5}$(1f)# □1| $\overset{\rightarrow}{1}$(5n)#| $\overset{\rightarrow}{1}$(2n)# N1|

|⇒ L1: x1(∅): x2(U): L2: x3(U): x4(U): x5(∅): x6(∅): x7(U): x8(∅): x9(U): x10(∅): x11(U): x12(U): R5: x13(∅): x14(U): x15(∅): x16(U): x17(U): x18(∅): x14(∅): x19(∅): x12(∅): x20(U): x21(∅): x7(∅): x6(U): x22(∅): x4(∅): x23(∅): x2(∅): x1(U): x24(U): x25(∅): x23(U): x3(∅): L5: x22(U): x5(U): x26(∅): x27(U): x8(U): x21(U): x20(∅): x11(∅): x19(U): x13(U): R2: x18(U): x17(∅): R1: x16(∅): x15(U): x10(U): x9(∅): x27(∅): x26(U): x25(U): x24(∅) ■

Fig. 29.IX		Fig. 29.X		Fig. 29.IX		Fig. 29.X
x1	→	x25	C.	x9	→	∅
x2	→	x3		x10	→	x13
x3	→	x26		x11	→	x15
x4	→	x27		x12	→	x10
x5	→	x8		x13	→	x9
x6	→	x19		x14	→	x4
x7	→	x21	C.	x15	→	∅
x8	→	x20		x16	→	x6
				x17	→	x7

The x9,x15-crossings disappear as the ℓ1n-string passes up and over the u1n-straight string (of Fig. 29.IX) to become the upper transverse (straight) string of Fig. 29.X. The crossings x1, x2, x5, x11, x12, x14, x16, x17, x18, x22, x23, x24 of Fig. 29.X are created by the movement "N1|" applied to Fig. 29.IX. The "1-△̲S " of the subsequent move are now apparent; the L1-△ has complex crossings {x1,x2}, {x24,x25} and the simple crossing x23 as vertices, while the R1-△ has {x15,x16}, {x17,x18} and x14 as vertices. In the next movement L2 hooks towards you over s;x23-x2, while R2 hooks towards you over s;x18-x14 -- passing down into these 1-△̲S in the process -- and continues its rotation down, away, and then back up towards you (below the 5∞) to its normal position; the original L2∞ (of Fig. 29.X) is naturally lost in the process of this rotation -- i.e 2 rotates out of its own loop, while picking up the "base" of the (inverted) 1-△.

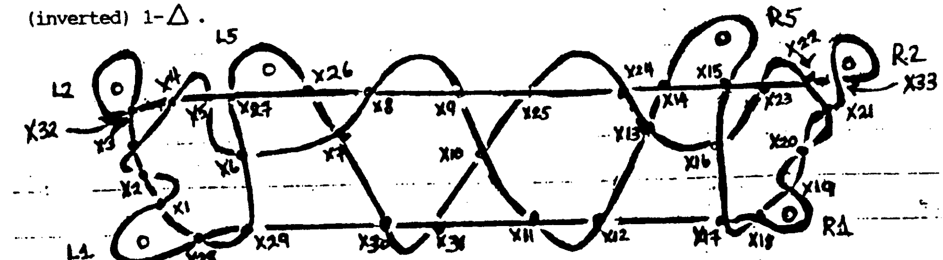


Fig. 29.XI: Osage Diamonds: O.A:□1| $\underset{\rightarrow}{1}$(5f)# $\overset{\rightarrow}{1}$(2f)#□5| $\overset{\leftarrow}{5}$(1f)#□1| $\overset{\rightarrow}{1}$(5n)#| $\overset{\rightarrow}{1}$(2n)# N1|:: $\overset{\leftarrow}{2}$↓(1-△̲S): <2(#)

⊨ L1: x1(∅): x2(U): x3(∅): x4(∅): x5(U): x6(U): x7(∅): x8(∅): x9(U): x10(∅): x11(U): x12(∅): x13(U): x14(U): R5: x15(U): x16(∅): x17(∅): x18(U): x19(U): x20(∅): x21(U): x33(∅): R2: x33(U): x22(U): x23(∅): x15(∅): x14(∅): x24(U): x25(∅): x9(∅): x8(U): x26(∅): x27(∅): x5(∅): x4(U): x32(U): L2: x32(∅): x3(U): x2(∅): x1(U): x28(U): x29(∅): x6(∅): x27(U): L5: x26(U): x7(U): x30(∅): x31(U): x10(U): x25(U): x24(∅): x13(∅): x16(U): x23(U): x22(∅): x21(∅): x20(U): x19(∅): R1: x18(∅): x17(U): x12(U): x11(∅): x31(∅): x30(U): x29(U): x28(∅) ■

Fig. 29.X		Fig. 29.XI	Fig. 29.X		Fig. 29.XI	Fig. 29.X		Fig. 29.XI
x1	→	x1	x10	→	x12	x19	→	x23
x2	→	x2	x11	→	x13	x20	→	x24
x3	→	x6	x12	→	x14	x21	→	x25
x4	→	x5	x13	→	x16	x22	→	x26
x5	→	x7	x14	→	x15	x23	→	x27
x6	→	x8	x15	→	x17	x24	→	x28
x7	→	x9	x16	→	x18	x25	→	x29
x8	→	x10	x17	→	x19	x26	→	x30
x9	→	x11	x18	→	x20	x27	→	x31

Crossings x3, x4, x21, x22, x32, x33 are, clearly, created by the move "⃖2↓(1-△s): < 2 (#)" applied to Fig. 29.X.

The figure Osage Diamonds, Fig. 28, is now obtained from that of Fig. 29.XI <u>via</u> "□5|", whose crossing-effect is obvious. In particular, {x26,x27} and {x14,x15} become extension-cancellable by Lemma 2.B. We remark that, in the canonical construction of Osage Diamonds, the last two movements, "< 2(#):□5|", are performed more or less simultaneously, while the palms of the hand naturally turn away from the body during the course of 2's rotation. Explicitly

	Fig. 29.XI		Fig. 28		Fig. 29.XI		Fig. 28		Fig. 29.XI		Fig. 28
	x1	→	x1		x12	→	x12		x23	→	x22
	x2	→	x2		x13	→	x13		x24	→	x23
	x3	→	x3	C.	x14	→	∅		x25	→	x24
	x4	→	x4	C.	x15	→	∅	C.	x26	→	∅
	x5	→	x5		x16	→	x14	C.	x27	→	∅
	x6	→	x6		x17	→	x15		x28	→	x26
	x7	→	x7		x18	→	x16		x29	→	x27
	x8	→	x8		x19	→	x17		x30	→	x28
	x9	→	x9		x20	→	x18		x31	→	x29
	x10	→	x10		x21	→	x19		x32	→	x25
	x11	→	x11		x22	→	x21		x33	→	x20

This completes the crossing-specific constructional analysis for the string-figure Osage Diamonds.

The corresponding "Heart"-sequence for Osage Diamonds bears some superficial similarities to that of the Brochos. In particular, from Fig. 29.II (O.A:□1|) -- in which each hand holds only two loops, a 2ꝏ and a 5ꝏ -- we may pass to Fig. 29.VII by the loop-specific manipulational equivalent sequence

5ꝏ(←) (2ꝏ):: > 5ꝏ⃗↓(2ꝏ): 5ꝏ(→) → 5|

That is, with respect to the string-position given by the schema of Fig. 29.II,

1(→) (<u>5f</u>)# 1⃗ (<u>2f</u>)#□5| ⃖5 (<u>1f</u>)#□1| ≡ 5ꝏ(←)(2ꝏ):: > 5ꝏ⃗↓(2ꝏ): 5ꝏ(→) → 5|

and Fig. 29.VII is produced by either sequence. [<u>Note</u> that, had we momentarily suspended the 5ꝏ -- brought below 2ꝏ to the near side of the figure -- on 1, the loop-specific manipulation, above, would have appeared as

5ꝏ(←)(2ꝏ)→ 1:: > 1ꝏ⃗↓(2ꝏ): 1ꝏ(→) → 5|

The final 1ꝏ-manipulation, here, is precisely that in the Brochos' "Heart"-sequence (<u>cf</u>. page 34).] Having reached the stage of construction of Osage Diamonds given by Fig. 29.VII, we may directly view the near-neighborhood of the geometric center of the complete, extended figure by performing the "peek"

2⃗ (5̄n)#: hook 1 away over ✗2n and press this string down, turning palms slightly away from body | (gently)

on this string-position. Here we see crossings x8, x9, x10, x11, x12, x23, x24, x28, x29 -- and all incident intermediary straight strings -- exactly as they appear in Fig. 28, the schema for the completed design. Thus, returning to Fig. 29.VII - <u>i.e</u>. □1#□ u2ꝏ| -- the final design is to be extended between 2n and 5n <u>via</u> an "interlacing" of these two strings which creates the extreme lateral "diamonds", with their complex boundary crossings. In fact, with respect to that figure (Fig. 29.VII), we find

1⃗ (<u>5n</u>)#| 1⃗ (<u>2n</u>)# N1|:: ⃖2↓(1-△s): < 2(#):□5| ≡
⃖5ꝏ→1: 2ꝏ→12:: >✗1ꝏ⃗→2: N2:: <2ꝏ→2|

the right-hand side being a loop specific manipulational equivalent of the left. We remark that, in the antepenultimate step -- >✗1ꝏ⃗→2 -- the ✗1ꝏ passes <u>over</u> the u1ꝏ (<u>i.e</u>. N1), performs a half-twist away from you, and is placed on 2 -- where it becomes the u2ꝏ. The final movement — <2ꝏ→2| -- is necessary only to produce the crossings x20, x25 of the completed design (Fig. 28). Combination, and minor simplification, of the above observations yields the "Heart"-sequence for Osage Diamonds:

O.A: □1| ::5ꝏ(2ꝏ): >5ꝏ↓(2ꝏ): 5ꝏ(2f): 5ꝏ →1:: 2ꝏ→12:: >1ꝏ→2: N2:: <2ꝏ→2| (gently).

Here, as is frequently the case, some rearrangement of the strings is necessary to produce the final design until correct intermediate tensions are mastered; even then, the "1-△s" movement invariably produces a cleaner, more dramatic final extension. In fact, we believe that it is this dramatic and surprising "Osage extension" which accounts, in large measure, for this figure's great popularity. We shall return to these matters after a brief pause to exhibit an Osage-extension of the Brochos:

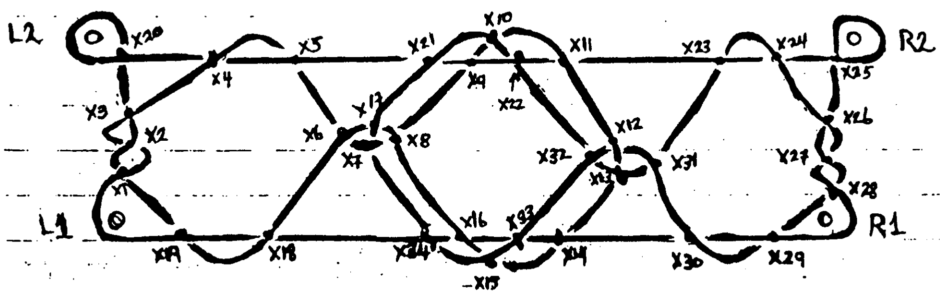


Fig. 30: Osage Extension of the Brochos

O.A: 1⃗(2f)# 2⃖(1f)#□1| 1⃗↓(2ꝏ): 1(5f)#□5| u2ꝏ→5:□2# 1ꝏ→2|(gently) 1⃗(5n)# 1⃗(2n)# N1|(gently):: 2⃖↓(1-△s): <2(#):□5|

|⇒ L1: x1(∅): x2(U): x3(∅): x4(∅): x5(U): x6(U): x7(∅): x8(U): x9(U): x10(U): x11(∅): x12(∅): x13(U): x14(U): x15(U): x16(∅): x8(∅): x17(U): x6(∅): x18(∅): x19(U): x1(U): x2(∅): x3(U): x20(U): L2: x20(∅): x4(U): x5(∅): x21(U): x9(∅): x22(∅): x11(U): x23(∅): x24(U): x25(∅): R2: x25(U): x26(U): x27(∅): x28(U): x29(U): x30(∅): x31(∅): x12(U): x32(∅): x33(∅): x15(∅): x34(U): x7(U): x17(∅): x21(∅): x10(∅): x22(U): x32(U): x13(∅): x31(U): x23(U): x24(∅): x26(∅): x27(U): x28(∅): R1: x29(∅): x30(U): x14(∅): x33(U): x16(U): x34(∅): x18(U): x19(∅) ■

We remark that during the construction of the figure, at the first occurrence of the movement "|(gently)," the central string-tensions should be adjusted so that the Brochos in the middle of the figure is equidistant from the hands, and has no loose constituent strings; that having been accomplished, the final "Osage-extension" will be every bit as satisfying as in the parent figure -- requiring little, if any, final adjustment. And, of course, the whole complex of figures -- pseudo-Brochos, Double Brochoi, pseudo-Double Brochoi, et cetera -- lend themselves well to Osage extension, and a variety of beautiful -- and, mostly, well-known -- string-figures result. For example, pseudo-Brochos plus Osage extension -- followed by 180° rotation in the plane of the figure -- is known on Santa Cruz island as "Nelo" (a bird) [H.C. Maude: Solomon Island String Figures, No.11]. The local construction is

Nelo: O.A: 1⃗(2f)# 2⃖(1f)#□1| 1⃗↓(2ꝏ): 1(5n)#□5| 5⃖(u2f): 5(2ꝏ): 5(1f)# 1⃗(u2n): N1: □ u2ꝏ|□2ꝏ and pull them free with opposite hand #|

With respect to the final string-position, i.e. Nelo, this is equivalent to the "direct" construction

Make the Brochos: <5ꝏ→2: <1ꝏ→1:: 5⃖↓(2ꝏ): 5(1f)# 1⃗(2n)# N1:□2|

where, in fact, the only difference between the above two sequences is the point at which the last 2ꝏ is released -- a matter of indifference to the final design. Subjectively, the early 2ꝏ-release of the latter sequence does real violence to the spirit of the figure Nelo: the beautiful design appearing at the penultimate | of Nelo -- with all its quasi-stability -- is simply elided. But this latter sequence is, indeed, instructive in the context of Brochos/pseudo-Brochos constructions of this type of final pattern.

We return, now, for an alternate look at what we have been calling the "Osage Extension". We set up, on the hands, the "Heart"-sequence for Osage Diamonds as far as Fig. 29.VII, viz.

O.A:□1| :: 5ꝏ(2ꝏ): >5ꝏ↓(2ꝏ): 5ꝏ→5|

where the 5ꝏ has been temporarily suspended back onto 5 subsequent to its journey towards you under, then twisted away down thru, and finally away under, the 2ꝏ.

Now, a movement quite analogous to that just completed by the 5∞ -- with an interposed 1-pickup move -- will (essentially) complete the Osage Extension of the figure. In words,

Bring $\overleftarrow{5\infty}$ (2∞) to near side of figure: > $\overrightarrow{5\infty}$ and place it over the 2n-string :: $\vec{1}$↓(5∞): 1(2n)# :: $\overrightarrow{5\infty}$↓(2∞): 5∞→5|

Note that, after 1 dips into the 5∞ -- as it (the 5∞) twists away from you over 2n -- and picks up that string (2n), the progress of the whole 5∞ (down into 2∞ and away under to 5) is impeded (as its near string is looped about s;1f-2n). Technically speaking, only the far string of the moving 5∞ completes the journey, the whole moving 5∞ being greatly "opened up" by virtue of its near string's snag. The movements "□2: < 5∞ →2|" complete the figure. The extension, here, compares favorably with that of the original construction -- particularly in a heavier cord -- and illuminates the "interlacing" of the 2n and 5n strings of Fig. 29.VII which creates the extreme lateral diamonds plus complex boundary crossings in the final design [Fig. 28; cf. page 64, where this matter is first introduced].

For a "source" figure employing this alternate view of the Osage extension, we present an unnamed figure from central Africa [cf.W.A. Cunnington: "String Figures and Tricks from Central Africa", No. 13]:

O.1: 15∞ → W:: $\vec{M}$ (Wn)# twist M∞ 180° clockwise # 1↑(M∞)#□M| W∞ →2:: 3 (1f & 2f)# 5 (2n)#□2|:□3# 2∞ →3: $\vec{1}$ (5n)#| $\overrightarrow{3\infty}$ →12345:: > $\overrightarrow{1\infty}$→3:□2345d#□5|(palms away).

Notes on the construction: (1) The resulting string-figure is Fig. 28 (extended on 1 and 3) except for crossings x20, x25 of that schema (ending the above construction with "< 3∞ →2" produces Fig. 28 exactly). (2) Twisting M∞ 180° counterclockwise results in Fig. 28 with the parity of crossing x10 reversed; this figure is not related to Osage Diamonds by a rigid motion. (3) The "source" construction employs the Great Toe subsequent to the first extension in line 1, above; we have employed an equivalent working which avoids this frame-element. (4) The 3∞ is to be kept well up on the tips of the fingers in "3∞ →12345"; i.e. do not allow this loop to slip down the hands to W. (5) The movement "□2345d" entails lifting this dorsal string over the tips of the fingers -- and the retained 3∞ -- and dropping it on the palmar side of the hands. (6) The

extension is fully as grand and dramatic as that of the original figure.

In terms of complex crossings, the schema for the string-figure Osage Diamonds exhibits only 15 such -- as compared with the 29 simple crossings of the original schema (Fig. 28).

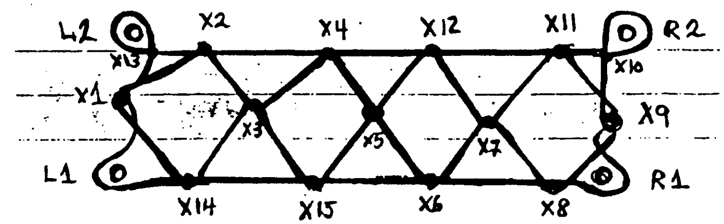


Fig. 31: Osage Diamonds, labeled for complex crossings

Here we have elided the "fine-structure" of the individual crossings, as only the global node-labeling is germane to the present discussion; whenever that fine-structure is needed, it is immediately recoverable from Fig. 28 via a comparison of labels:

Fig. 31		Fig. 28	Fig. 31		Fig. 28
x1	—	x1, x2, x3	x9	—	x17, x18, x19
x2	—	x4, x5	x10	—	x20
x3	—	x6, x7	x11	—	x21, x22
x4	—	x8, x9	x12	—	x23, x24
x5	—	x10	x13	—	x25
x6	—	x11, x12	x14	—	x26, x27
x7	—	x13, x14	x15	—	x28, x29
x8	—	x15, x16			

Note that x5, x10, x13 are all simple crossings, and that -- if xn is any crossing of Fig. 31 -- we have α(xn) = 2 uniformly. Also χ(x1) = χ(x9) =3; while χ(x5) = χ(x10) = χ(x13) = 1, since these are simple crossings -- but if xn is any other crossing of Fig. 31 -- we have χ(xn) = 2. That is, for n ≠ 1, 5, 9, 10, 13, we have χ(xn) = 2.

To exhibit the linear sequence associated to Fig. 31, we must add four new entries to the X-Dictionary — corresponding to the four new types of complex crossings encountered in Osage Diamonds(1):

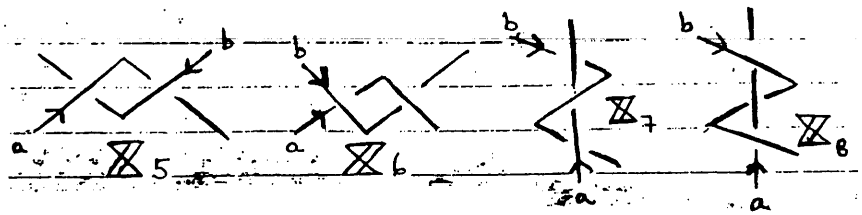


Fig. 32: ✕-Dictionary (continued)

We remark that these four new entries -- though obviously related in pairs -- are essentially distinct, and necessary to the (complex) crossing analysis of Osage Diamonds. The constituent arcs in a given complex crossing have been labeled for their ✕-Dictionary entry in terms of where, and how, they arise in the linear sequence associated (canonically) to the string-figure which first gives rise to them -- that is, lexicographically. Here, the linear sequence associated to Fig. 31 is

|⇒ L1: x1(7,a+): x2(5,a+): x3(5,b-): x4(5,a+): x5(∅): x6(5,b-): x7(6,a+): x8(6,b+): x9(8,a+): x10(U): R2: x10(∅): x11(6,b-): x12(6,b-): x4(5,b+): x2(5,b+): x13(∅): L2: x13(U): x1(7,b+): x14(5,b-): x3(5,a+): x15(6,b+): x5(U): x12(6,a+): x7(6,b+): x11(6,a+): x9(8,b+): R1: x8(6,a-): x6(5,a-): x15(6,a-): x14(5,a-) ■

Now let us construct a schema from the above linear sequence(!) We shall do this by suppressing information in the sequence, selectively -- producing what we shall call a subsequence of the given linear sequence -- from which valid inferences may be drawn concerning the configuration of the associated schema.

... ≡ suppressed information

Step 1: Suppress all non-frame nodes of the sequence. This gives the subsequence

|⇒ L1 ... R2 ... L2 ... R1... ■

from which we infer that the associated schema has (only) the frame-nodes L1, L2, R1, R2 -- i.e. the figure is extended on 1 and 2. Further, there is one loop on each of these fingers.

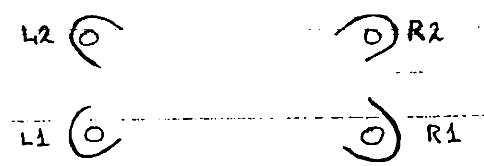


Fig. 33.I: Schema-so-far (after Step 1)

Step 2: The subsequence

|⇒ ... x10(U): R2: x10(∅) ... x13(∅): L2: x13(U) ... ■

gives the "gross" schema

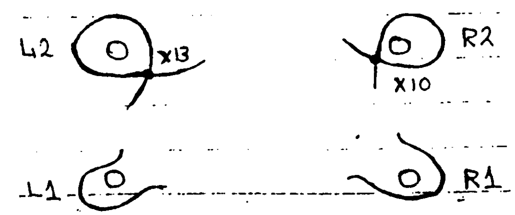


Fig. 33.II: Schema-so-far

The adjective "gross", here, is used to denote that the parity of the simple crossings x10, x13 cannot be determined from the information so far employed Of course, they are simple crossings by virtue of their arguments.

Step 3: The subsequences

|⇒ L1: x1(7,a+) ... L2: x13(U): x1(7,b+) ... ■

and

|⇒ ... x9(8,a+): x10(U): R2 ... x9(8,b+): R1 ... ■

provide the additional information

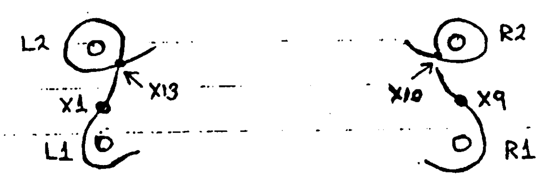


Fig. 33.III: Schema-so-far

which is gross with respect to the complex crossings x1, x9, but is now complete with respect to x10, x13 (which will, henceforth, be suppressed). From this point forward we shall concern ourselves only with "gross" information -- to see how something at the level of specificity of Fig. 31 can be "seen" in the given linear sequence; but we remark that the complete detail of Fig. 28 may be recovered with insignificantly more effort.

Step 4: The subsequence

⊫ L1 ...

far straight string} R2: x10: x11: x12: x4: x2: x13: L2 ...

near straight string} R1: x8: x6: x15: x14 ∎

fleshes out our evolving schema to

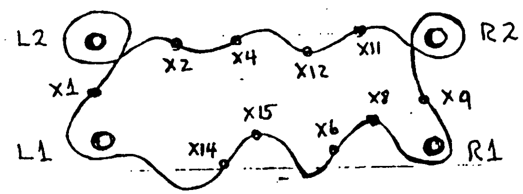


Fig. 33.IV: Schema-so-far

where we cannot say that the 1n and 2n strings are "straight", or that -- say -- the (ordered!) crossings x14, x15, x16, x8 are equally spaced on the 1n-string: indeed, schemata are indifferent to such matters. What we may assert is that the 1n and 2n strings do not intersect each other (there would be a common crossing if they did), and that the crossings on these strings appear in the stated order (i.e. that of the given linear sequence). Thus, in terms of the (class of) schemata associated to the linear sequence under consideration -- if, indeed, there are any such -- we may as well pick the one below (Fig. 33.IV') as "canonical":

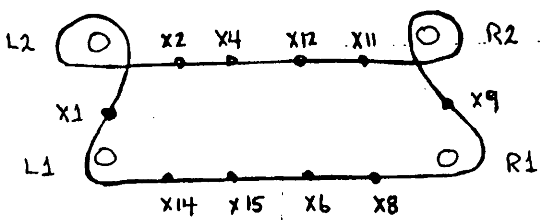


Fig. 33.VI': Schema-so-far

Step 5: Each of the subsequences, in turn

◊ I ⊫ ... x1: x2: x3 ... x1: x14: x3 ... ∎

◊ II ⊫ ... x3: x4: x5 ... x3: x15: x5 ... ∎

◊ III ⊫ ... x5: x6: x7 ... x5: x12: x7 ... ∎

◊ IV ⊫ ... x7: x8: x9 ... x7: x11: x9 ... ∎

{ "New" crossings receive a red underline

provides a "central diamond", and our evolving gross schema becomes

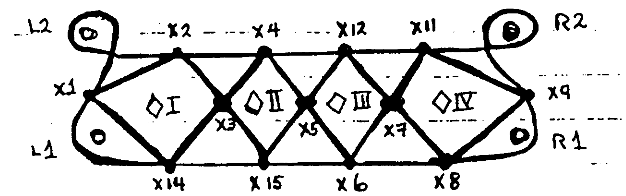


Fig. 33.V: Schema-so-far

where the placement of crossings x3, x5, x7 are arbitrary to the extent that they may suffer some "local" movement, but not so much as to create any "new" string-crossings (as all crossings in the given linear sequence are, at this point, accounted for). The above placement of these crossings (Fig. 33.V) may again, be thought of as "canonical".

Now, at this point in the process of schema-construction, it will be observed that Fig. 33.V represents a pretty fair approximation to the "gross" schema of Fig. 31 -- with no reference to the Ж-Dictionary(!) And the only legitimate conclusion which may now be drawn from our 5-Step analysis of the given linear sequence is that if there is a schema corresponding to the linear sequence, then it has the gross representation of Fig. 33.V (≡ Fig. 31). The existence (or nonexistence) problem is decided via a closer attention to the fine-structure of the crossing-types during the derivation [for further discussion, see Appendix A, pages 96-105; Warning: "Appendix" material is of a somewhat advanced level of technical difficulty and, perhaps, should be postponed until the reader just can't stand not knowing what's going on at that level a moment longer].

We pause in our discussion of Osage Diamonds to present a construction of a "pseudo-Osage Diamonds" figure directly analogous to the Brochos/pseudo-Brochos dichotomy. The figure is known as the "Imperial Pigeon (Carpophaga)"

[K. Haddon: Cat's Cradles From Many Lands, Fig. 46]:

O.A: 1∞ →2:: 1→(2n): 1→(p)#| M→(5f)#|: give M∞ a 180°-twist counter-clockwise: 1←↑(M∞):□M#| N1| 1∞→ →2:: 1→(ℓ2f)# 1→↑(u2∞)#|:: 3←↓(u2∞): 3←(ℓ2n)# N1|□2:□5#|(palms away).

The resulting figure is Fig. 28 (Osage Diamonds), rotated 180° in its own plane. While the actual working of this figure is somewhat "constricted" -- even when fully mastered -- the final extension is magnificent; different from, but thoroughly the equal of, that of Osage Diamonds, itself.

The string-figure Osage Diamonds has a multiplicative -- or repetitive -- aspect similar to that encountered with the Brochos; but here the matter is slightly more complicated [cf. L.A. Dickey: String Figures From Hawaii, pp. 18-34; also A.R. Amir-Moéz: Mathematics and String Figures, Chapter 1, "Figures with Lozenges"]. Taken as a whole, the sequence provides a genuinely satisfying complex of string-figures clustering about the parent figure, Osage Diamonds.

We begin the explication of the repetitive process with a discussion of string-figures of Osage-type constructions whose gross schemata are all given by

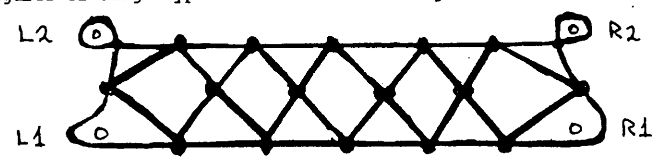


Fig. 34: Osage 5-Diamonds; gross schema

Here the indicated crossings may be either simple or complex, although 12 of these (5 central on 2n, 5 on 1n, and 1 each on the lateral s;1n-2f) are certainly complex. The gross schema of Fig. 34 cannot be labeled for these complex crossings at this time -- because each different complex crossing determines the direction of passage through itself and, hence, the labeling of future crossings. In the subsequent discussion, exactly two complex crossing labeling (gross) schema will emerge, and these will be denoted by right and left -- for reasons soon to become readily apparent.

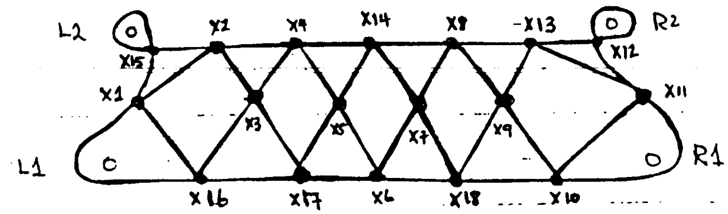


Fig. 34.A: Osage 5-Diamonds, right

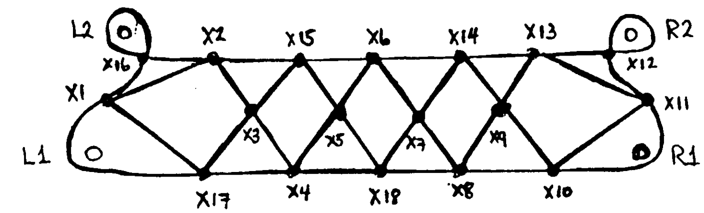


Fig. 34.B: Osage 5-Diamonds, left

We shall do the analysis for Osage 5-Diamonds, right, in some detail; the analysis for Osage 5-Diamonds, left, will follow from the symmetry of the situation.

Consider the manipulation sequence

O.A:□1| 1→(5f)# 1→(2f)#□5| 5←(1f)#□1| >>R2: >>R5: 1→(5n)#| 1→(2n)# N1|:: 2←↓(1-Ⓢ): <2(#)□5|(palms away).

This produces the string-figure we shall call Osage 5-Diamonds, right (>>R2 >>R5), whose schema is

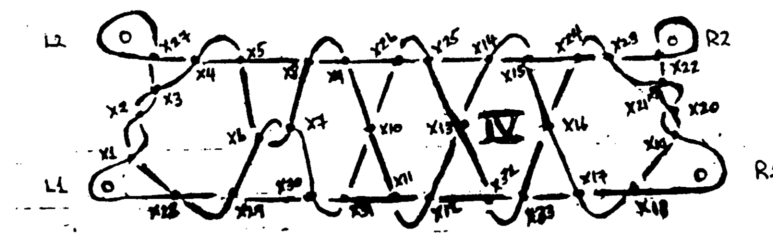


Fig. 34.A.i: Osage 5-Diamonds, right (>>R2, >>R5)

Note that this is a fine-schema substitution instance of Fig. 34.A; i.e. a "right" variant. The associated linear sequence is

|⇒ L1: x1(Ø): x2(U): x3(Ø): x4(Ø): x5(U): x6(U): x7(Ø): x8(Ø): x9(U): x10(Ø): x11(U): x12(Ø): x13(U): x14(Ø): x15(U): x16(Ø): x17(Ø): x18(U): x19(U): x20(Ø): x21(U): x22(U): R2: x22(Ø): x23(U): x24(Ø): x15(Ø): x14(U): x25(U): x26(Ø): x9(Ø): x8(U): x5(Ø): x4(U): x27(Ø): L2: x27(U): x3(U): x2(Ø): x1(U): x28(U): x29(Ø): x6(Ø): x7(U): x30(Ø): x31(U): x10(U): x26(U): x25(Ø): x13(Ø): x32(U): x33(Ø): x16(U): x24(U): x23(Ø): x21(Ø): x20(U): x19(Ø): R1: x18(Ø): x17(U): x33(U): x32(Ø): x12(U): x11(Ø): x31(Ø): x30(U): x29(U): x28(Ø) ■

And the corresponding complex-crossing linear sequence associated to Fig. 34.A is, directly,

|⇒ L1: x1(7,a+): x2(5,a+): x3(5,b-): x4(5,a+): x5(Ø): x6(5,b-): x7(U): x8(5,a+): x9(Ø): x10(6,b+): x11(8,a+): x12(U): R2: x12(Ø): x13(6,b-): x8(5,b+): x14(6,b-): x4(5,b+): x2(5,b+): x27(Ø): L2: x27(U): x1(7,b+): x16(5,b-): x3(5,a+): x17(6,b+): x5(U): x14(6,a+): x7(Ø): x18(5,b-): x9(U): x13(6,a+): x11(8,b+): R1: x10(6,a-): x18(5,a-): x6(5,a-): x17(6,a-): x16(5,a-) ■

There are a total of four right-variant substitution instances for the gross-schema, Fig. 34, of Osage 5-Diamonds. These are obtained, respectively, by replacing the red-underlined phrase -- >>R2: >>R5 -- in the construction (page 74) by <<R2: <<R5; <<R2: >>R5; and >>R2: <<R5: that is, by varying the directions of the twists on R2∞ and R5∞. The schemata resulting from these four distinct constructions are all identical to Fig. 34.A.i with the exception of the parities of the four (simple) boundary crossings -- i.e. x14, x15, x32, x33, -- of the fourth central diamond (labeled IV) of Fig. 34.A.i. Reproducing this local piece of the schema in each of these four cases, we have

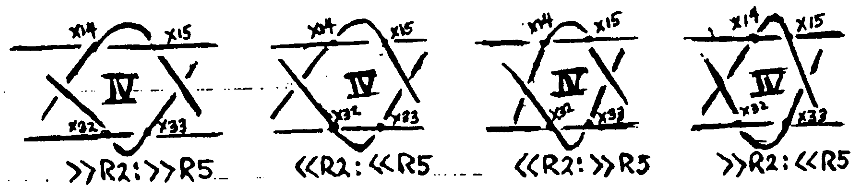


Fig. 34.A.ii: All substitution instances of Osage 5-Diamonds, right.

Thus the corresponding associated linear sequences are all identical to that of Fig. 34.A.i (page 75, top) with the exception of the eight entries underlined in red therein. Reproducing this schema in each of these four cases -- using the "subsequence suppression" convention -- we obtain, respectively,

>>R2: >>R5 { |⇒ ... x14(Ø): x15(U) ... x15(Ø): x14(U) ... x32(U): x33(Ø) ... x33(U): x32(Ø) ... ■

<<R2: <<R5 { |⇒ ... x14(U): x15(Ø) ...x15(U): x14(Ø) ... x32(Ø): x33(U) ... x33(Ø): x32(U) ... ■

<<R2: >>R5 { |⇒ ... x14(Ø): x15(U) ... x15(Ø): x14(U) ... x32(Ø): x33(U) ... x33(Ø): x32(U) ... ■

>>R2: <<R5 { |⇒ ...x14(U): x15(Ø) ... x15(U): x14(Ø) ... x32(U): x33(Ø) ... x33(U): x32(Ø) ... ■

Turning, now, to the complex-crossing schema for this ("right") situation -- Fig. 34.A -- we have

Fig. 34.A		Fig. 34.A.i
x8	⟶	x14, x15
x18	⟶	x32, x33

Thus the four corresponding (complex) linear schema associated to Fig. 34.A are all identical to that appearing on page 75, bottom, with the exception of the four entries underlined in red. Using "subsequence suppression" we obtain, respectively,

>>R2: >>R5 { |⇒ ... x8(5,a+) ... x8(5,b+) ... x18(5,b-) ... x18(5,a-) ...
<<R2: <<R5 { |⇒ ... x8(6,a+) ... x8(6,b-) ... x18(6,b+) ... x18(6,a-) ...
<<R2: >>R5 { |⇒ ... x8(5,a+) ... x8(5,b+) ... x18(6,b+) ... x18(6,a-) ...
>>R2: <<R5 { |⇒ ... x8(6,a+) ... x8(6,b-) ... x18(5,b-) ... x18(5,a-) ...

We see that a full (360°) twist of R2 [on Fig. 29.VII] creates complex crossing x18 of Fig. 34.A, while a full twist of R5 creates x8. Further, >>R2 forces x18 to be an X_5-type crossing, while <<R2 forces x18 to be an X_6-type crossing; similarly, >>R5 forces x8 to be an X_5-crossing, while <<R5 forces x8 to be an X_6-crossing. This may be directly verified by a complete crossing-analysis, proceeding from, e.g.

Fig. 29.VII: >>R2: >>R5#|

to Fig. 34.A.i in five steps ~~(analogous to the original analysis of the construction of Osage Diamonds)~~, etc. We shall content ourselves with the above discussion of Osage 5-Diamonds, right, and briefly turn our attention to the case of Osage 5-Diamonds, left (Fig. 34.B).

As an introduction to Osage 5-Diamonds, left, consider the manipulation sequence

O.A:□1| $\underrightarrow{1}$(5f)# $\overrightarrow{1}$(2f)#□5| $\overleftarrow{5}$(1f)#□1| >>L2: >>L5: $\overrightarrow{1}$(5n)#| $\overrightarrow{1}$(2n)#
N1|:: $\overleftarrow{2}$↓(1-△s): <2(#)□5|(palms away).

This produces the string-position whose schema is

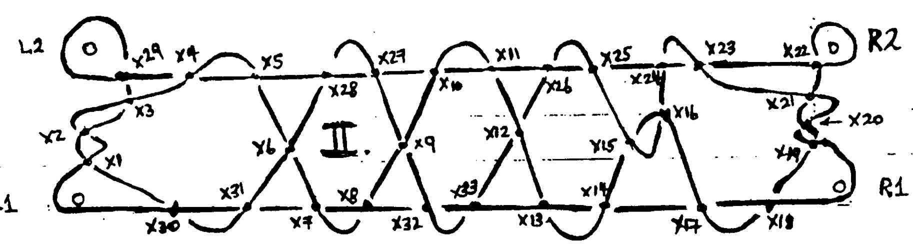


Fig. 34.B.i: Osage 5-Diamonds, left (>>L2: >>L5).

This is a fine-schema substitution instance of Fig. 34.B; i.e. a "left" variant, whose associated linear sequence is

|⇒ L1: x1(∅): x2(U): x3(∅): x4(∅): x5(U): x6(U): x7(∅): x8(U): x9(∅): x10(∅): x11(U): x12(∅): x13(U): x14(∅): x15(U): x16(∅): x17(∅): x18(U): x19(U): x20(∅): x21(U): x22(U): R2: x22(∅): x23(U): x24(∅): x25(U): x26(∅): x11(∅): x10(U): x27(U): x28(∅): x5(∅): x4(U): x29(∅): L2: x29(U): x3(U): x2(∅): x1(U): x30(U): x31(∅): x6(∅): x28(U): x27(∅): x9(U): x32(∅); x33(U): x12(U): x26(U): x25(∅): x15(∅): x16(U): x24(U): x23(∅): x21(∅): x20(U): x19(∅): R1: x18(∅): x17(U): x14(U): x13(∅): x33(∅): x32(U): x8(∅): x7(U): x31(U): x30(∅) ■

There are a total of four left-variant substitution instances for the gross schema, Fig. 34, of Osage 5-Diamonds. These are obtained, respectively, by replacing the red-underlined phrase -- >>L2: >>L5 -- in the construction above by <<L2: <<L5; <<L2:>>L5; and >>L2:<<L5: that is, by varying the directions of

the twists on L2∞ and L5∞. The schemata resulting from these four distinct constructions are all idential to Fig. 34.B.i with the exception of the parities of the four (simple) boundary crossings -- i.e. x7, x8, x27, x28 -- of the second central diamond (labeled II) of Fig. 34.B.i. Reproducing this local piece of the schema in each of the four cases, we find

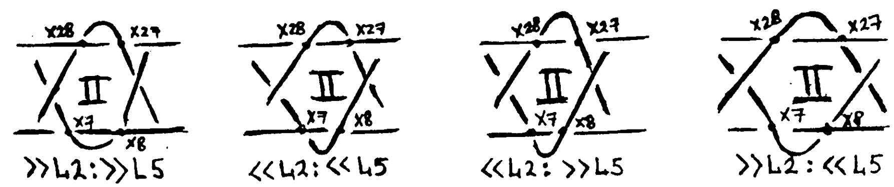


Fig. 34.B.ii: All substitution instances of Osage 5-Diamonds, left.

And, considering the complex-crossing schema for this ("left") situation -- Fig. 34.B -- we have

Fig. 34.B		Fig. 34.B.i
x15	⟶	x27, x28
x4	⟶	x7, x8

and we see that a full(360°) twist of L2 [of Fig. 29.VII] creates complex crossing x4 of Fig. 34.B, while a full twist of L5 creates x15. Further, >>L2 forces x4 to be an $\mathbf{X}_6$-type crossing, while <<L2 forces x4 to be an $\mathbf{X}_5$-type crossing; similarly, >>L5 forces x15 to be an $\mathbf{X}_6$-type crossing, while <<L5 forces x15 to be an $\mathbf{X}_5$-type crossing. Thus the situation for Osage 5-Diamonds, left, is seen to be entirely symmetric to the corresponding case of Osage 5-Diamonds, right, whence both analyses may be considered to be complete.

It should, also, be clear from a combination of the above right- and left-specific analyses, that the manipulational sequence,say,

O.A:□1| $\underrightarrow{1}$(5f)# $\overrightarrow{1}$(2f)#□5| $\overleftarrow{5}$(1f)#□1| >>R2:>>R5:>>L2>>L5; $\overrightarrow{1}$(5n)#|
$\overrightarrow{1}$(2n)# N1|:: $\overleftarrow{2}$↓ (1-△s): <2(#)□5|(palms away)

produces a string-position with six central diamonds, all (five) of whose mutual, central, boundary-crossings are simple. In fact, we produce

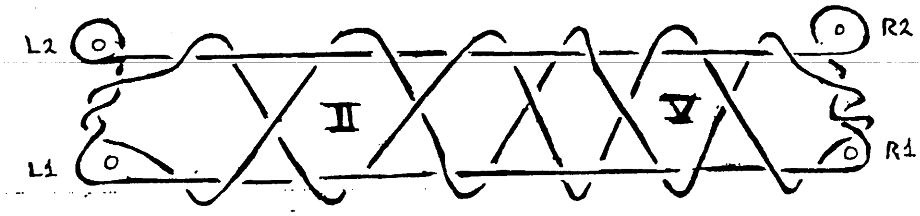


Fig. 35: Osage 6*- Diamonds

which we shall call "Osage 6^*-Diamonds". We shall not go on to label crossings in Fig. 35, et cetera, as all analyses collapse to the various cases of Osage 5-Diamonds, with respect to (the red-labeled) central diamonds II and V, the remaining crossings being constant, and immediately derivable therefrom. Further, there are a total of $16 = 2^4$ such figures (Osage 6^*-Diamonds), corresponding to the various directions of the twists on L2, L5, R2, R5. These are obtained, respectively, by replacing the red-underlined phrase --(1)>>R2:>>R5:>>L2>>15 -- in the construction (page 78) by

(2) >>R2:>>R5:>>L2:<<L5,
(3) >>R2:>>R5:<<L2:>>L5,
(4) >>R2:>>R5:<<L2:<<L5,
(5) >>R2:<<R5:>>L2>>L5,
(6) >>R2:<<R5:>>L2:<<L5,
(7) >>R2:<<R5:<<L2:>>L5,
(8) >>R2:<<R5:<<L2:<<L5,
(9) <<R2:>>R5:>>L2:>>L5,
(10) <<R2:>>R5:>>L2:<<L5,
(11) <<R2:>>R5:<<L2:>>L5,
(12) <<R2:>>R5:<<L2:<<L5,
(13) <<R2:<<R5:>>L2:>>L5,
(14) <<R2:<<R5:>>L2:<<L5,
(15) <<R2:<<R5:<<L2:>>L5,
(16) <<R2:<<R5:<<L2:<<L5.

Finally, after our comprehensive discussion of the various cases of Osage 5-Diamonds, the analysis of these 16 variations of Osage 6^*-Diamonds must be considered to be complete.

Another method of constructing six diamonds in the Osage tradition is by affixing a magical prefix to the construction of Osage Diamonds. In symbols,

this prefix is the phrase

$\underline{O}.A{:}\square 1|\ \underset{\rightarrow}{1}(\overline{5f})\#\ \overrightarrow{1}(\underline{2f})\#\square 5|\ \overleftarrow{5}(\underline{1f})\#\square 1|$

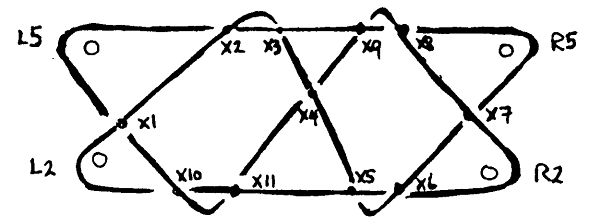


Fig. 36: Osage prefix.

$\mapsto$ L2: x1(∅): x2(∅): x3(U): x4(∅): x5(U): x6(∅): x7(U): R5: x8(U): x9(∅): x3(∅): x2(U): L5: x1(U): x10(∅): x11(U): x4(U): x9(U): x8(∅): x7(∅): R2: x6(U): x5(∅): x11(∅): x10(U) ■

When this figure is taken as the starting point -- i.e. in place of Fig. 29.II for the Osage Diamonds construction, we obtain a string-position we shall call Osage 6-Diamonds:

$\underline{O}.A{::}\square 1|\ \underset{\rightarrow}{1}(\overline{5f})\#\ \overrightarrow{1}(\underline{2f})\#\square 5|\ \overleftarrow{5}(\underline{1f})\#\square 1|{::}\ \underset{\rightarrow}{1}(\underline{5f})\#\ \overrightarrow{1}(\underline{2f})\#\square 5|\ \overleftarrow{5}(\underline{1f})\#$
$\square 1|\ \overrightarrow{1}(\underline{5n})\#|\ \overrightarrow{1}(\underline{2n})\#\ N1|{::}\ \overleftarrow{2}\downarrow(1-\triangle_{S}){:}\ <2(\#){:}\square 5|$ (palms away).

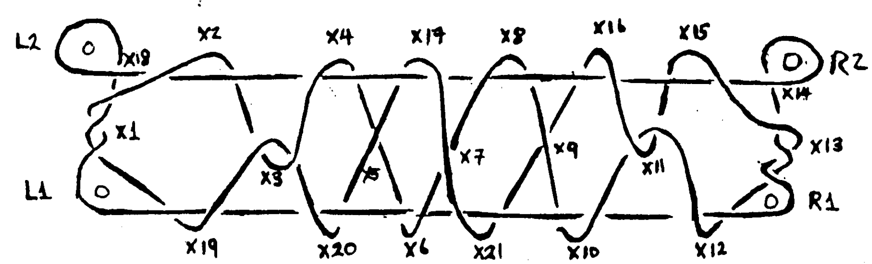


Fig. 37: Osage 6-Diamonds,

Fig. 37 has been labeled for complex crossing-types -- eliding the simple-crossing labeling -- and has associated complex linear sequence

⊨> L1: x1(7,a+): x2(5,a+): x3(5,b-): x4(5,a+):x5(U): x6(5,b-): x7(U):
x8(5,a+): x9(∅): x10(5,b-): x11(6,a+): x12(6,b+): x13(8,a+): x14(U):
R2: x14(∅): x15(6,b-): x16(6,b-): x8(5,b+):x17(6,b-): x4(5,b+):
x2(5,b+): x18(∅): L2: x18(U): x1(7,b+): x19(5,b-): x3(5,a+): x20(6,b+):
x5(∅): x17(6,a+): x7(∅): x21(6,b+): x9(U): x16(6,a+): x11(6,b+):
x15(6,a+): x13(8,b+):R1: x12(6,a-): x10(5,a-): x21(6,a-): x6(5,a-):
x20(6,a-): x19(5,a-) ■

Here -- to oversimplify the matter, slightly -- the two diamonds of the prefix maintain themselves, centrally, throughout the Osage Diamonds construction, and end up flanked by the two (symmetric) lateral halves of that figure.(2) And because the extreme lateral diamonds are constructed by the Osage-extension, itself, this figure shares the previously discussed multiplicative process with the parent figure; specifically, by way of example,

O.A::□1| 1↳(5f)#1⃗(2f)#□5| ⃖5(1f)#□1|:: 1↳(5f)# 1⃗(2f)#□5| ⃖5(1f)#□1|
>>R2:>>R5: 1⃗(5n)#| 1⃗(2n)# N1|:: ⃖2↓(1-△): <2(#):□5|(palms away)

produces an Osage 7-Diamonds, right -- whose rightmost central crossing is now simple -- by exact analogy with the passage from Osage Diamonds to Osage 5-Diamonds, right. Further, any of the four possible combinations of full twists on R2 and R5 (see page 75, bottom) produce a right-variant of Osage 6-Diamonds, in a manner that is -- by now -- well understood.

Similarly, replacing the red-underlined phrase, above, by ">>L2:>>L5"-- or, indeed, any of the four possible combinations of full twists on L2 and L5 -- we produce the figure(s) Osage 7-Diamonds, left.

And finally, replacing the red-underlined phrase, above, by ">>R2:>>R5: >>L2:>>L5" -- or, indeed, by any of the 16 distinct full-twist combinations listed on page 79 -- we produce the figure Osage 8*-Diamonds, all of whose mutual, central, boundary-crossings are simple. Stated in brief, the magical Osage-prefix is completely compatible with the (full-twist) augmentation procedure for Osage Diamonds, previously discussed. And that's not the only thing about this "prefix" which is magical.

Let us consider the string-position resulting from -- loosely speaking -- doubling the prefix. Specifically, we consider the phrase

O.A:□1| [1↳(5f)# 1⃗(2f)#□5| ⃖5(1f)#□1|]^2.

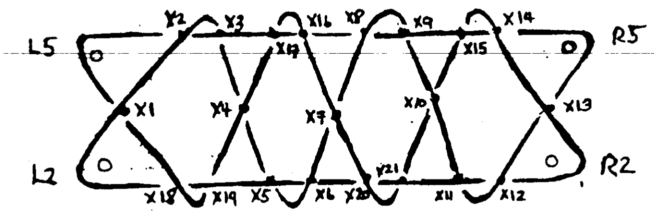


Fig. 38: Osage [prefix]^2.

⊨> L2: x1(∅): x2(∅): x3(U): x4(U): x5(U): x6(∅): x7(U): x8(∅): x9(U):
x10(∅): x11(U): x12(∅): x13(U): R5: x14(U): x15(∅): x9(∅): x8(U):
x16(U): x17(∅): x3(∅): x2(U): L5: x1(U): x18(∅): x19(U): x4(∅):
x17(U): x16(∅): x7(∅): x20(∅): x21(U): x10(U): x15(U): x14(∅):
x13(∅): R2: x12(U): x11(∅): x21(∅): x20(U): x6(U): x5(∅): x19 (∅):
x18(U) ■

At first glance, this is simply a 4-diamond analogue of Fig. 36, and -- at least functionally with respect to the Osage Diamonds construction -- it is! That is, when Fig. 38 is taken in place of Fig. 29.II in the Osage Diamonds construction, we obtain Osage 8-Diamonds:

O.A: 1| [1↳(5f)# 1⃗(2f)#□5| ⃖5(1f)#□1|]^2: 1↳(5f)# 1⃗(2f)#□5| ⃖5(1f)#□1|
✱ 1⃗(5n)#| 1⃗(2n)# N1|:: ⃖2↓(1-△): < 2(#):□5|(palms away).

Further, the Osage-[prefix]^2 is also compatible with the previous augmentation procedures: specifically, any combination of full twists on R2 and R5 -- inserted at the ✱ appearing in the above construction of Osage 8-Diamonds -- produces an Osage 9-Diamonds, right; any combination of full twists on L2 and L5, inserted at ✱, produces an Osage 9-Diamonds, left; and any combination of full twists on R2, R5, L2, L5, inserted at ✱, produces an Osage 10*-Diamonds. Et cetera.

If, now, we extend our "power"-notation for a bracketed expression from the string-figure calculus to include n=0 and n=1, viz.

[....]^0 ≡ do not perform the manipulations indicated within the brackets

[....]^1 ≡ perform the manipulations indicated within the brackets exactly once

then we may combine the results of our previous investigations into a collection of assertions about an extremely general Osage-type construction③:

|⇒ O.A:□1| $\left[\right.$ 1→(5f)# 1⃗(2f)#□5| 5⃖(1f)#□1| $\left.\right]^n$:

1→(5f)# 1⃗(2f)#□5| 5⃖(1f)#□1|: (>>R2:>>R5)^i : (>>L2:>>L5)^j :

1⃗(5n)#| 1⃗(2n)# N1|:: 2⃖↓(1-△s): < 2(#):□5|(palms away)

where n=0,1,2,3,4, ...; i=0,1; and j=0,1. The total number of central diamonds produced by this construction is 2n+(i+j)+4. Specifically, in tabular form

# Diamonds	n	i	j	Name
4	0	0	0	Osage Diamonds
5	0	1	0	Osage 5-Diamonds, right
	0	0	1	Osage 5-Diamonds, left
6	0	1	1	Osage 6*-Diamonds
	1	0	0	Osage 6-Diamonds
7	1	0	1	Osage 7-Diamonds, right
	1	1	0	Osage 7-Diamonds, left
8	1	1	1	Osage 8*-Diamonds
	2	0	0	Osage 8-Diamonds
9	2	1	0	Osage 9-Diamonds, right
	2	0	1	Osage 9-Diamonds, left
10	2	1	1	Osage 10*-Diamonds
	3	0	0	Osage 10-Diamonds
11	3	1	0	Osage 11-Diamonds, right
	3	0	1	Osage 11-Diamonds, left
12	3	1	1	Osage 12*-Diamonds
	4	0	0	Osage 12-Diamonds
13	4	1	0	Osage 13-Diamonds, right
	4	0	1	Osage 13-Diamonds, left

et cetera. Since the expression 2n+(i+j)+4 runs over all natural numbers greater than or equal to 4, as n, i, and j run over their respective ranges, it follows that for every natural number n≥4 there exists a string-figure "Osage n-Diamonds"; in fact, for n>4, there exists more than one such (distinct) figure. And our analysis of this sequence of figures gives much characteristic information about the individual elements of the sequence. Rather than pursue this further, we now turn our attention to the case n<4, for completeness of the discussion.

The figure Osage 2-Diamonds is constructed by the manipulational sequence

O.A:□1|✱ 1⃗(5f)#| 1⃗(2n)# N1| 2⃖↓(1-△s): < 2(#):□5|(palms away).

Note that the third movement has "5f" for its argument, in contradistinction to the "usual" Osage-extension.

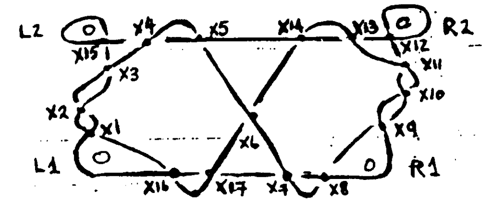


Fig. 39: Osage 2-Diamonds

|⇒ L1: x1(∅): x2(U): x3(∅): x4(∅): x5(U): x6(∅): x7(∅): x8(U): x9(U): x10(∅): x11(U): x12(U): R2: x12(∅): x13(U): x14(∅): x5(∅): x4(U): x15(∅): L2: x15(U): x3(U): x2(∅): x1(U): x16(U): x17(∅): x6(U): x14(U): x13(∅): x11(∅): x10(U): x9(∅): R1: x8(∅): x7(U): x17(U): x16(∅) ■

We now attempt the full-twist augmentation procedure on this figure, by inserting the appropriate twisting move at the ✱ appearing in the above construction. The results of this experiment include

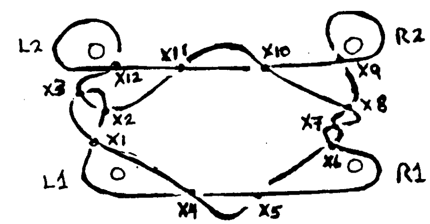


Fig. 40: Osage 1-Diamond (<<R2:<<R5).

|⇒ L1: x1(U): x2(∅): x3(U): x1(∅): x4(∅): x5(U): x6(U): x7(∅): x8(U): x9(U): R2: x9(∅): x10(U): x11(∅): x12(∅): L2: x12(U): x3(∅): x2(U): x11(U): x10(∅): x8(∅): x7(U): x6(∅): R1: x5(∅): x4(U) ■

Also

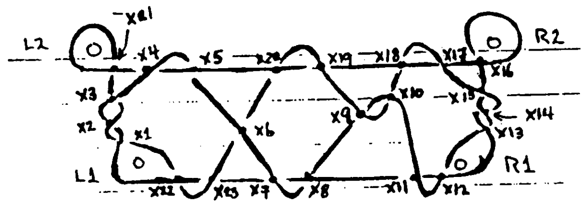


Fig. 41: Osage 3-Diamonds (>>L2:>>L5).

|=> L1: x1(Ø): x2(U): x3(Ø): x4(Ø): x5(U): x6(Ø): x7(Ø): x8(U): x9(U): x10(Ø): x11(Ø): x12(U): x13(U): x14(Ø): x15(U): x16(U): R2: x16(Ø): x17(U): x18(Ø): x19(U): x20(Ø): x5(Ø): x4(U): x21(Ø): L2: x21(U): x3(U): x2(Ø): x1(U): x22(U): x23(Ø): x6(U): x20(U): x19(Ø): x9(Ø): x10(U): x18(U): x17(Ø): x15(Ø): x14(U): x13(Ø): R1: x12(Ø): x11(U): x8(Ø): x7(U): x23(U): x22(Ø) ■

We remark that, for example, the inserted phrase "<<L2:<<L5" at ✱ produces Fig. 41 except that the parities of the (simple) crossings x6, x7, x8, x19, x20 of that figure are reversed. Inserting the phrase ">>R2:>>R5" produces a degenerate "two-diamond-like" figure, whose leftmost diamond twists doubly about each of the near (1n) and far (2n) strings. Finally, we remark that insertion of the phrase ">>R2:>>R5:>>L2:>>L5" at ✱ in the above construction produces a 4-diamond-type of figure all of whose mutual, central, boundary crossings are simple. We may refer to this figure as Osage 4*-Diamonds, by analogy with our previous discussion(cf. augmentation procedure, page 79, Fig. 35). In general then, -- as in the case for the Brochoi (see page 40) -- for each natural number, n, we may produce a string-figure "Osage n-Diamonds", length of string permitting.

There are many more figures surrounding the Osage Diamonds complex which are interesting, beautiful, and directly analyzable by the methods developed here. We shall content ourselves with a single example thereof. Suppose that we preface the Osage 2-Diamonds construction with the Magical Osage-prefix, i.e.

O.A: □1| 1→(5̄f)# 1→(2f)# □5| 5←(1f)# □1|: 1→(5f)#| 1→(2n)# N1| 2←↓(1-△s): < 2 (#): □5| (palms away).

The resulting string-position is that of Fig. 28; that is, Osage Diamonds.

In fact, a "Heart"-sequence analysis of the present construction reveals that it differs from the "Heart"-sequence for the previous construction only as to at what point in the manipulation sequence the twist(s) on the 5∞'s occurs -- a matter of indifference to the (schema for) the final figure. Our position, in these notes, is that this construction should not be called "Osage Diamonds" but, rather, an equivalent of that figure -- that is, that a string-figure consists of both its construction and its final design. Thus, the final design of the string-figure Osage Diamonds admits two distinct -- but closely related (via the "Heart"-sequence) -- constructions. We shall have more to say about this matter when it arises subsequently, in context.

We conclude our discussion of Osage Diamonds with a presentation of two "derivative" string-figures -- one, amusing and "catchy"; the other beautiful and profound. The first is a simple continuation -- requiring some dexterity -- of the parent figure, called "Flies on Flypaper" [M. & M. Jackson: String Games, pages 26-27]:

Construct Osage Diamonds and -- rotating the figure 90° in its plane (either direction) -- lay figure down on a flat surface in front of you, and release both hands from the string.

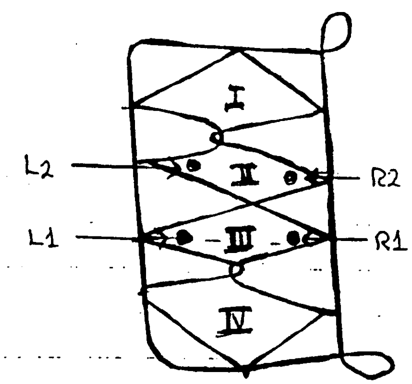


Fig. 42.A: Flies, preparatory to "catching" them.

Turn hands with fingers pointing down, and insert 2 in diamond II and 1 in diamond III, as indicated in Fig. 42.A, above. Now, pulling crisply and sharply to the sides -- while simultaneously spreading 1 and 2 widely apart -- raise the figure off the flat surface to an extension in front of you, palms facing away from body.

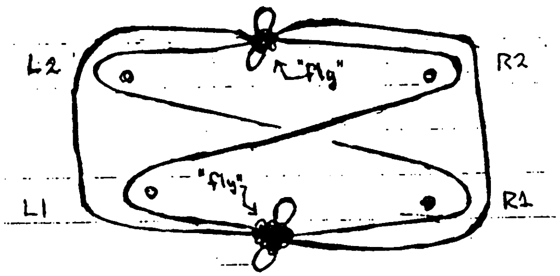


Fig. 42.B: Flies, "caught".

Depending on speed, timing, and coordination, you will catch 0, 1, or 2 flies, as above.

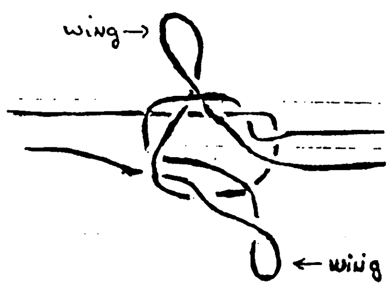


Fig. 42.C: Detail of "Fly" (knot).

The second figure, "Osage Diamonds on a Bridge", is contemporarily known as "Amanohashidate" [Bridge to Heaven -- cf. L. Dickey: String Figures From Hawaii, Fig. 66; Ana Paakai (Salt Cave)].

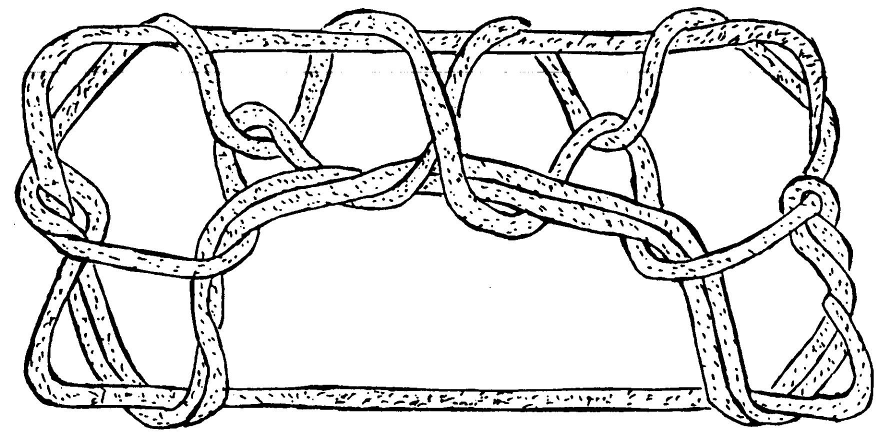


Fig. 43: Bridge to Heaven.

O.A: $\overleftarrow{2345}$ (1n)#□1| $1_{\rightarrow}$(5f)# $\vec{1}$(2f)#□5| $\overleftarrow{5}$(1f)#□1| $\vec{1}$(5n)#| $\vec{1}$(2n & 2345n)# N(ℓ1∞):: $\overleftarrow{2}$↓(1-△S): <2 (#):□5|(palms away): There is now a far hanging-loop over the 2n string; toss this towards you over central figure so that it crosses the 1∞ on either side: hook $\vec{1}$ over this loosely hanging string: $\vec{1}$↓(1∞) -- allowing these (former) 1∞'s to slip off thumbs |(palms away).

Notes on the construction: ① The 2345∞, created at step 2 of the construction, must remain on 2345 during subsequent manipulations; i.e. do not allow this to slip over 1 down to wrists (as in Fig.26, The Sun -- page 48). ② The move "N(ℓ1∞)" means, of course, to release ℓ1∞ from 1 over both u1∞'s. ③ We shall, later, introduce notation to encompass "hooked" finger-positions and "hanging" loops; this will obviate the final littoral steps in the construction, and reduce the entirety of the manipulation sequence to the Calculus.

We remark that any of the Osage n-Diamonds figures may be put on a bridge. Below is the result for the case n=6:

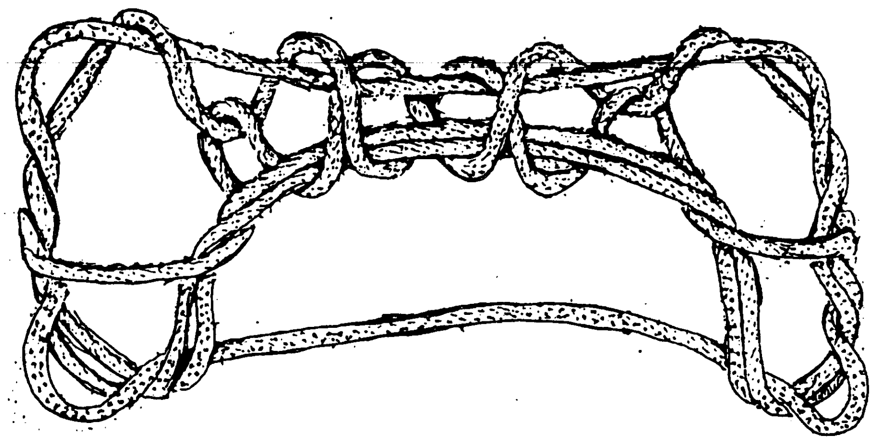


Fig. 44: Osage 6-Diamonds, "on a bridge".

Finally, we mention that many figures may be "put on a bridge": a contemporary example, "Earthrise" (below), is -- in reality -- our old friend, the Brochos (plus Osage-extension ≡ Nelo) on a bridge. The construction, below, is not precisely "as collected" but, rather, represents the author's later attempt to reconstruct the figure.

O.A:□5| ⇆5 (2f)#| O.A(with 3): ⇆2345(1n)#□1| 1→(5f)#□5|: 3⃗∞ →5: 1⃗ (5n)#
⇆2 (1f)#□1| 2∞ →1 3: 1⃗ (2n & 2345n)# N(ℓ1∞):: ⇆2↓(1-△S): 2⃗↓(2∞)::
hook 3 down over that string now held down by 2, where that string
crosses the 3∞: 3⃗↓(3∞):: < 2 & 3 [releasing former 2∞ & 3∞]:□5|
(palms away): toss far hanging loop towards you to near side of figure:
hook 1⃗ over near hanging string: 1⃗↓(1∞)|(palms away). Extend fingers
widely, 3 well above 2:□3|.

Notes on the construction: The role of 3 in the latter stages of this construction is to lift figure well up onto "bridge", ensuring a clean extension. The final release of 3 drops central "Sun" figure onto this bridge with no "sag" in lateral support strings. As is often the case with more complicated figures,

some practice and experimentation is necessary to achieve the intermediate string-tensions necessary for a crisp final extension. We believe that this figure is well worth the time and effort involved.

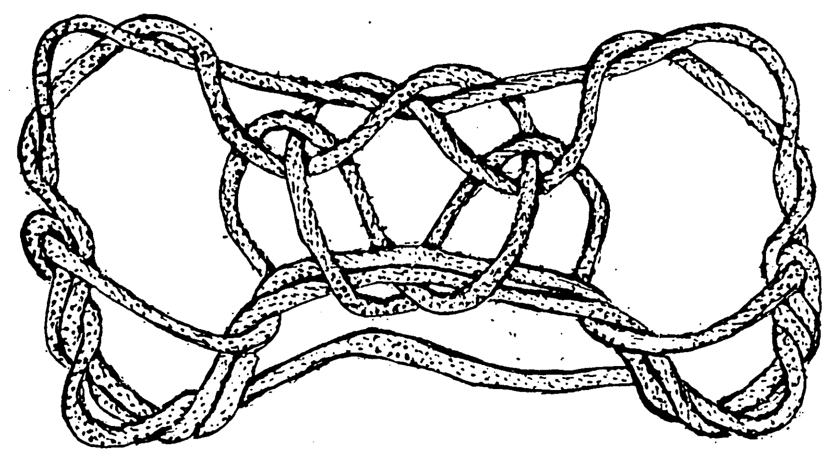


Fig. 45: Earthrise.

---- End Osage Diamonds Discussion ----

OSAGE DIAMONDS NOTES

① (page 68) Note that the 𝕏-crossings $\mathbb{X}_7$, $\mathbb{X}_8$ may equally well be represented in the 𝕏-Dictionary by their symmetric self-equivalents

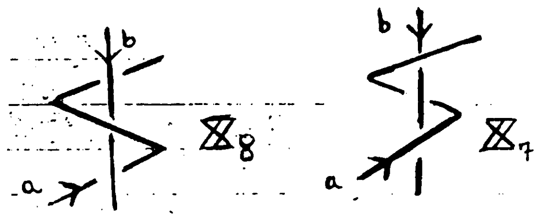


Fig. 32': Alternate crossing patterns

e.g. by "straightening out" b, and examining a's twining about b.

② (page 81) For example, tie (any kind of) knot at the crossing, x1, of Fig. 29.II:

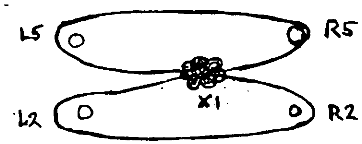


Fig. 29.II': Crossing x1 replaced by a knot

Now complete the construction of Osage Diamonds. The result is Osage Diamonds -- i.e. Fig. 28 -- with the central crossing, x10, replaced by the knot in question.

③ (page 83) Note this is not the most general construction relative to our previous investigations. For that, one would replace line 3 of the given construction by

$$(>>R2)^{i_1}(<<R2)^{i_2}(>>R5)^{k_1}(<<R5)^{k_2}(>>L2)^{j_1}(<<L2)^{j_2}(>>L5)^{\ell_1}(<<L5)^{\ell_2}$$

where $0 \le i_1 + i_2 = k_1 + k_2 \le 1$ and $0 \le j_1 + j_2 = \ell_1 + \ell_2 \le 1$; that is, a given finger may twist (or not) in either direction, at most once. Further, R2 suffers a twist if and only if R5 does; and L2 suffers a twist if and only if L5 does. We shall content ourselves with the easier (and less general) formula displayed in the text.

BIBLIOGRAPHY: OSAGE DIAMONDS

Abraham, A.J.: STRING FIGURES
Calabash Net — Fig. 49

Abraham, R.M.: EASY TO DO ENTERTAINMENTS AND DIVERSIONS WITH COINS, CARDS, STRING, PAPER AND MATCHES.
Fence — #147

Amir-Moéz, A.R.: MATHEMATICS AND STRING FIGURES.
Jacob's Ladder — Fig. 23,24

Amir-Moéz, A.R. & J.D. Hamilton: ART AND MATHEMATICS OF STRING FIGURES.
Jacob's Ladder — Fig. 13

Andersen, J.C.: MAORI STRING FIGURES.
Tahitinui — #12

Ariki, T.: AYATORI NYŪMON.
Yodan Bashigo (Four rung ladder) — p. 14

Armstrong, A.: MAORI GAMES AND HAKAS.
Tawhiti Figure — #6

Ball, W.W. Rouse: FUN WITH STRING FIGURES.
Fishing Net — #12

Beaglehole, P.: STRING FIGURES FROM PUKAPUKA.
Te-Kivi-A-Te-Kakai (The Kiva of the People) — Fig. 41

Blixen, O.: FIGURAS DE HILO TRADICIONALES DE LA ISLA DE PASCUA.
Oho Kero — Fig. 13

Brewster, P.G.: SOME STRING FIGURES AND TRICKS FROM THE UNITED STATES.
Jacob's Ladder — Fig. 1

Campbell, J.: THE WAY OF THE ANIMAL POWERS
No name (photo) — #181

Campbell, R.: LA HERENCIA MUSICAL DE RAPANUI.
Oho Kero — N.O.B. 1, p.418

Cunnington, W.A.: STRING FIGURES AND TRICKS FROM CENTRAL AFRICA.
No name — #1
Sumbo (Fishing Net) — #3
No name — #13

Davidson, D.S.: SOME STRING FIGURES OF THE VIRGINIA INDIANS.
Jacob's Ladder — Fig. 142

de Palavecino, M.D.M.: ALGUNOS JUEGOS INFANTILES CRIOLLOS E INDIGENAS DEL NORTE ARGENTINO.
No name (photo) — Plate 2

Dickey, L.A.: STRING FIGURES FROM HAWAII.
Four Eyes A, Method #1,2,3 — pp. 27-30

Elffers, J. & M. Schuyt: CAT'S CRADLES AND OTHER STRING FIGURES.
Diadem — pp. 148-150

Emory, K.P. & H.C. Maude: STRING FIGURES OF THE TUAMOTUS.
Kahegahega — #64B

Farabee, W.C.: THE CENTRAL CARIBS.
Waitai (Fish Trap) — #1

Gardner, M.: MATHEMATICAL GAMES: THE ANCIENT LORE OF STRING PLAY.
Jacob's Ladder — p. 150

Gryski, C.: CAT'S CRADLES, OWL'S EYES.
Fishnet — pp. 40-43

Haddon, A.C.: STRING FIGURES FROM SOUTH AFRICA
Amadande (East African Fish Trap) — #5

Haddon, K.: CAT'S CRADLES FROM MANY LANDS.
Calabash Net — #23
Imperial Pigeon — #46

Haddon, K. & H.A. Treleaven: SOME NIGERIAN STRING FIGURES.
A Bridge — #7

Hagen, G. von: DIE BANA.
No name (photo) — Fig. 13

Held, G.J.: THE PAPUANS OF WAROPEN.
Manggamusa (The Axe) — p. 366

Hornell, J.: STRING FIGURES FROM FIJI AND WESTERN POLYNESIA.
Thiukambi — Fig. 43

STRING FIGURES FROM SIERRA LEONE, LIBERIA, AND ZANZIBAR.
Gama Nani (Four Eyes) — Mende 10

STRING FIGURES FROM GUJARAT AND KATHIAWAR.
Machhi Jal (Fishing Net) — #27

STRING FIGURES FROM THE ANGLO-EGYPTIAN SUDAN.
Shabaka (Fishing Net) — #11

Howard, D.: STRING GAMES OF AUSTRALIAN CHILDREN.
Harbour Bridge (photo) — p. 385

Jackson, M. & M. & K. WATKINS: STRING GAMES.
The Bridge — Fig. 28

Jayne, C.F.: STRING FIGURES.
Osage Diamonds — Fig. 50

Junod, H.A.: THE LIFE OF A SOUTH AFRICAN TRIBE, Vol. I.
No name (drawing) — p. 176

Kamiya, K.: VARIATIONS OF JACOB'S LADDERS

Kattentidt, I.: FADENSPIELE AUS ALLER WELT.
Die Himmelsleiter — pp. 8-9

Koch-Grünberg, T.: VOM ROROIMA ZUM ORINOCO, Vol. III.
Kauăra'tŭ Menŭdu — Tafel 55, #1

Leeming, J.: FUN WITH STRING.
Osage Diamonds — pp. 153-154

Lutz, F.E.: STRING FIGURES FROM THE PATOMANA INDIANS OF BRITISH GUIANA.
The River — Fig. 10

Martinez-Crovetto, R.: JUEGOS DE HILO DE LOS ABORIGENES DEL NORTE DE PATAGONIA
Estribo en Escalera — #12

Maude, H.C.: SOLOMON ISLANDS STRING FIGURES.
Two Men Cut a Beam (cont.) — #50

Maude, H.C. & H.E.: STRING-FIGURES FROM THE GILBERT ISLANDS.
Aua Ni Ba-n Itai (Four Itai Leaves) — #20

McCarthy, F.D.: THE STRING FIGURES OF YIRRKALLA.
Boobook Owl — #42

Métraux, A.: ETUDES SUR LA CIVILISATION DES INDIENS CHIRIGUANO.
Dessins de Serpent — Fig. 90, #3

QUELQUES JEUX DE FICELLE L'AMERIQUE DU SUD.
Quatre Etoiles — Fig. 2

Michelson, T.: ALGONQUIAN INDIAN TRIBES OF OKLAHOMA AND IOWA.
Bridge — Fig. 160

FOX PHYSICAL ANTHROPOLOGY.
Bridge — unpublished

Mindt-Paturi, F.R.: THREE STRING TRICKS FROM THE WOLOF TRIBE IN SENEGAL.
Four Eyes — p. 15

Nimuendaju, C.: THE TUKUNA
Hammock Ornament (photo) — Plate 10, c

Noguchi, H.: STRING FIGURES IN JAPAN, II.
Four Diamonds #31

Oorschot, A. van: TOUWFIGUREN.
Haarnet p. 13

Parkinson, J.: YORUBA STRING FIGURES.
Calabash Net #1

Ransom, W.R.: PASTIMES WITH STRING AND PAPER.
Flashed Fence p. 122

Robinson, A.: SOME FIGURES IN STRING FROM THE MAKUSHIS ON THE IRENG AND TAKUTU RIVERS.
Kuru-eri (A Fish) Fig. 14

Roth, W.E.: STRING FIGURES, TRICKS, AND PUZZLES OF THE GUIANA INDIANS.
Four Fish Marching #671

Rydén, S.: SOUTH AMERICAN STRING FIGURES.
Four Stars Fig. 12, #5

Saito, T.: AYATORI ITOTORI, Vol. II.
Yodan Bashigo (Four rung ladder) Cover, #3

Sbrzesny, H.: DIE SPIELE DER !KO-BUSCHLEUTE.
Die Hängematie (photo) Fig. 2

Senft, B. & G. NINIKULA FADENSPIELE AUF DEN TROBRIAND-INSELN, PAPUA-NEUGUINEA.
Die Hauptlingsfrau (Halfling's Woman) -cont. Fig. 58B

Stachewski, F.: DIE BANJANGI.
Essik p. 53, #6

Stannus, H.S.: THE WAYAO OF NYASALAND.
The Net Plate 23

Tessmann, G.: DIE BUBI AUF FERNANDO POO.
Diŏdi Fig. 162

Tracey, H.: STRING FIGURES (MADANDI) FOUND IN SOUTHERN RHODESIA.
Mamburi (Hunting Net) #4, picture 5

Walker, J.: CAT'S CRADLES AND OTHER TOPOLOGIES FORMED WITH A TWO-METER LOOP OF FLEXIBLE STRING (The Amateur Scientist).
Fishing Net p. 143

Wedgwood, C.H. & I. Schapera: STRING FIGURES FROM BECHUANALAND PROTECTORATE.
A Gate Fig. 2

Yuasa, S. & T. Ariki: TANOSHII AYATORI ASOBI.
Yodan Bashigo (Four rung ladder) #26

Appendix A: Two "deep" problems arising in passage from linear sequences to associated schema.

1. The discussion on page 72 apparently implicitly asserts that some perfectly reasonable-appearing candidates for linear sequences have no schema associated to them. We shall show this to be, indeed, the case via an example derived from the Osage Diamonds analysis. The following discussion is pitched at the "gross" level of organization, i.e. with all crossing "fine structure" suppressed: it will then follow that no schema corresponds to our given example for any fine-structure substitution instance.

To that end consider the "gross" linear sequence

↦ L1: x1: x2: x3: x4: x5: x6: x7: x8: x9: x10: R2: [x12: x11:] x4: x2: x13: L2: x13: x1: x14: x3: x15: x5: x12: x7: x11: x9: R1: x8: x6: x15: x14 ■

which is the linear sequence associated to Osage Diamonds (cf. page 69), except

i). All crossing fine-structure has been suppressed,

ii). Adjacent elements x11, x12 in the original sequence have been transposed ([ ], above).

We show that no ("gross") schema can correspond to this "gross" linear sequence.

Pursuing the 5-Step analysis for this sequence (cf. pages 69-72) we find, at Step 5, that if there is a schema associated to this sequence, it must look like

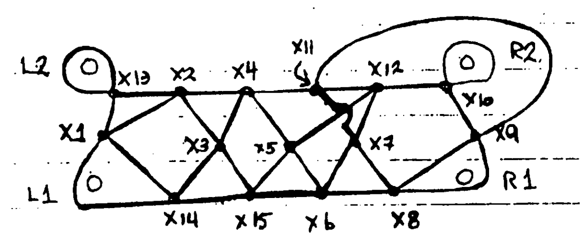


Fig. A1: Schema-so-far.

And we are left with the problem of connecting x11 to x7 without crossing any lines of the existing configuration (note the adjacent pair x7:x11 in the original linear sequence). Clearly this is not possible for this configuration of nodes and (connecting) lines as we've drawn them. The (big) question is: Is it possible to find a configuration of 19 labeled points, like this one, joined to each other, as above, in such a way that two lines cross only at a node of the configuration? In general, this is a hard problem -- but a branch of mathematics, called Graph Theory, has successfully addressed it. The following is an introduction to that beautiful subject.

By a graph we shall mean a collection of points -- called nodes -- and a collection of line-segments -- called lines -- joining some of these, in pairs. The nodes will be named (labeled) from the sets

$$\begin{cases} L1,\ L2,\ \ldots;\ R1,\ R2,\ \ldots \\ x1,\ x2,\ x3,\ \ldots \end{cases}$$

and -- if nodes xi and xj are joined by a line -- we shall denote this line by the unordered pair $\{xi, xj\}$, and say that nodes xi and xj are adjacent. In this case we write either

xi adj xj or xj adj xi;

i.e. "adjacency" is a symmetric relation. If there is no line joining nodes xi and xj, we say that xi and xj are nonadjacent, and write xi $\not{\text{adj}}$ xj. Finally, the total number of lines meeting a given node xi is called the degree of xi, and written deg(xi).

Now a linear sequence prescribes a set of nodes (the distinct elements of the sequence) and a set of adjacencies between these nodes. Specifically, if xi and xj occur in the given linear sequence, then

xi adj xj $\equiv$ xi and xj occur as adjacent entries somewhere in the given sequence.

Thus, in this case, the concept of "graphical adjacency" is a direct translation of the adjacency (juxtaposition) between sequence elements.

After these definitions and observations, the geometric configuration "Schema-so-far" of Fig. A1 is, clearly, a graph*:

*Technically -- because of the dual adjacencies between L2 and x13 (or R2 and x10) -- this is a "multigraph", a minor "nicety" of no consequence to our present discussion.

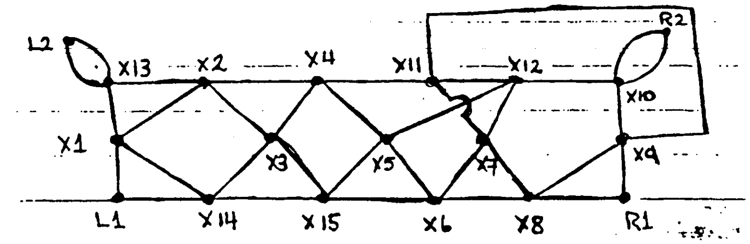


Fig. A2: The Graph of Schema-so-far.

A graph is said to be planar if it can be drawn in the plane in such a way that two lines cross only at a node of the graph. For example, consider the graph, K_4, in the following diagram:

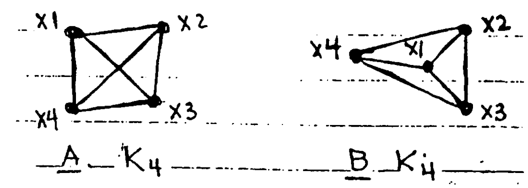


Fig. A3: The planar graph K_4.

Note that, in part A of Fig. A3, the graph K_4 has been drawn in such a way that two of its edges -- $\{x1, x3\}$ and $\{x2, x4\}$ -- cross at a point which is not one of the four nodes of the graph. Thus K_4 may be drawn in such a way that it does not "appear" to be planar. That K_4 is, indeed, planar is verified by the companion configuration, Fig.A3.B; this is a graph with exactly the nodes and adjacencies of Fig. A3.A -- i.e. K_4 -- and, since these define a graph, this is K_4 also. Finally since two lines of Fig. A3.B meet only at nodes of this graph, K_4 is planar -- by definition. We may think of node x1 of Fig. A3.A as being "slid" down into the triangle formed by nodes x2, x3, x4 to create Fig. A3.B; this is analogous to how strings slide across one another during construction of a figure.

We wish to show, returning to the example, that the graph of Fig. A2 is not planar and, hence, that the proposed schema of Fig. A1 is not a schema at all; as any drawing thereof entails strings which must intersect at nonexistent crossings [i.e. crossings not explicitly appearing in the original linear sequence]. Thus, no schema can correspond to that linear sequence, and the analysis of this example will be complete.

Two distinguished nonplanar graphs, K_5 and $K_{3,3}$, play a central role in our discussion of planarity:

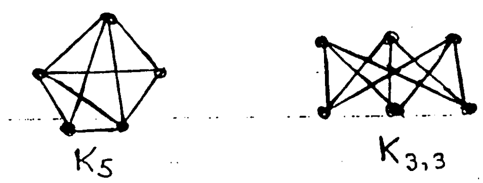


Fig. A4: The nonplanar graphs K_5 and $K_{3,3}$.

And we shall need to introduce two more definitions; the first is the idea of a subgraph of a given graph. This is any graph obtained from the given graph by any sequence of erasures of the latter's lines and/or nodes [Note: when a node is deleted, all lines incident to that node must also be deleted]. So, for example

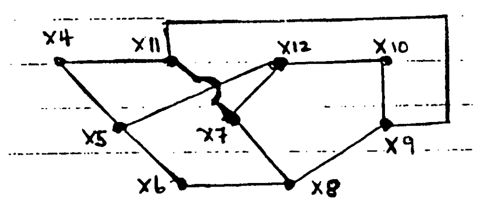


Fig. A5: A subgraph of Fig. A2

the graph of Fig. A5 is obtained from that of Fig. A2 by the erasure of nodes

L1, L2, R1, R2, x1, x2, x3, x13, x14, x15

and of lines {x6, x7}, {x11, x12}; hence this graph is a subgraph of the graph of Fig. A2 -- by definition.

The second (and last!) concept needed to finish our present foray into the subject of planarity of graphs is that of homeomorphism between two graphs. This is defined in terms of insertion and deletion of nodes of degree 2 into a given graph.

Fig. A6: Insertion of a node (x3) of degree 2.

To insert a node of degree 2 into a given graph, we create a new node on any existing line of the graph. Similarly,

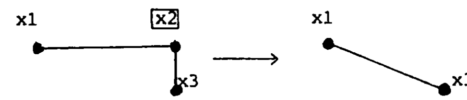


Fig. A7: Deletion of a node (x2) of degree 2.

To delete a node of degree 2 from a given graph, we choose any node of degree 2 in the graph (e.g. node x2 in Fig. A7, above), erase it, and create a new adjacency between the two nodes which were adjacent to the given node (e.g. nodes x1, x3 in Fig. A7, above; the new adjacency {x1, x3} is created in the deletion graph). We remark that, so defined, the above two operations on graphs are "inverses" of each other.

We now define two graphs to be homeomorphic if one can be derived from the other by a finite sequence of insertions and/or deletions of nodes of degree 2; in this case, there is said to be an homeomorphism between the two graphs in question. For example, the nodes x4, x6, x10 of Fig. A5 are all of degree 2 whence, suppressing them, we get

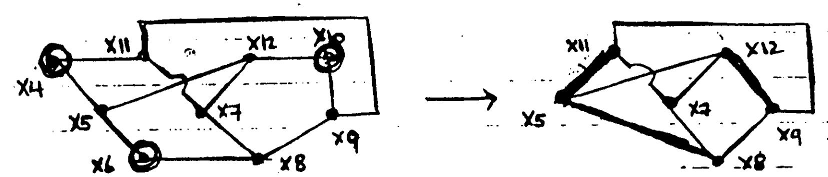


Fig. A8: A graph homeomorphic to Fig. A5

and the graph on the right is homeomorphic to that on the left (Fig. A5). The nodes of degree 2 -- to be deleted -- on the left are O'ed, the new lines (adjacencies) created by the deletion process appear in red on the right. Note that the right-hand graph may alternately be drawn

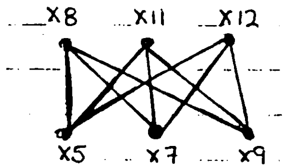


Fig. A9: A homeomorph of Fig. A5.

which is more easily recognizable as the non-planar graph $K_{3,3}$ (Fig. A4). We may now state the fundamental necessary and sufficient condition for planarity of a given graph [K. Kuratowski: "Sur le probleme des courbes gauches en topologie", Fundamenta Mathematica, Vol. 15 (1930), pp. 271-283]:

Theorem A: A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$.

The implication of this beautiful theorem for our present example, the graph of Fig. A2, is that -- since the graph of Fig. A5 is a subgraph which is homeomorphic to the graph of Fig. A9, i.e. $K_{3,3}$ -- the given graph is nonplanar. Hence, every attempt to draw this graph in the plane must result in lines which cross at points other then nodes of the graph. But, if this were the graph of a legitimate string-figure schema, all line-crossings would have to be listed as nodes of the graph -- by definition. This contradiction shows that the original gross linear sequence cannot give rise to a string-figure schema and, hence, there is no way a fine-structure of crossing types based on this gross linear sequence can lead to a legitimate schema. The method of analysis is entirely general, and may be used in all cases to investigate the "legitimacy" of proposed linear sequences.

Appendix A (Cont'd.)

2. A second problem concerning perfectly reasonable-appearing candidates for linear sequences is illustrated by the following example. Consider the proposed linear sequence (this time, with fine structure),

|⇒ L1: x1(U): x2(U): L5: x3(U): x4(∅): R1: x5(U): x1(∅): L2: x2(∅): x3(∅): R5: x4(U): x5(∅) ■

Here a (much easier) analogue of the 5-Step Analysis (cf. pages 69-72) leads us directly to the proposed schema

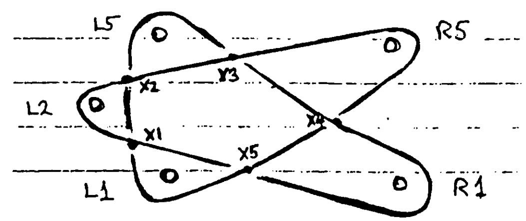
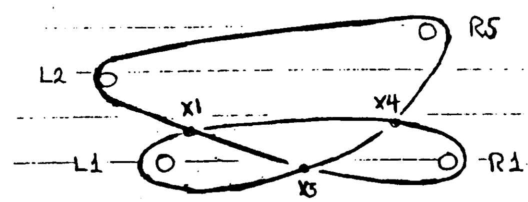


Fig. A10: Schema-so-far

which is reminiscent of the schemata of Fig. 6 (page 7), and appears to be entirely reasonable. However, attempting to lace the closed loop of string about your fingers in conformance with the above diagram quickly convinces you that something is very much amiss here!

To discover what this is, we analyze Fig. A10:



Note: x2, x3 of Fig. A10 become extension-cancellable, upon □L5, by Lemma 2.B

Fig. A11: Fig. A10:□L5#.

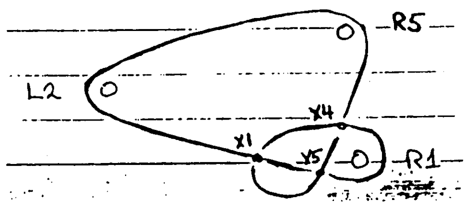


Fig. A12: Fig. A10: □L5#□L1#.

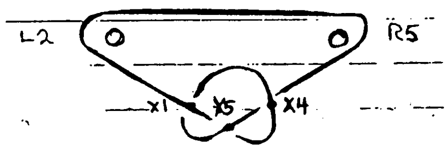


Fig. A13: Fig. A10:□L5#□L1#□R1.

And here we recognize the configuration {x1, x4, x5} of simple crossings as the overhand knot*. But, the closed loop cannot be continuously deformed into the configuration of Fig. A13 -- with its horrid constituent knot! Mathematicians say that the two configurations are not "topologically equivalent".

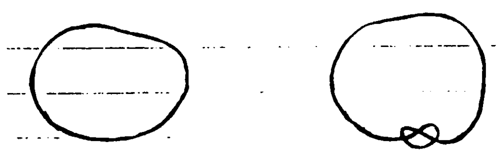


Fig. A14: Topologically inequivalent configurations.

Hence, the proposed linear sequence currently under consideration is not a linear sequence for string figures formed from the usual simple closed loop of string. We shall have to address this situation more completely when it next arises in our developing analysis of string-figures. For the time being, however,

*Also known as the "half-hitch"

we shall be content with calling attention thereto by means of a single example.

As a closing remark to the present section, we opine that it is an intuitive recognition of the topological inequivalence of the plain and knotted strings which lends charm and memorability ① to the classic, widely-known string trick the "Impossible Knot" [cf. R.M. Abraham: Diversions and Pastimes, No. 243 which uses a ring in the working]:

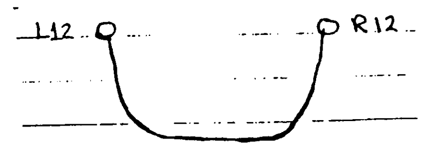


Fig. A15.I: Impossible Knot.

I. A plain piece of string (i.e. not a loop) is held at either end between L1 & L2 and R1 & R2, and allowed to hang loosely in front of you.

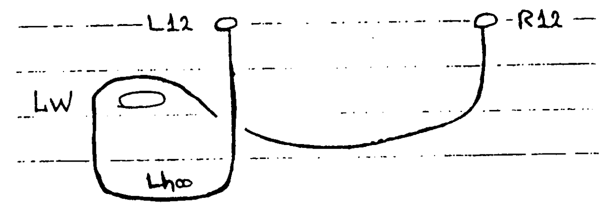


Fig. A15.II: Impossible Knot.

II. Pass R12 toward, and to the near side of LW, then around behind, and to the far side of LW, and back to its initial position. This will create a left-hand hanging loop (Lh∞), composed of the L12 string, running down, then up, to become LWn.

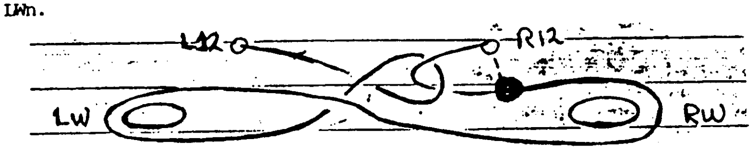


Fig. A15.III: Impossible Knot.

III. Again pass R12 toward, and to the near side of LW; then pass R12 away from you through Lh∞ and pick up on the back of ℝ (the whole right hand) the LWf-string. ℝ returns(#) back towards you through Lh∞ to its initial position. [The string picked up by ℝ becomes RW∞].

You are now ready to tie the Impossible Knot. The three previous moves have been somewhat slowly and openly done, and the resulting Fig. A15.III displayed to the audience. You will notice that the R12 string passes to the center of the figure, through a small ring -- whose strings are a continuation of the L12-string -- and thence to the far side of RW.

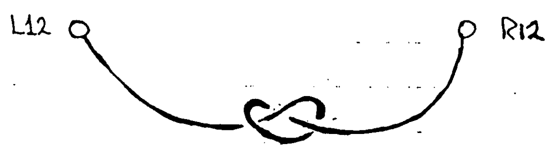


Fig. A15.IV: The Impossible Knot.

IV. >B, releasing both W∞'s in the process, while simultaneously □R12 and regrasp this same string after it has passed through the L12-ring (i.e. at the point marked O in Fig. A15.III). Continue >B back up to initial position | (gently).

This last move is to appear as a "shrugging off" of the W∞'s by a smooth rotation of the hands (>>B). The release and regrasp of R12 is entirely natural, as the point marked O comes directly into R during the movement ②.

---- End Appendix A ----

APPENDIX NOTES

① (page 104) Or the underwhelming "Can You Make It?" knot trick [F.J. Rigney Cub Scout Magic, p. 76]. We quote,

> First fold your arms and then take the rope ends.... Unfold your arms and the knot is made. You may have to shake it off your wrists.... [See Fig. A15.IV].

I may have to be helped across the street.

② (page 105) This is also to be found in J. Ould: Hindu Rope Book, "The Puzzle Knot", page 9; but this reference is almost impossible to obtain in recent years.

<u>BIBLIOGRAPHY</u>: OSAGE DIAMONDS, APPENDIX

<u>A.1</u>: Harary, F.: GRAPH THEORY. Addison-Wesley; Reading, Massachusetts (1969).

<u>A.2</u>: Crowell, R.H. & R.H. Fox: INTRODUCTION TO KNOT THEORY. Ginn and Company, New York (1963).

Reidmeister, K.: KNOTENTHEORIE. Chelsea, New York (1948).

<u>IV. CROWS FEET</u>

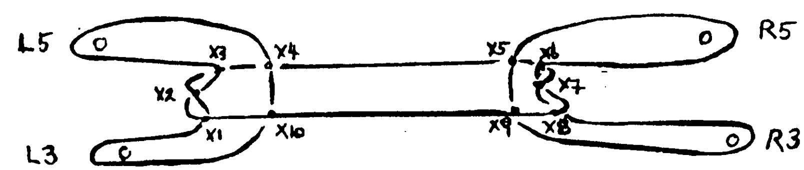
The second widely known string figure -- in addition to the foregoing Osage Diamonds -- is the design Crows Feet [P.G. Brewster: "String Figures and Tricks from the United States", Fig. 2 -- "Crow Foot"]. Although <u>O</u>.A is frequently used to initiate this figure, the construction below begins with what has traditionally come to be known as "Japanese Opening A" -- <u>O</u>.JA.

$$\underline{O}.JA \equiv \underline{O}.1{:}\ \overleftarrow{R3}(\underline{Lp})\#\ \overrightarrow{L3}\downarrow(R3\infty){:}\ \underline{L3}_{\Rightarrow}(\underline{Rp})\#|.$$

That is, Japanese Opening A is "Opening A with the middle-fingers". The construction, below, is very easy:

$$\underline{\text{Crows Feet}}{:}\ \underline{O}.JA{:}\ \overleftarrow{2345}(\underline{1n})\#\ \Box 1|\ \overrightarrow{2345\infty}\to W{:}\ \overleftarrow{3\infty}\to 1{:}\ \overrightarrow{W\infty}\to 3{::}$$
$$\overleftarrow{5}(3f){:}\overrightarrow{5}(\underline{5n})\#\ N5{:}\ \Box 1|$$

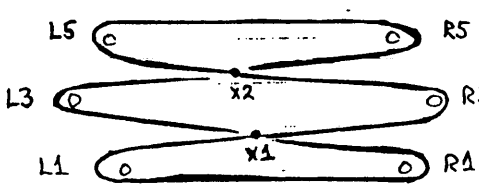
<u>Construction Note</u>: In the antepenultimate (complex) 5 move, 5 hooks towards you over 3f from above, and pulls this string away over its own near string (5n), which it then dips down under and picks up -- from below -- on its back. The 3f string is thus released from 5 upon #. At this point we have a new, unusual, situation for the two loops on 5: the u5f-string proceeds to 5, across the back and to the near side of 5, and down into the 3∞ (<u>i.e.</u> over 3f); from there it proceeds directly back to (the near side of) 5 -- to become the ℓ5∞ -- across the back of, and to the far side of 5, where it becomes the ℓ5f-string.



<u>Fig. 45</u>: Crows Feet.

|⇒ L3: x1(U): x2(∅): x3(U): x4(U): x5(U): x6(U): x7(∅): x8(U): R3: x9(U): x5(∅): R5: x6(∅): x7(U): x8(∅): x9(∅): x10(∅): x1(∅): x2(U): x3(∅): L5: x4(∅): x10(U) ■

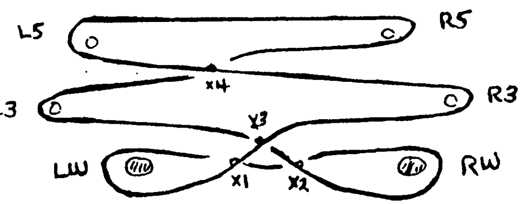
We proceed with an abbreviated constructional analysis, again recording the crossing <u>alias</u> transformations as we go:



<u>Fig. 46.I</u>: Crows Feet; <u>O</u>. JA.

⊨ L1: x1(∅): R3: x2(∅): L5: R5: x2(U): L3: x1(U): R1 ■

For the next string-position of the construction, we require a frame-node corresponding to W; this we append as the near-most node of the schema.



<u>Fig. 46.II</u>: Crows Feet: <u>O</u>. JA: $\overleftarrow{2345}$(<u>1n</u>)#□1| $\overrightarrow{2345\infty}$ →W.

⊨ LW: x1(U): x2(U): RW: x2(∅): x3(U): L3: x4(U): R5: L5: x4(∅): R3: x3(∅): x1(∅) ■

<u>Note</u> that, in reality, only the manipulation "$>\overrightarrow{1\infty}$→ W" has taken place between Fig. 46.I and Fig. 46.II. A "complete" intermediary analysis devolves into an exercise in the renaming of frame-nodes in the above schema.

<u>Fig. 46.I</u>		<u>Fig. 46.II</u>
x1	⟶	x3
x2	⟶	x4

The crossings x1, x2 are created by the movement $>\overrightarrow{1\infty}$→W applied to Fig. 46.I.

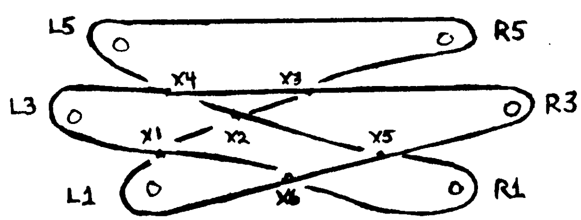
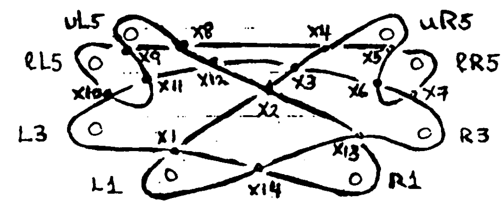


Fig. 46.III: Crows Feet; <u>O</u>.JA: $\overleftarrow{2345}$(<u>1n</u>)#□1| $\overrightarrow{2345\infty}$ →W: $\overleftarrow{3\infty}$→1: $\overrightarrow{W\infty}$→3.

⊨ L1: x1(U): x2(U): x3(U): R5: L5: x4(U): x2(∅): x5(U): R1: x6(U): x1(∅) L3: x4(∅): x3(∅): R3: x5(∅): x6(∅) ■

<u>Fig. 46.II</u>		<u>Fig. 46.III</u>
x1	⟶	x1
x2	⟶	x5
x3	⟶	x6
x4	⟶	x2

The crossings x3, x4 are created by the movement(s) $\overleftarrow{3\infty}$→1: $\overrightarrow{W\infty}$→3 applied to Fig. 46.II.



<u>Fig. 46.IV</u>: Crows Feet: <u>O</u>.JA: $\overleftarrow{2345}$(<u>1n</u>)#□1| $\overrightarrow{2345\infty}$→W: $\overleftarrow{3\infty}$→1: $\overrightarrow{W\infty}$→3:: $\overleftarrow{5}$(3f): $\overrightarrow{5}$(<u>5n</u>)#.

⊨ L1: x1(U): x2(U): x3(∅): x4(∅): uR5: x5(∅): x6(∅): x7(U): ℓR5: x5(U): x4(U): x8(U): x9(U): ℓL5: x10(U): x11(∅): x9(∅): uL5: x8(∅): x12(∅): x2(∅): x13(U): R1: x14(U): x1(∅): L3: x10(∅): x11(U): x12(U): x3(U): x6(U): x7(∅): R3: x13(∅): x14(∅) ■

<u>Fig. 46.III</u>		<u>Fig. 46.IV</u>
x1	⟶	x1
x2	⟶	x2
x3	⟶	x7
x4	⟶	x10
x5	⟶	x13
x6	⟶	x14

The crossings x3, x4, x5, x6, x8, x9, x11,x12 are created by the movement(s) $\overleftarrow{5}$(3f): $\overrightarrow{5}$(<u>5n</u>)# applied to Fig. 46.III.

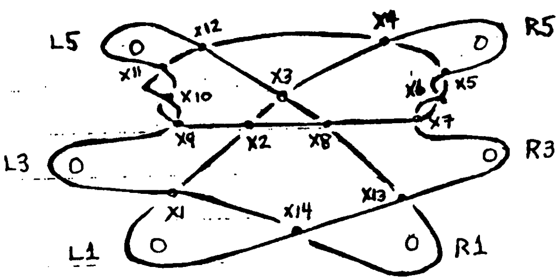


Fig. 46.V: Crows Feet; O.JA: $\overleftarrow{2345}$(1n)#□1| $\overrightarrow{2345\infty}$ →W: $\overleftarrow{3\infty}$→1: $\overrightarrow{W\infty}$→3:: $\overleftarrow{5}$(3f): $\overrightarrow{5}$(5n)# N5:

⊨ L1: x1(U): x2(U): x3(U): x4(∅): R5: x5(∅): x6(U): x7(∅): x8(∅): x2(∅): x9(∅): x10(U): x11(∅): L5: x12(∅): x3(∅): x8(U): x13(U): R1: x14(U): x1(∅): L3: x9(U): x10(∅): x11(U): x12(U): x4(U): x5(U): x6(∅): x7(U): R3: x13(∅): x14(∅) ■

Fig. 46.IV		Fig. 46.V	Fig. 46.IV		Fig. 46.V
x1	→	x1	x8	→	x2
x2	→	x3	x9	→	x9
x3	→	x4	x10	→	x10
x4	→	x8	x11	→	x11
x5	→	x7	x12	→	x12
x6	→	x5	x13	→	x13
x7	→	x6	x14	→	x14

Here all crossings are accounted for; the alias transformation between Fig. 46.IV and Fig. 46.V is one-to-one.

The figure Crows Feet (Fig. 45) is now obtained from that of Fig. 46.V via "□1|" whose alias crossing transformation is apparent. In particular, x13, x14 become extension-cancellable by Lemma 2.B and -- upon performing this cancellation -- x1, x3 become extension-cancellable (also by Lemma 2.B). Thus, explicitly,

	Fig. 46.V		Fig. 45		Fig. 46.V		Fig. 45
C.	x1	→	∅		x8	→	x10
	x2	→	x9		x9	→	x1
C.	x3	→	∅		x10	→	x2
	x4	→	x5		x11	→	x3
	x5	→	x6		x12	→	x4
	x6	→	x7	C.	x13	→	∅
	x7	→	x8	C.	x14	→	∅

This completes the crossing-specific constructional analysis for the string-figure Crows Feet.

The "Heart"-sequence for this figure is particularly short, and sweet:

O.A: >$\overrightarrow{1\infty}$→3: { $\overrightarrow{L2\infty}$ →R5 / R2∞↑(R3∞): R2∞→L5 }: N5|

Note that, in particular, the R2∞ passes under the (moving) L2∞ on its way to L5; further, L5 enters this loop from the same direction (i.e., below) as did R2. Et cetera. In fact, the manipulation sequence O.A: >$\overrightarrow{1\infty}$→3 results, directly, in the schema of Fig. 46.III, with the nodes L1, R1 relabeled L2, R2, respectively. We find "source" cognizance of the equivalence of the manipulational sequences

$\overleftarrow{2345}$(1n)#□1| $\overrightarrow{2345\infty}$ →W: $\overleftarrow{2\infty}$→1: $\overrightarrow{W\infty}$→3 ≡ >$\overrightarrow{1\infty}$→3: $\overleftarrow{2\infty}$→1

on the string-position O.A in the figure "A Drum" [K. Haddon and H.A. Treleaven: "Some Nigerian String Figures", No. 1]:

A Drum: O.A: >$\overrightarrow{1\infty}$→3:: $\overrightarrow{1}$↓(3∞): 1→(5n)#| $\overleftarrow{5}$(1n)# □1| N5: □2|

And the further equivalence

$\overleftarrow{5}$(3f): $\overrightarrow{5}$(5n)# ≡ $\overrightarrow{1}$↓(3∞): 1→(5n)#| $\overleftarrow{5}$(u1n)# □1

on Fig. 46.III -- producing Fig. 46.IV -- is also demonstrated by this construction. ① The figures Crows Feet and A Drum are identical (as to final design), and constructionally-equivalent (having identical "Heart"-sequences).

Similarly, in the figure "Duck's Feet II" [P.H. Buck: "Samoan Material Culture", page 558] we have

Duck's Feet II: O.A: 2∞→1: >ℓ$\overrightarrow{1\infty}$→3#|:: $\overleftarrow{5}$↓(3∞): 5→(5n)# N5|

-- where the movement " >ℓ$\overrightarrow{1\infty}$→3" implies "N1" -- whose final extension is achieved by the latter equivalence. Further, here, the initial segment,

O.A: 2∞→1 >ℓ$\overrightarrow{1\infty}$→3#|

results in a string-position which is the same as that of the schema of Fig. 46.III except that the parities of crossings x5, x6 are reversed. This is, again, a matter of no consequence to the final design as, at the final step of the construction --□1| -- these crossings are extension-cancellable, as before. [Now the sequence is {x1, x6} → ∅ followed by {x2, x5} → ∅]. In both cases -- i.e. Crows Feet and Duck's Feet II -- the crossings x1, x2, x5, x6 are pairwise extension-cancellable upon □1; only the order of the cancellation changes. The final designs are, thus, identical. [This in distinction to the similar-looking contemporary figure

Hen's Feet: O.A: 2∞ ⟶1:>ℓ$\overrightarrow{1\infty}$⟶3#|:: $\overleftarrow{5}$(3∞): $\underset{\Rightarrow}{5}$ (5n)# N5|□1#|

whose final design lacks ">>5" from being identical to that of Crow's Feet].

In terms of complex crossings, the schema for the string-figure Crows Feet exhibits only 2 such -- as compared with the 10 simple crossings of the original schema (Fig. 45).

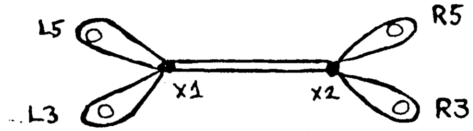


Fig. 47: Crows Feet, labeled for complex crossings.

Again, this is a gross-schema whose fine-structure is recoverable from the composition

Fig. 47		Fig. 45
x1	—	x1, x2, x3, x4, x10
x2	—	x5, x6, x7, x8, x9.

Thus, in Fig. 47, we find $\alpha(x1) = \alpha(x2) = 3$, $\chi(x_1) = \chi(x_2) = 5$. To exhibit the (complex) linear-sequence associated to the schema of Fig. 47, we must add two new entries to the $\mathbb{X}$-Dictionary -- corresponding to the two new types of complex crossings encountered in this schema:

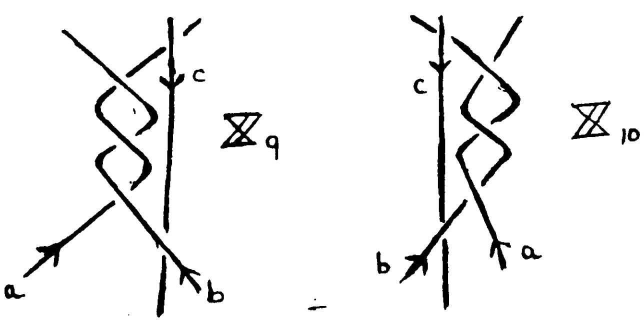


Fig. 48: $\mathbb{X}$ - Dictionary (Continued).

These two new entries are essentially distinct -- 'though obviously related, geometrically -- and are necessary to the complex crossing-analysis of the Crows Feet. In terms of them, the associated (complex) linear sequence may be written

|⟹ L3: x1($\underline{\underline{9}}$,a+): x2($\underline{\underline{10}}$,a−): R3: x2($\underline{\underline{10}}$,c−): R5: x2($\underline{\underline{10}}$,b−): x1($\underline{\underline{9}}$,b+): L5: x1($\underline{\underline{9}}$,c+) ■

We remark that, for example, the complex crossing $\mathbb{X}_9$ may present itself in either of two equivalent forms

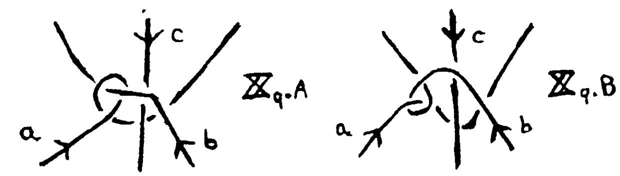


Fig. 49: Equivalents of $\mathbb{X}_9$

depending on thickness of string, and individual string tensions; a symmetric pair of presentations exists for $\mathbb{X}_{10}$. As $\chi(\mathbb{X}_9.A) = \chi(\mathbb{X}_9.B) = 6 > 5 = \chi(\mathbb{X}_9)$, these are not the simplest presentations of this complex crossing, and -- when met with -- we shall always assume (or arrange strings so) that the crossing is in the "canonical" presentation of $\mathbb{X}_9$. Et cetera. The analogous statements for the related complex crossing $\mathbb{X}_{10}$ also apply and will be assumed, by symmetry.

Let us now take a closer look at the complex crossings $\mathbb{X}_9$, $\mathbb{X}_{10}$ of the Crows Feet, with an eye to their composition. In terms of known entries of the $\mathbb{X}$-Dictionary, we find

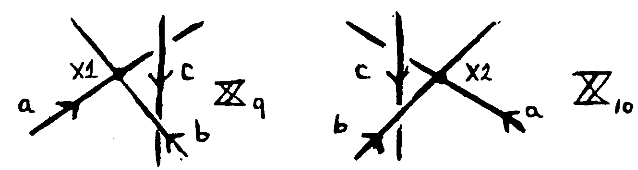


Fig. 50: $\mathbb{X}_9$ and $\mathbb{X}_{10}$, structurally

where crossing x1, above, is the complex crossing $\mathbb{X}_8$, with the direction of b reversed; we shall denoted this by $\mathbb{X}_8$ (a-b,c). Similarly, in these terms, complex crossing x2 is $\mathbb{X}_7$(a,-b,c). Thus, these two crossings, $\mathbb{X}_7$, $\mathbb{X}_8$ -- which arose in connection with the string-figure Osage Diamonds -- are fundamental to the discussion of the Crows Feet. But we know how X_7, X_8 arise from the Osage extension and, in fact, have two separate ways to view their construction [cf. page 64, bottom and pages 66-67]. Each viewpoint gives rise to a class of constructions for the figure Crows Feet, as discussed below.

We discuss a left bilaterally-specific construction of complex crossings X_7, X_8; the right-constructions will then follow from the symmetry of the situation. Form the string-position resulting from the Calculus string

O.1: $\overleftarrow{R2}$(Lp)#|

Which leaves L with two loops: an L1∞ and an L5∞.

I. Now perform

A. $\overleftarrow{R12}$↓(L5∞): Bring R12 and seize between them the L1f-string: draw this string away from you, up through the L5∞, then towards you and hang it on L1 (□R12's tenure of this string): N(L1)#|

The result is the creation of complex crossing X_7 on the Lp-string. Similarly, perform the symmetric

B. $\overleftarrow{R12}$↓(L1∞): Pass R12 and seize between them the L5n-string: pull this string towards you, up through the L1∞, then away from you and hang it on L5 (□R12's tenure of this string): N(L5)#|

This time the result is the creation of complex crossing X_8 on the Lp-string. We should not be surprised to see a class of constructions for the figure Crows Feet which are basically analyzable as

Use Method I to create X_7 or X_8 on the palmar string; release some loop on the opposite hand, which passes through the center of the figure to engage this complex crossing (like the string labeled "c" in the two cases of Fig. 50).

Indeed, the constructions met with so far are all of this type, for Method I.B.

Using Method I.B, we see that Crows Feet admits the bilaterally specific construction

O.1: 1. $\overleftarrow{R2}$(Lp)# >$\overrightarrow{L1∞}$→L3:: $\overleftarrow{L1}$↓(L3∞): L1(L5n)# $\overleftarrow{L5}$(L1n)#□L1|: N(L5):□R2|: Repeat Step 1, interchanging L and R

We shall adopt the notation

∼ ---- repeat

↶R⌣L↗ ---- interchange "L" and "R"

whence, as with Osage Diamonds, the string-figure Crows Feet may be constructed "around" a central knot. That is, starting with "O.1 + knot", viz.

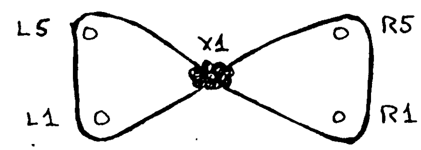


Fig. 51: O.1 + knot

-- where any kind of a knot has been tied at x1 -- and proceeding with any of the previous (Method I) constructions of this section, we shall produce "Crows Feet + knot", viz.

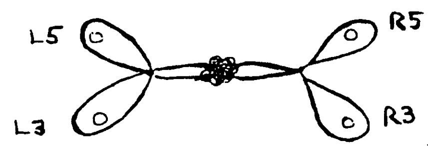


Fig. 52: Crows Feet + knot.

Further, if F is the generic functor (finger), and we are given any string-position in which there exists an F∞, then Method 1.B may be employed to create a "Crows Foot" out of the F∞: We illustrate this for the L2∞ in O.A: Crows Feet ending,

O.A: $\overrightarrow{L2∞}$→L15:$\overleftarrow{R2}$(Lp)#| >u$\overrightarrow{L1∞}$→L3::$\overleftarrow{L1}$↓(L3∞):L1(uL5n)# $\overleftarrow{L5}$(uL1n)# □uL1∞:(mL5∞)NL5::$\overleftarrow{L3∞}$→L2: u$\overleftarrow{L5∞}$→L3# □uR2∞|.

This is the original string-position (i.e. O.A), with the L2∞ replaced by a "Crows Foot" on L2 and L3, [cf. J. Elffers and M. Schuyt: Cat's Cradles and Other String Figures, p. 195]. The procedure is, of course, entirely general.

The symmetric bilaterally specific construction of Crows Feet based on Method I.A is given by

O.1: 1. $\overleftarrow{R2}$(Lp)# <$\overleftarrow{L5∞}$→L3::$\overleftarrow{L5}$↓(L3∞):L5(L1f)# $\overrightarrow{L1}$(L5f)#□L5|: N(L1):□R2|:∼1 (↶R⌣L↗)#|

And "source" confirmation for this construction is to be found in the Maori string figure "Matamata Karehu" [A. Armstrong: Maori Games and Hakas, No.5]:

O.1: $\overrightarrow{2}$(5f)# $\overrightarrow{1}$(2f)# N1: □5|

which lacks "<1∞|" of being the canonical pseudo-Crows Feet.② [Of course, all constructions have their symmetric counterparts (cf. "looking at the construction from the ⌜wrong⌝ side of the hands" -- page 36). In the absence of a "source" referent, we shall generally elide these.]

We return to the original string-position,

O.1: $\overleftarrow{R2}$(Lp)#| .

II. Now perform

A. L5$\overleftarrow{\infty}$→L4:: Pass $\overrightarrow{L1\infty}$ and place it over the L4n-string: pass $\overrightarrow{L1}$↓ this loop and pick up L4n from below #(L1):: $\overrightarrow{L1\infty}$↓(L4∞): L1∞→L5::□L4#|

This creates $\mathbf{x}_7$ on the Lp-string. Similarly

B. $\overrightarrow{L1\infty}$→L2:: L5$\overleftarrow{\infty}$ and place it over the L2f-string: pass $\overleftarrow{L5}$↓ this loop and pick up L2f from below #(L5):: $\overleftarrow{L5\infty}$↓(L2∞): L5∞→L1::□L2#|

This time the result is the creation of $\mathbf{x}_8$ on the Lp-string. Again, we should not be surprised to see a class of constructions for the figure Crows Feet which are basically analyzable as

Use Method II to create $\mathbf{x}_7$ or $\mathbf{x}_8$ on p; release some loop from the opposite hand.

We choose three figures of Crows Feet-type which arise from the Method II.B construction of the fundamental $\mathbf{x}_8$-type crossing. The first of these is "Mashiri Mana" [H. Tracey: "String Figures (Madandi) Found in Southern Rhodesia", No. 9]:

O.A: $\overrightarrow{1}$(5n)# □5|::$\overleftarrow{5}$↓(u1∞):$\overrightarrow{5}$(ℓ1f)# N1: □2|.

The second figure is "Matamata Karehu" [J.C. Andersen: Maori String Figures, No. 17]:

O.A: 2$\overrightarrow{\infty}$→W: $\overrightarrow{1}$(5n)# $\overleftarrow{3}$(1f)# N1|:□5:□W|.

Note that a figure with this name was previously given as an example of a Method I.B-type construction [top of page]. The present figure is identical as to final design, but is quite distinct as to construction. Identification of the two dissimilar-appearing viewpoints represented by Methods I.B and II.B is required to analyze the two figures' similarity, and this is non-trivial.

The final Method II.B-type construction we shall give is the amazing "Two

Devilfish" [J. Averkieva: Kwakiutl String Games, Nos. 37B, 38B]:

O.A:: $\overrightarrow{3}$(5n):3↓(1∞):<3(#) [thus picking up 1f on 3, and □5n-string from 3]:: 1(2∞):$\overrightarrow{1}$(5n; central to its crossing by s;1f-3f):>1(#):□5# □2|.

This final design lacks "<1∞:>5∞|" of being identical to the final design of Crows Feet. [Note that the antepenultimate movement, ">1(#)", of the construction picks up the 5n-string (after it is crossed by s;1f-3f) on the back of 1 from above.]

While we know of no "source" string-figure ["pseudo-Crows Foot"] corresponding to construction Method II.A, one is easy to "invent":

O.A: 2$\overrightarrow{\infty}$→W:$\overleftarrow{5}$(1f)# $\overrightarrow{2}$(5n)# N5|:□1:□W|.

This is the pseudo-Crows Feet construction analogous to "Matamata Karehu" [Andersen] -- of page 117, bottom. We have merely employed the symmetry of the situation to turn a Method II.B-type construction into its corresponding Method II.A-type construction; this may be done with both of the remaining Method II.B-type constructions, in like manner.

Next we perform the gedanken-experiment of visualizing the figure Crows Feet as an independent entity in three-dimensional space, divorced from its supportive frame (cf. the gedanken-experiment of page 35 for the Brochos). The basic diagram for the figures under consideration will all have the gross-schema

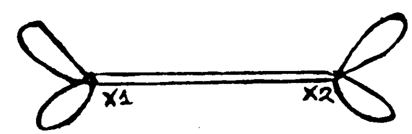


Fig. 47.A: Crows Feet, a "gedanken"-schema
[Note: no frame nodes]

and we shall consider the complex-crossing types of x1, x2 arising from our various constructions. Basically, there are four such -- considering bilateral specificity -- which, without loss, we shall derive from the Method I construction (page 115)

Crow (L): O.1: $\overleftarrow{R2}$(Lp)# >$\overrightarrow{L1}$∞→L3::$\overrightarrow{L1}$↓(L3∞):L1(L5n)# $\overleftarrow{L5}$(L1n)# N(L5):□L1|□R2|.

We shall call the bilateral mate to this construction "Crow (R)"; it is obtained from the above via the uniform interchange of the symbols "L" and "R", and "⇒" and "⇐", throughout. These, together, constitute the Method I.B construction.

The symmetric mate to the above construction, we know, produces pseudo-Crows Feet; whence we define

$$\underline{\text{pseudo-Crow (L)}}: \underline{O}.1: \overleftarrow{\overleftarrow{R2}}(\underline{1p})\#\ \langle\overleftarrow{L5\infty}\rightarrow L3::\overleftarrow{L5}\downarrow(L3\infty):\underleftarrow{L5}(\underline{L1f})\#$$
$$\overrightarrow{L1}(\underline{L5f})\#\ N(L1): \square L5 \mid \square R2 \mid$$

and its bilateral mate, "pseudo-Crow (R)", obtained from the above by the exact same uniform symbol-interchanges, as before. We now examine the four distinct "substitution instances" of the schema of Fig. 47.A which can arise from mixing the above four constructions: specifically,

A. Crow (L) -- Crow (R)
B. Crow (L) -- pseudo-Crow (R)
C. pseudo-Crow (L) -- Crow (R)
D. pseudo-Crow (L) -- pseudo-Crow (R).

We consider A. first: we know from the text discussion that

Crows Feet ≡ Crow (L) -- Crow (R).

Let us examine this figure: We know that the crossing x1 (of Fig. 47.A) is an $\mathbf{X}_9$-type, while x2 is an $\mathbf{X}_{10}$-type. Now construct the figure Crows Feet on the hands, then turn the hands with fingers pointing down, and lay the figure flat (i.e. "turn the figure over"): this is, again, a substitution instance of Fig. 47.A. In fact, the (turned-over) crossing x1 is, again, an $\mathbf{X}_9$-type; while x2 is, again, an $\mathbf{X}_{10}$-type. A string-by-string analysis confirms that this is, indeed, the Crows Feet! The line joining x1 to x2 in Fig. 47.A is a line of symmetry for the Crows Feet, and the figure maps into itself under a 180°-rotation in this line (i.e. being "turned over"). Next we examine rotation of the figure by 180°, in its own plane, about a central point of the figure: Again we get a substitution instance of Fig. 47.A. This time, of course, x1 is an $\mathbf{X}_{10}$-type and x2 is an $\mathbf{X}_9$-type -- the crossings having changed places (in their plane) under the rotation. Now, in a separate string, construct

Pseudo Crows Feet ≡ pseudo-Crow (L) -- pseudo-Crow (R).

A string-by-string comparison confirms that the figures in the two strings are identical! That is, the Crows Feet goes to pseudo-Crows Feet under 180°-rotation in the plane of the figure. Of course, we may now infer that pseudo-Crows Feet

maps into itself under a 180°-rotation in its line of symmetry; that is, pseudo-Crows Feet turns over into pseudo-Crows Feet -- as is directly verifiable. Thus, with respect to the original gedanken perspective, there is one figure, Crows Feet,⊛ which has exactly two distinct presentations: Crows Feet and Pseudo-Crows Feet. This completes the analysis of constructions of type A. and D., above.

Next we discuss the gedanken figures of type B. construction,

Bogus Crows Feet ≡ Crow (L) -- pseudo-Crow (R).

Strictly speaking, this string-"figure" has gross-schema

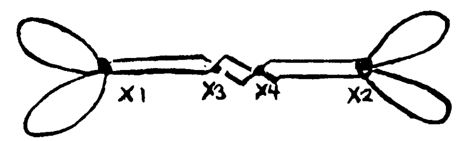


Fig. 47.B: gedanken-schema, Bogus Crows Feet

and is not a substitution instance of Fig. 47.A [cf. the illustration for "Morna Pag", in J. Hornell: "String Figures from Gujarat and Kathiawar", Fig. 4. -- illustrating a classic Crows Feet construction -- for an interesting example of a non-standard schema for such figures.] Here we know from the construction that crossing x1 of Fig. 47.B is of $\mathbf{X}_9$-type and, from the previous analysis, that x2 is also of $\mathbf{X}_9$-type! Further, Bogus Crows Feet turns over into Bogus Crows Feet (only an examination of what happens to crossings x3, x4 is needed to confirm this, after our previous discussion), and -- under 180° rotation in its own plane -- it maps into a figure whose gross-schema is, again, that of Fig.47.B; and, obviousl both crossings x1, x2 remain of X_9-type. (Of course, this latter figure also turns over into itself). Thus, Bogus Crows Feet exhibits a remarkable "stability" in that it always presents itself as the same figure, regardless of being turned over or rotated by 180° in its own plane. Finally, if we "turn over" the left "foo only -- and do this once more -- then we get a genuine substitution instance of Fig. 47.A, in which crossings x1, x2 are both of $\mathbf{X}_9$-type. Calling this figure Crows Feet (9,9), we see that, a fortiori, all of the above remarks hold for this new figure (there being no x3, x4-crossings to consider). A more reasonable construction for Crows Feet (9,9), perhaps, is given by prefixing the move ">>L" to Construction B: specifically,

B'. Crows Feet (9,9): O.1: >>L15∞: Crow (L) -- pseudo-Crow (R)I*
It remains to discuss Construction C.

Motivated by the previous discussion of (the modified) Construction B, we propose a similar modification to Construction C; namely the prefixing of "<<IL" to this manipulational sequence. Specifically,

C'. Crows Feet (10,10): O.1: <<L15∞: pseudo-Crow (L) -- Crow(R)I*
where we fully expect to produce a substitution instance of Fig. 47.A, which exhibits X_{10}-type crossings at both nodes x1, x2 of that figure. Indeed, this is found to be the case. Further, our previous discussion implies that this figure exhibits the same remarkable "double-stability" enjoyed by Crows Feet(9,9), as is directly verifiable.

Thus, from the perspective of our original gedanken-experiment, Construction A produces the string-figure Crows Feet ≡ Crows Feet (9, 10), while D produces pseudo-Crows Feet ≡ Crows Feet (10, 9); these are, in reality, two presentations of the same figure (under rotation by 180° in the plane). Construction B' produces Crows Feet (9, 9), while C' produces Crows Feet (10, 10); both are stable under 180° plane rotation, and they are distinct. Further, all three distinct figures are stable with respect to being "turned over". The author knows of no "source"-figures of either type Crows Feet (9, 9) or Crows Feet (10, 10).

This completes the gedanken-experiment.

Next we shall present four extended constructions of the Crows Feet, which will appear as the final design in a series of figures. That is, among the intermediate string-positions leading to the production of the final design, some will be distinguished as "string-figures" in their own right. Such figures are termed "serial" and, since the distinguished intermediate string-position are almost invariably displayed by an extension, we shall amend our "|" notation to include

I ---- extend hands to display distinguished intermediate or final string position.
[briefly, "extend to figure".]

That is, we shall use the symbol "I" to distinguish those string-positions with "source"-recognition as string-figures. It will be useful to extend our string-figure Calculus before launching into these new, lengthier constructions.

*See discussion of "I" at bottom of page.

Frequently, in longer series especially, it is difficult to keep multiple loops/strings on a functor distinct during the actual working. And, many times, this is unnecessary, as the required string is distinguished in some characteristic (i.e. easily discernible) way. For example, perform the manipulation sequence

O.A: $\vec{1}$(5n)#|

and suppose that the subsequent move in a given construction involves passage of $\overleftarrow{2}$(ul∞) and picking up from below, on its back, one of the 1n-strings. Of course, as written above, the two 1n-strings are distinct, by convention; but let us suppose the contrary -- that we have allowed the newly picked-up 5n-string to slide to the base of 1 where it is confusable with the (former) 1n-string inherited from O.A. At least in this example, we may distinguish between these two strings via the referents

1n-s --- the 1n-string running straight from thumb to thumb

versus

1n-o --- the 1n-string running obliquely to the central configuration (i.e. not a "straight" string).

Thus, the two manipulational sequences

O.A: $\vec{1}$(5n)#|: $\overleftarrow{2}$(1n-s)#|

and

O.A: $\vec{1}$(5n)#|: $\overleftarrow{2}$(1n-o)#|

cover both possible eventualities. In general, for generic functor, F, the F-s string proceeds directly from LF to RF and, as such, "F-s" may be thought of as a shorthand for our earlier-introduced "s;LF-RF". Et cetera. In the interest of notational precision, we shall introduce (but rarely use) the symbol "ǂ(F)" when the strings/loops on functor F need not be maintained as distinct (Note: another functor may have distinct strings/loops even though F does not). By abuse of notation, "ǂ" -- by itself -- will be used to imply that no multiple loops in a given string position need be kept distinct.

ǂ --- multiple loops need not be distinct.

Thus, the original string-position of the present example would be rendered

O.A: $\vec{1}$(5n)ǂ #|.

We shall also adopt the convention of referring to multiple strings/loops in a given string-position by a superscript consisting of a parenthesized natural number, indicative of their cardinality. For example

①. $\overrightarrow{1}$ (2∞$^{(2)}$): 1↘(<u>5n</u>) --- 1 away over both 2∞'s, away under all intermediate strings, and pick up 5n from below.

②. $\overleftarrow{5}$ (<u>1f</u>$^{(3)}$) --- 5 towards you, over, and pick up all three 1f-strings from below.

③. >$\overrightarrow{2\infty^{(2)}}$ →5 --- Twist both 2∞'s 180° away to 5.

<u>Et cetera</u>. We remark that this notational specificity is extremely helpful, especially as a "consistency check" in the mastering of a given construction. More explicitly, if you have but one loop on 2 when you meet an instruction like ③, above, you have not arrived at the current, correct string-position for the string-figure under construction.

Often, in complex string manipulations, effecting a given contorted movement on one hand requires the help of the fingers of the other hand. Usually, (but not invariably) this will involve the thumb and forefinger of the opposite hand seizing between them a string or loop of the working hand and accomplishing its required passage. We shall indicate such a seizure, say, of L2n by R1 and R2 by the notation

$\overleftarrow{\text{R1*R2}}$ (<u>L2n</u>);

Thus, the manipulative sequence

<u>O</u>.A: $\overrightarrow{\text{L2}\infty}$ →L4|

might be actually effected by

<u>O</u>.A: $\overleftarrow{\text{L4}}$ (<u>L2f</u>)#□L2|

or by

<u>O</u>.A:: $\overleftarrow{\text{R1*R2}}$ (<u>L2n</u>):□L2:$\overrightarrow{\text{(R1*R2)}\infty}$ →L4#|

where the loop seized between R1 and R2 is to be placed directly on L4 (<u>i.e</u>. oriented as it was on L2).

In many complex string-figures we encounter intermediate positions which have "hanging loops" (<u>cf</u>. "Amanohashidate", Fig. 43, page 88). Often these are treated merely as "loose strings", to be absorbed by a subsequent "|"; but, occasionally -- as in "Amanohashidate"-- these are subject to various manipulations before such absorption. We shall legitimize the notion of "hanging loops" -- notationally, "h∞" -- by treating gravity as a "weak finger" (Functor) and appending nodes h_1, h_2, ... to our schemata corresponding to each such hanging loop of a given

string position. And we shall operate on h∞'s <u>via</u> the string-figure calculus in a perfectly ordinary way. (<u>Note</u>: but the gravity-node h cannot move to pick up a string, for example; hence the epithet "weak" for this functor. Further, we shall never see the symbology "□h"; the symbol "|" will serve to cancel the functor h and its loop.)

Finally, we shall encounter distinguished hand-positions other than the "normal position", #; these are the "hooked" hand-positions. These frequently arise from the "hooking down" of a given string by a finger; the generic notation for "hook" is "H". For example, perform the manipulational sequence

<u>O</u>.A:□5: $\overleftarrow{2\infty}$→1:: 5↙↑(ℓ1∞): pass 5 away under u1f, then hook 5 towards you over this string, from above, and pull it down thru the lower 1∞ and hold it securely hooked against palm of hand :: $\overleftarrow{2}$↓(u1∞): 2↙($\overline{\ell 1n}$)#(2): □1I.

The result is "Two Diamonds." <u>Note</u> the symbol "#(2)" here means "return 2 to <u>its</u> normal position"; the hands are <u>not</u> #, as 5 remains hooked to the palms. We introduce a notation for the complex 5-move in the above construction:

5↙↑(ℓ1∞): $\overleftarrow{\text{H5}}$($\overline{\text{u1f}}$)#(H5)

all but whose final symbol, #(H5), is self-explanatory in terms of the above. The symbol "#(H5)" shall mean that the hands (and fingers) are to be held in the normal position <u>except</u> for 5, which is to be held hooked securely to the palm. The meanings to be ascribed to such expressions as, <u>e.g</u>.

#(H45), #(H2), #(H34)

are, similarly, easily inferred. And these will usually be followed by the symbol "|", which -- as usual -- means "absorb all slack (in this hand-position)." In our notation, then, we have

<u>Two Diamonds:</u> <u>O</u>.A: □5: $\overleftarrow{2\infty}$→1:: 5↙↑(ℓ1∞):$\overleftarrow{\text{H5}}$($\overline{\text{u1f}}$)#(H5)|:: $\overleftarrow{2}$↓(u1∞): <u>H2</u>↘(<u>ℓ1n</u>):<2(#): □1#(H5)I

Here the final complex 2-movement has been rendered into the "Hook"-notation (although the former is perfectly satisfactory). Note that -- unlike the previous 5-move -- the index, here, reaches down and hooks <u>up</u> the ℓ1n-string (from below). The subsequent "<2" returns 2 to its normal position -- and the modifier "(#)" <u>on 2</u> emphasizes that no other strings are to be picked up, <u>i.e</u>. that 2 is to return "the way it came".

With these few additions to the string-figure Calculus we are ready to launch into the four serial string-figures whose final design is the Crows Feet.

I -- extend to display string-figure

s -- straight string

o -- oblique string

ǂ -- loops not distinct*

1n$^{(2)}$ -- double 1n-strings

$\overrightarrow{1*2}$(5n) -- seize 5n between 1 and 2 (pinch).

h -- hanging

H -- Hooked

#(H5) -- return to normal position except for hooked 5.

The first "extended" construction is the string-figure "Two Hogans" [A.C. Haddon: "A Few American String Figures and Tricks", p. 217].

Two Hogans: O.A: $\overrightarrow{M}$ (both central crossings) | : □2.□1→h∞:
5∞→W: : $\overrightarrow{5}$↓(Wf) : 5↑(h∞) : $\overleftarrow{5}$ (Wf) : 5↓(o;W∞) : : □M].

This lacks ">5∞" of being the Crows Feet, extended on W and 5. Notes on the construction: (1). The pick-up move with M refers to the crossings x1, x2 of Fig. 46.I (O.A) -- thus four strings are picked up by M. As there can be no confusion in this case, we may encode this movement in the Calculus as "$\overrightarrow{M}$ (c-x $^{(2)}$)" --Mouth picks up both central crossings. (2). At the stage "□1→ h∞" we have the gross schema

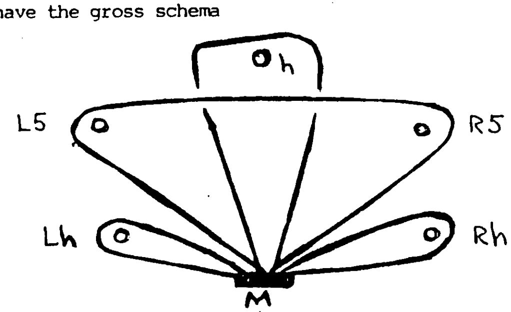


Fig. 53.I: Multiple h∞'s

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* Note the three distinct -- but similar appearing --
  ± -- parity of a simple crossing ("plus or minus")
  ≠ -- is not equal to ("denial of equality")
  ǂ -- loops not distinct

where nodes have been introduced for the mouth (M) and the three distinct hanging loops. The move "□2" has created Lh∞ and Rh∞ (which we have not bothered to name, in the construction, as they play no part therein), while "□1" has created the h∞. (3). The move "5∞→W" produces the above schema with the frame nodes L5, R5 relabeled LW and RW, respectively. (4). Basically, we create the schema

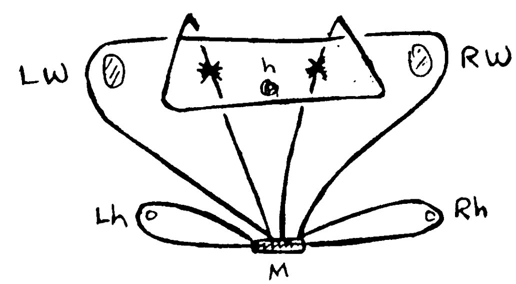


Fig. 53.II: A basic configuration, I

from the h∞, and then reach 5 over, and down, to pick up the lateral strings of the h∞ (before they loop about Wf) at the point(s) marked ✱ in the above figure. These strings would usually be described as s;M-Wf -- in the previously developed notation -- here we have chosen to use them as a vehicle to introduce the alternate perspective, that they are the unique string cutting the W∞ (on either side). Thus we introduce the notation

o;W∞ -- the oblique string cutting W∞

the choice "s;W∞" being inappropriate, as no starting point nor destination is given for this construct. This is a second usage of the concept "o -- oblique" [cf. page 122, middle] -- being a "default" definition; the string in question fails to meet the criteria for an "s"-designation -- hence, it receives an "o"-designation. (5). The basic central triangular configuration (with minimal variations) is fundamental to several advanced, serial, constructions of figures of "Crows Feet"-type; it will be met with again in the subsequent constructions. (6). Often, as in this case, an alternate final extension is sometimes given for a Crows Feet-type figure. To effect this, seize (one or) both central strings of the final figure -- running from "foot" to "foot" -- with M, pull hands away from M, and extend with both palms facing you.

The second construction is the string-figure "New Guinea Birds Feet" [H.C. Maude and C.H. Wedgwood: "String Figures From NOrthern New Guinea", Fig. 2 -- "Four Gourds"]

New Guinea Birds Feet: O.A: □R2|: $\overrightarrow{R}$ (R5n)# $\overleftarrow{R}$5 (R1f)#| $\overleftarrow{R2}$↓(L2∞): □L2#|:: $\overleftarrow{L2}$↓(R2∞): L2 (Rp$^{(2)}$)#|□R1:□R5] $\overrightarrow{M}$ (L2n$^{(2)}$): □2#:: R1↑(L5∞):□L5#| 1∞→5|: $\overrightarrow{2}$ (s,M-5f):□M#].

This lacks "<5∞" of being the Crows Feet. Notes on the construction: (1). At the first occurrence of "]" in the above sequence of manipulations, we reach the string-position whose "schema" is given by

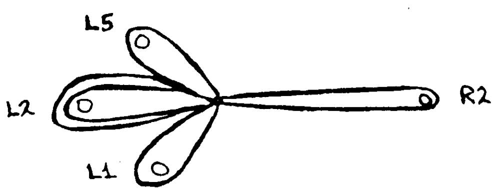


Fig. 54.I: Intermediate "schema" for New Guinea Birds Feet.

Note that, strictly speaking, the two loops on L2 preclude this from being an actual schema, as currently defined. Of course we could designate the L2∞$^{(2)}$ as an uL2∞ and a ℓL2∞ and resolve Fig. 54.I into a traditional schema; but this is to abandon the "spirit" of the construction; the L2∞$^{(2)}$'s are carried as one "doubled loop" -- and, at some later time -- we must extend the definition of a "Schema" to embrace this actuality. [Note that the "associated linear sequences" will also have to be generalized at that time]. (2). At the antepenultimate step of the construction, "1∞→5|", we again meet a "basic configuration" depending from M. Here

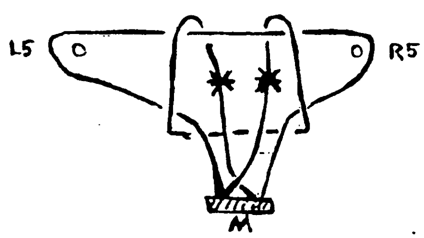


Fig. 54.II: A basic configuration, II

and 2 is to bend away and down to pick up this s;M-5f at the point(s) marked ✱ on either side (i.e. L2 picks up left-most ✱ , R2 picks up right-most ✱ -- as in the previous, and all(!) such, occurrences of a basic configuration). Had the intent been for L2 and R2 to pick up the opposite strings at ✱ , we should have encoded this "$\overrightarrow{2}$ (s;M-5f, as in O.A)", for example.

The third extended construction of the Crows Feet is the serial figure, the "Fu-fu Stick" [J. Parkinson: "Yoruba String Figures", Fig. 4]:

Fu-fu Stick: O.A: $\overrightarrow{1}$ (2f & 5n)‡#□5| $\overleftarrow{2345}$ (1n$^{(3)}$)#□1|: $\overrightarrow{1}$ (u2n$^{(2)}$)# <<2345(□2345d$^{(3)}$): [$\overleftarrow{2345}$ (1n$^{(2)}$)#□1|: $\overrightarrow{1}$ (u2n$^{(2)}$)# □2345d$^{(2)}$#|]^2: $\overleftarrow{2345}$ (1n$^{(2)}$)#□1|:: $\overrightarrow{1}$↓(ℓ2∞): 1 (ℓ2f)#| $\overleftarrow{345}$ (1n)#□1|: 1 (ℓ2n)# $\overleftarrow{2345}$ (1n)#□1#(H5)].

This gives the figure "Pang-pa-ta" ["facemark"].

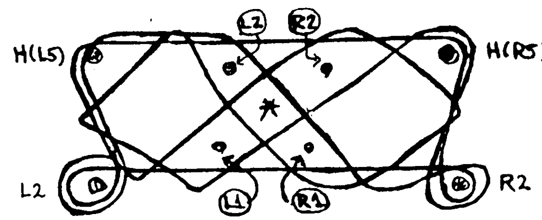


Fig. 55: Pang-pa-ta.

Lay figure flat:□B:: insert (L1), (R1), (L2), (R2) down into central configuration as indicated in Fig. 55: pass L1, R1, L2, R2 to center under the respective intermediary strings, and bring them directly up through central quadrilateral, at point marked ✱ in Fig. 55#].

This produces pseudo-Crows Feet as the final design.

Notes on the construction: (1). This is one of the most repetitive constructions known to the author. It is truly remarkable in this regard. (2). In the final extension of the intermediate figure, "Pang-pa-ta", the little fingers naturally hook forward and down over the 2n$^{(2)}$'s to the inside of its lateral crossing by two of the 2345-d strings (which loop about 2n$^{(2)}$-s). The extension is on "#(H5)". (3). String-figures with gross "schema" identical to that of "Pang-pa-ta" are met with throughout the world, and these will be discussed further in the following. For the nonce we remark that in none of these is there a "source" report of a continuation to (pseudo-or ordinary) Crows Feet. The extraction of pseudo-Crows Feet from this figure is, in fact, little short of miraculous.

The final figure in our serial Crows Feet tetralogy is "Tutae Takahuri 2" [J.C. Andersen: Maori String Figures, No. 19]. It is to be considered "invented" (or, rather, "contrived") in that it arose from the author's misconstruing of the final extension of the ultimate design in the "source" string-figure series. To emphasize, there is absolutely no "source" confirmation for this construction of a Crows Feet-type figure; it is included here merely as an interesting, beautiful way to produce this final design. [Some strings in intermediate schema appear in red to emphasize the bilateral symmetry of the situation. This is not discussed further in the present exposition].

Tutae Takahuri 2: O.A::: Pass 𝕃 to near side of ℝ: with 𝕃 fingers pointed away, L1*2(ℝ-s;all):□ℝ: >𝕃:: R1↑(L1*2nꝏ): R2↑(L1*2mꝏ): R5↑(L1*2fꝏ)::□L1*2#|1ꝏ→2:: 1↓(ℓ2ꝏ): 1(5n)#□5|: u2ꝏ→1#| [2345(u1n): 2345(ℓ1n)#□ℓ1ꝏ(N1)| 2345ꝏ→1]^{2}: 1. H45↓(1ꝏ$^{(2)}$)| 1(2n)#: (ℓ1ꝏ$^{(2)}$)N1: <2ꝏ→1: 2(o$^{(2)}$;1ꝏ$^{(2)}$)#(H45):□1](<ℝ).

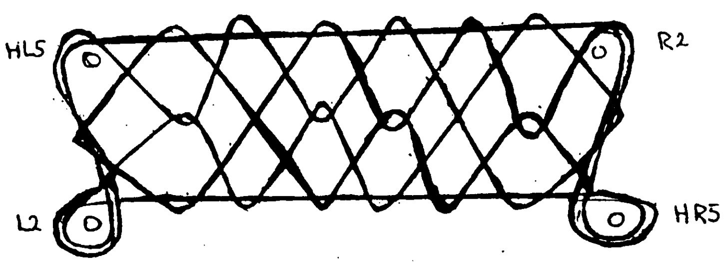


Fig. 56.I: Tutae Takahuri 2; intermediate figure #1.

We continue with the construction:

2. >ℝ: <2ꝏ$^{(2)}$:: 2↓(2ꝏ$^{(2)}$): H2(o-n;2ꝏ$^{(2)}$): >2(#) [□former 2ꝏ$^{(2)}$ in the process]:: 1↑(H5ꝏ$^{(2)}$)#(1)□5#|: H345(all strings)::>𝔹: 1*2(all strings-c):□345|(widely, keeping tips of 1,2 together pointing down): <2(#): <1(below 2ꝏ)#|::∿1.].

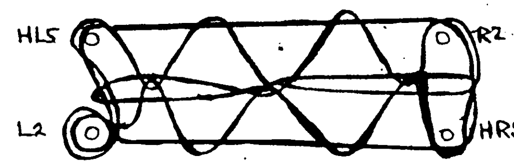


Fig. 56.II: Tutae Takahuri 2; intermediate figure #2.

We continue with the construction:

3. >ℝ:□2#(H5): <HR5ꝏ$^{(2)}$(straightening out this loop): 1↑(H5ꝏ$^{(2)}$)#(H5)] [Tangata-tahae, "the thief"].

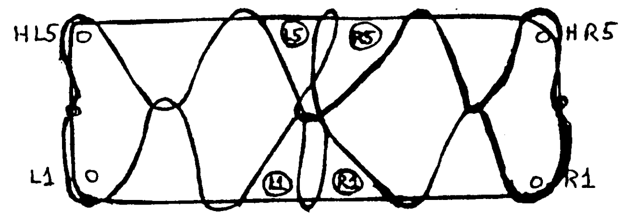


Fig. 56.III: Tutae Takahuri 2; intermediate figure #3, Tangata-tahae.

We contine with the construction:

4. With fingers pointing up, lay figure flat:: Pass L1,R1 away under near string of figure and bring them up at (L1), (R1) of Fig. 56.III, above: Pass L5,R5 towards you under far string of figure and bring them up at (L5), (R5) of Fig. 56.III above::#| >1ꝏ→2: 1(2n & 5n)#□2ℙ.

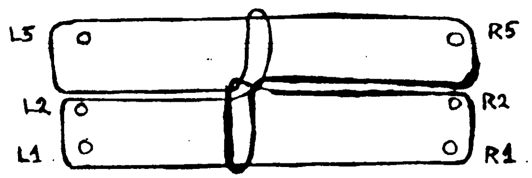


Fig. 56.IV: Tutae Takahuri 2; intermediate figure #4.

This figure is also "source"-identified as "Tangata-tahae". To extract Crows Feet from this figure, we continue:

5. □1:<$\overleftarrow{2\infty}$→1::$\overrightarrow{M}$(L1n):□L1:$\overleftarrow{L5\infty}$→L1::L5↑(R5∞):□R5# $\overrightarrow{R1\infty}$→R5# R1↑(M∞)#□M].

This figure lacks ">1∞:<5∞" of being pseudo-Crows Feet. To extract pseudo-Crows feet from Fig. 56.III directly, construct that figure, and lay flat, as before.

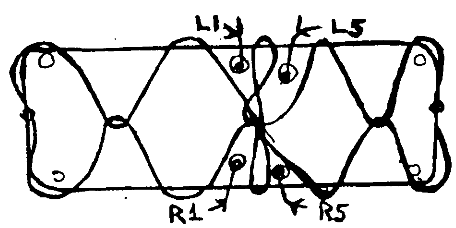


Fig. 56.V: Tutae Takahuri 2, intermediate figure #3'; Tangata-tahae.

4'. This time pass L1,L5 towards you under far string of figure, and pass R1,R5 away from you under near string of figure, and bring them up in spaces so designated (in Fig. 56.V). Extend fingers widely].

Again, this lacks ">1∞:<5∞" of being pseudo-Crows Feet.

Notes on the construction: (1) The first complex move of the construction, from ":::" to first "#|", has the function of removing all R loops -- not allowing them to twist or change relative positions -- turning them directly over, away from you, and replacing them on R1, R2, R5; the former R1 loop is turned over to R5, the former R2 loop is turned over to R2, the former R5 loop is turned over to R1. (2) The subsequent sequence in "brackets squared" has come to be known as the "Gilbert Movement", GM: specifically,

G.M. ≡ [$\overleftarrow{2345}$(u1n):2345(ℓ1n)#□ℓ1∞(N1)|$\overleftarrow{2345\infty}$→1]^2.

It occurs frequently in the string-figures of Oceania, where it is frequently followed by "<<1". (3) The sequence of moves (1.) designated as Step 1 of the figure -- a method of grouping long symbol-strings which bear later repetition -- has come to be known as the "Gilbert Extension", I_G: Specifically

I_G = H45↓(1∞$^{(2)}$) L1(2n)#(H45):(ℓ1∞$^{(2)}$)N1:<$\overleftarrow{2\infty}$→1:$\overleftarrow{2}$(o$^{(2)}$;1∞$^{(2)}$) #(H45):□1].

(4). The extended, complex movement of Step 2 which begins with the symbols "$\overleftarrow{H345}$(all strings)", and proceeds to "#|" has the effect of "<<1"! Thus it is a fancy Maori way of twisting the 1∞$^{(2)}$ a full-twist towards you. It is one of those movements that "feels good in the hands" to do, and is exceptionally satisfying. (5). The move "<HR5∞$^{(2)}$" in Step 3 of the construction means "remove the double loops from HR5, give them a half-twist towards you, and replace them on HR5." The R5-finger retains its Hooked-position throughout the movement. (6). In Step 4 of the figure, the initial movement, "With fingers pointing up, lay figure flat", it is to be remarked that, with the fingers in Hooked-position, they are pointing up when the knuckles are turned down, away from you. That is, the figure is laid flat by turning it directly away, and down, to a flat surface, and then □B. (7). The symbol-string from "::" to the end,"□M]", of the construction in Step 5 of the figure has the function of rotating the loops of the figure one finger, counterclockwise. Diagrammatically,

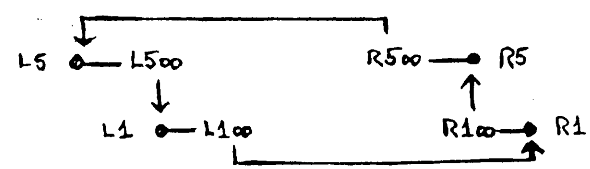


Fig. 57: Rotating loops one finger counterclockwise.

Thus, this move effectively rotates the whole figure 90° counterclockwise in its own plane.

We next present two constructions of the pseudo-Crows Feet -- one a "source" figure, the other contemporary -- from opposite sides of the Earth.

The figures begin on the feet, and knees, respectively -- and, though distinct, exhibit an obvious interrelation. The first ③ is "Wooden Spoon" [W.A. Cunnington: "String Figures and Tricks from Central Africa, No. 5]; The feet (F) are required in the construction:

Wooden Spoon: Loop on both feet: $\overrightarrow{H5}$($\overline{Fn}$):: 2(H5n): $\overrightarrow{H2}$($\overline{Ff}$): pull Ff on H2 back to near side of H5∞:: reverse 2's direction in its own loop: $\overrightarrow{H2}$($\overline{Ff}$): pull this string back thru former 2∞:□F].

The figure is extended on H2, H5, knuckles down, fingers curled up. This is pseudo-Crows Feet, extended on H2, H5.

The second figure requires the knees for its construction:

Feet of a Certain Bird: Loop on both knees (with minimal slack): B(Near knee string): 1*2(far knee string): pull this back towards you under near knee string: (1*2)-string→W:: $\overrightarrow{H5}$($\overline{\text{far knee string}}$): pull this back thru former W∞:: rotate 1 to inside and down (by raising the elbows): 1(near knee string):: □ knees#].

This lacks ">1∞" of being pseudo-Crows Feet. These two bizarre constructions occupy their own interesting, distinguished niche in the hierarchy of the Crows Feet complex of figures. See also C.F. Jayne, p. 358.

We next turn our attention to the construction of "Crows Feet"-type figures with >2 toes per foot. The first of these is contemporary, and is the sole example of a three-toed figure known to the author. [cf. C.F. Jayne, Fig. 825].

Raven, His Feet: O.1: $\overrightarrow{M}$(5f)#|(moving B well away from M): □M→h∞: $\overrightarrow{M}$(1n-c)#|:: $\overleftarrow{R2}$↓(M∞): R2(Lp)#:: $\overrightarrow{L2}$↓(R2∞ & M∞): L2(Rp)#□M|[this is O.A thru the M∞]: >1∞→1: <5∞→5: X2(R)#](sharply).

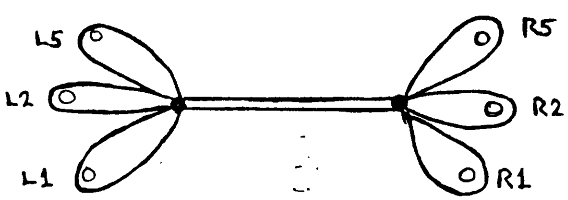


Fig. 58: Raven,His Feet.

Recall that "X2(R)" means "exchange 2∞'s, passing Right over Left" (see page 27 for discussion). The final extension must be "snapped" sharply, in order for the three distinct "toe"-loops on either side to properly engage and form the respective central (complex!) crossings. There is a well-defined "knack" to this proper extension, which generally requires two or three attempts to master; once accomplished, the figure is -- indeed -- satisfying. Until then, however, one or both "feet" of the figure will collapse into a pitiful tangle about the base of the fingers of the respective hand. The same comments also apply to the following, four-"toed" figure.

The following is a four-"toed" figure ④ known as the "Wapishana Bush" [F.E. Lutz: "String Figures From the Patamona Indians of British Guiana", pages 6-7]:

The Bush: O.1: 1. $\overleftarrow{HR2}$($\overline{Lp}$):>R2: $\overrightarrow{L15}$↓(R2∞):□R2#| 2.∿1. (√L\R↗): $\overleftarrow{R2}$↑(L1p∞): $\overleftarrow{R3}$↑(L5p∞):: $\overrightarrow{L2}$↓(R2∞): L2↑(R1p∞): $\overrightarrow{L3}$↓(R3∞): L3↑(L5p∞)#| 3. (1n-s)N1: (5f-s)N5#| X2&3(R)#](sharply).

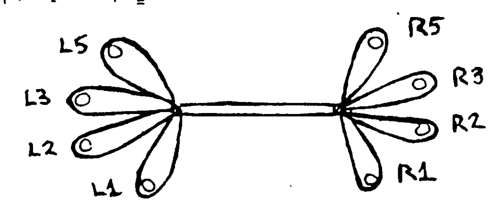


Fig. 59: Wapishana Bush.

Notes on the construction: ①. The complicated "Opening" required for this figure comprises Steps 1 and 2 of the construction. ②. In Step 2 of the construction, R2 passes to the left and picks up, from below, the s;L1n-L5n near the base of L1; as this string (and its continuation) appear to form a loop on the palmar aspect of L1, we have encoded this move, simply, as "$\overleftarrow{R2}$↑(L1p∞)", et cetera. ③. Thruout the figure the 2 & 3-fingers move in concert; i.e. simultaneously. Thus the final move, "X2 & 3 (R)", finds these two sets of exchanges done in the same movement.

Of related, but definitely peripheral, interest are figures consisting of one "Foot" only -- on either hand -- with multiple "toes". Such designs are generally a level of difficulty less, as to their construction, than is met with in the the Crows Feet complex of figures -- and the popular, commercial, string-figure literature fairly abounds with them. We present two examples; the first is the "Fish Spear" [J. Leeming: Fun With String, page 139]:

Fish Spear: O.1::R2(Lp):>>R2# L2↓(R2∞):L2(Rp)#|□R1 & R5].

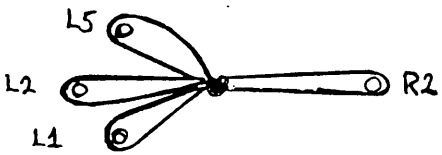


Fig. 60: A Fish Spear.

A four-"toed" version (on R) is called the "Parachute" [R.M. Abraham: Easy-To-Do Entertainments and Diversions, No. 117]: The opening is O.1 on R only–

Parachute: O.1(R)::L1*2(Rp): draw Rp back between R1 and R2; across the back, and to the far side of R; then towards you and up through the h∞, and hang it (as a loop) on R2 (without twisting):□L1*2::L1*2(R5n): draw R5n towards you across Rp (and over R2∞) and hang it (as a loop) on R1: □L1*2:: Turn R palm down, fingers pointing away and slightly left: L1*2 (R2345d): draw R2345d over the tips of the R fingers, and off R: pull this string directly downward] ("see-sawing" R).

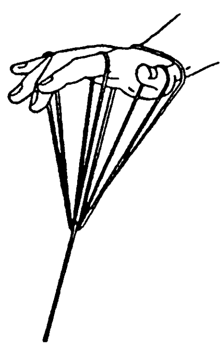


Fig. 61: The Parachute.

The above examples are typical of their genre in that their constructions do not easily extend to a "two-handed" figure. That is, the figure consisting of a "left-hand Parachute joined to a right-hand Parachute at either end of the doubled central string" has no easily-derivable construction from that of the parent figure.

For the final complex of figures of the present section, we return to more substantial matters very closely allied to the Crows Feet complex under discussion. The figures in question all have gross-schema

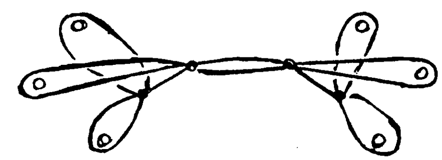


Fig. 62: The Pepper Plant.

The frame-nodes in Fig. 62, above, have not been labeled, as the three constructions are extended on various functors; all four complex-crossings involved are type X_9 or X_{10}. One of the constructions, below, appears with a different name -- "Pumpkin" -- but we have adopted the older "source"-designation for the triad of designs. (5)

#1. "Pepper Plant" [cf. J. Elffers and M. Schuyt: Cat's Cradles and Other String Figures, p. 195]:

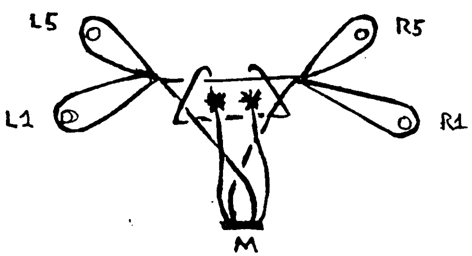
Pepper Plant #1: Make the Crows Feet:5∞→W:1∞→2:Crows Feet ending (on L2 and R2)].

For the Crows Feet ending, see page 116; the "setup" opening, O.A, is -- of course -- omitted from the sequence of manipulations which comprise the Crows Feet ending.

#2. "Pepper Plant" [D. Jenness: "Papuan Cat's Cradle", No. 9, "Mashiri Mana"]:

Pepper Plant #2: O.A::H2↓(5∞):>2(#):□5| H5(2f$^{(2)}$)#(H5)|:M(1n): M(c-X)#(H5):□1| 1(u2n-s):□2 & 5#| >1∞→15|:O.A:1(5n)# 2(1f)# N1:□5| 5(u2f)#□2|(gently)::2↓(M∞):2(s;M-1f)#□M]

We remark that, at the point "□2|(gently)", we have arrived at the schema



[A basic configuration, III]

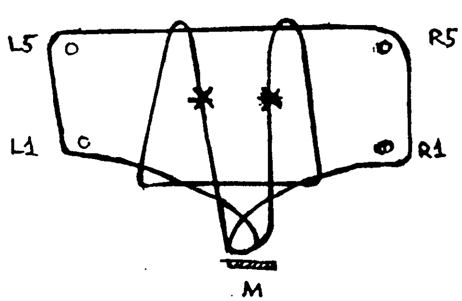
Fig. 62.A: Pepper Plant #2, after "□2|(gently)"

where the previously discussed "basic configuration" for the Crows Feet (cf. Fig. 53.II and Fig. 54.II) once again distinguishes itself. The subsequent move of the construction -- 2 picks up s;M-1f at ⚹ -- must surely, by now, have been well-anticipated. And, we encounter this "basic configuration" once again in the distinct -- but closely related -- figure in our construction trilogy.

#3. "Pumpkin" [P.D. Noble: String Figures of Papua New Guinea, No. 103]:

Pepper Plant #3: O.A:1. H2↓(5∞):>2(#):□5: 2. 5↑(2∞$^{(2)}$)#|: M(1n):M(c-X)#□1|: 1(u2n-s):□ 2 & 5#| >1∞→15|: O.A: ∾1.:5(1f)#| 1(ℓ2n-s)# N1|:3↓(M∞):3(s;M-1f)#□M:□2].

We remark that, after completion of the movement ">1∞→15|" in the above two constructions, we have produced a string-position whose schema is given by



[A basic configuration, IV]

Fig. 62.B: Pepper Plant #3, after ">1∞→15|".

This is the "basic configuration", which maintains itself until the final "pickup" move of the construction. And that move is to pick up the s;M-5f at ⚹ in both figures. Note that, if we perform this move (say, with 2) on the "basic configuration" of Fig. 62.B itself, then the subsequent move, "□M", results in the Crows Feet, itself. extended on 15 and 2. The remainder of the manipulations in Pepper Plant #2 and #3 -- from the schema of Fig. 62.B to the final pickup move -- are two distinct executions of the Method II.B-construction of the Crows Feet, executed on the near loop of the Crows Feet arising from the "basic configuration". Viewed in this way, these latter two figures are very closely related, indeed -- and not so different from the simple-minded construction of the first entry in this sequence, after all. Further, the basic importance of the figure "Two Hogans" (page 125) to the present line of analysis cannot be overstated -- it was this figure we chose as a vehicle for the introduction of the "basic configuration" of the Crows Feet complex.

We conclude our discussion of the Crows Feet complex of figures by briefly addressing the extensions of the theory necessary for the introduction of multiple loops/strings into the schemata and their associated linear sequences [as promised earlier, cf. page 127]. In terms of our earlier discussion, we should anticipate that this extension will be analogous to the passage from simple crossings to complex crossings -- essentially through the introduction of an X-Dictionary -- witnessing the performer's evolving recognition of ever more complex sub-constructs in the tangled mass of string between his hands; that is, a "lift" of consciousness level in which diverse simple constructs with well-defined, consistent, interrelations between them come to be "recognized" as complex objects (at a higher level) through repeated encounters with them. With that in mind, we return to our earlier discussion -- which introduced "complex crossings" and the X-Dictionary -- with an eye to an "easy" extension thereof to embrace the complexities of the current, more general, situation. (6)

In terms of the "ant crawling along the string" analogy, we generalize the concept of a "freeway interchange" -- i.e. complex crossing -- to include conduits essential to the structure of the "interchange", but ones through which he cannot pass during his round trip: the "frame-elements" (functors). One may think of the "railroad tracks" which pass through many such interchanges in this country (e.g. the one in South Buffalo, New York) by way of analogy; they wind through the interchange -- and influence its structure -- but the motorist has no access to them.⑦ So viewed, a string-frame interaction may be considered, merely, as **a substitution instance of a "generalized-complex crossing" (a g-crossing) from** a generalized $\mathbb{X}$-Dictionary (an $\mathbb{X}^{(g)}$-Dictionary). As before, an entry in the $\mathbb{X}^{(g)}$-Dictionary will consist of several labeled constructs -- the first⑧ of which will be the functor involved, by convention -- each with an arbitrarily assigned orientation; in a schema which delineates their precise mutual interactions. For example, perform the manipulational sequence

O.A: give L1f an extra twist about L1 by bringing it towards you across the palm of L1, then passing it left, behind, and then back to the far side of L1 (i.e. "clockwise"): $\overrightarrow{L1}$ (L5n) # |

and consider the complex string-frame interaction at L1. It is well-describable by the $\mathbb{X}^{(g)}$-Dictionary entry-candidate

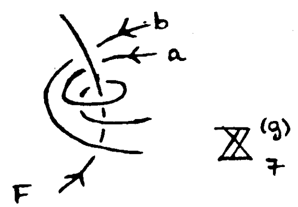


Fig. 63: Potential $\mathbb{X}^{(g)}$-Dictionary entry

which has been given a "fictitious" subscript, "7". Now let us suppose that the ant, crawling along our previous string-position has just left R1 along the ln-s string, bound for L1. The next entry in the linear sequence associated to his journey -- his "trip log" -- would be

L1(7,+,a-);

that is, he is approaching L1 as an $\mathbb{X}_7^{(g)}$-type of intersection, L1 is in # (the "+" in the second argument), and his route is along the "road" labeled "a" in the $\mathbb{X}_7^{(g)}$-diagram, in the direction opposite to that in the given schema. Note that the functor, F, in $\mathbb{X}_7^{(g)}$ is identified as the L1 of the present notation, and the second argument -- the functor's "status" -- can be

+ — #(F)
- — #(HF)
o — weak.

Until we have need of the full generality of the above notation, we shall simply refer to "functor plus status" by the symbology

F — #(F)
HF — #(HF)
h — weak

as labels for the various frame-nodes.

Now consider the case of multiple loops on a given functor; specifically, perform the sequence

O.A: $\overrightarrow{1}$ (5n) ǂ #□5 |: <2∞# |

in the string.

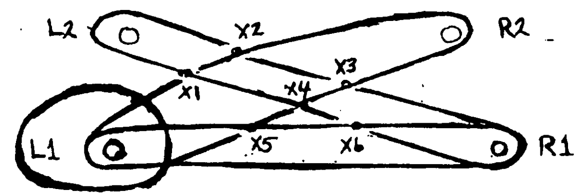


Fig. 64: "Schema" for multiple l∞'s.

We agree to accept as (generalized) schemata representations of string-positions which entail multiple loops. This generalization carries an unavoidable cost -- the loss of information about structure near the node supporting the multiple loops. E.g. Which L1f-string is the continuation of the ln-s string in Fig. 64? In the absence of further, more detailed information, there is no way of knowing. Of course the (fictitious) appropriate $\mathbb{X}^{(g)}$-Dictionary entry $\mathbb{X}_4^{(g)}$ (below)

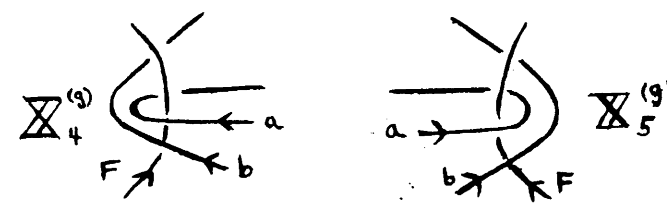


Fig. 64.A: Potential $\mathbb{X}^{(g)}$-Dictionary entries

contains the answer: L1(4,+,b+) -- but, as with the gross-schemata in general -- the multiple-loop schemata, per se, elide such information. And, just as the complex schemata -- with their unavoidable information loss -- have proven to be useful in a variety of "visualizational" contexts; so, too, will the multiple-loop schemata have their worthwhile place in the scheme of things. As a final remark on these present matters, we present the linear sequence associated to the string-position of Fig. 64: We choose a point on the ln-s string as the initial point of this sequence (Note: our convention about such matters does not cover the present example).

$\mapsto$ L1(4,+,b+): x1(U): x2(∅): R2: x3(∅): x4(U): x5(U): L1(4,+,a+): x5(∅): x6(∅): R1(5,+,a-): x6(U): x4(∅): x1(∅): L2: X2(U): x3(U): R1(5,+,b-) ■

Note that L1 and R1 both occur twice in the associated linear sequence, above, as the number of multiple loops on each is two. In general, a functor incident with exactly n multiple loops in a given string-position will occur exactly n times in the linear sequence associated to this position.

We conclude the present section with an example (of a multiple-loop schema): Perform the manipulational sequence

O.A: give L5f an extra twist about L5: $\overleftarrow{R5}$(L5p) ǂ # $\overleftarrow{5\infty}^{(2)}\rightarrow$ 2|

The (g)schema for this string-position is

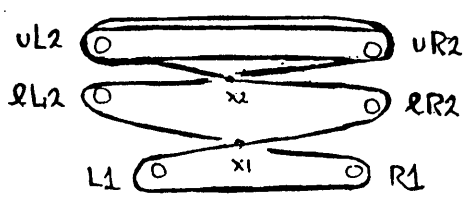


Fig. 65: (g)Schema, an Example.

Note in particular, that the u2∞$^{(2)}$ are not distinct from one another, but that they are distinct from ℓ2∞. Further, there are two u2f-strings; had they twisted around one another in any way, this would have had to have been recorded, in terms of schema-crossings.

---- End Crows Feet Discussion ----

CROWS FEET NOTES

① (page 112) cf. also "Duck Feet I" [P.H. Buck: "Samoan Material Culture", p. 558]:

Duck Feet I: O.A:>$\overrightarrow{1\infty}$→3::$\overleftarrow{5}$(3f):$\overrightarrow{5}$(5n)# N5:□1|

which further exemplifies these equivalent sequences of string manipulations.

② (page 117) Note that the complex crossing x1 in the gross-schema of Fig. 47 is now basically an X_7-type crossing, as earlier anticipated; thus the appellation "pseudo-Crows Feet". All previous figures in this section have entailed the X_8-type crossing at x1 (of Fig. 47), as in the original construction of the design. [cf. Fig. 50]

③ (page 133) See also "Matemo" (the Hoes) [M.D. & L.S.B. Leakey: Some String Figures From North-East Angola, p. 9].

④ (page 134) A nice photograph for this "source"-figure is to be found in C.E. Keeler: Cuna Art, page 145, where the name given is the curious "Two Shrimp Holes."

⑤ (page 136) The most recent "source"-reference for the string-figure "Pepper Plant" is the large collection of G. and B. Senft: Ninikula Fadenspiele auf den Trobriand-Inseln, Papua-Neuguinea, No. 42, where the name given is "Mweyadoga". Most unfortunately, the value of this collection is severely limited by its failure to include constructions for its figures, but only final designs. We remark that attempts after-the-fact to obtain such constructions from the literature -- or from string-figure authorities -- are unrealistic, and misguided with respect to the subject, itself. This last assertion is, we feel, perhaps one of the most important, implicit, corollaries of our present investigations.

⑥ (page 138) This discussion is to be found on pages 45-48; the interested reader may wish to refresh himself with respect to this material before continuing with the current discussion. We remark, however, that the previous discussion was written with the current level of complexity (and future levels) in mind, so the more technically sophisticated reader should find that the present discussion is self-contained.

(7) (page 139) The phrase, "You can't get there from here", heard often in Buffalo, perhaps refers directly to this phenomenon.

(8) (page 139) Our next generalization will allow for multiple functors in a given $\mathbf{x}^{(g)}$-Dictionary entry. In fact, this level of generalization is (at present) required for a precise treatment of "P" and "L1*L2", for example.

(9) (page 146) This table provides a particularly interesting, and useful, tabulation of the geographic distribution of the Crows Feet complex of figures. These matters are outside of the scope of the discussion in these notes on the subject, except for literature citation herein.

BIBLIOGRAPHY: CROWS FEET

Abraham, A.J.: STRING FIGURES.
Leashing of Lochiel's Dogs — Fig. 33

Andersen, J.C.: MAORI STRING FIGURES.
Matamata Karehu (a Tuhoe woman) — #17

Ariki, T.: AYATORI NYŪMON.
Futatsu Matsuba (Two Pine Needles) — p. 128

Armstrong, A.: MAORI GAMES AND HAKAS.
Matamata Karehu — #5

Averkieva, J.: KWAKIUTL STRING GAMES.
Two Devilfish — #37B, #38B

Ball, W.W. Rouse: FUN WITH STRING.
Crow's Feet — #8

Brewster, P.G.: SOME STRING FIGURES AND TRICKS FROM THE UNITED STATES.
Crow Foot — Fig. 2

Brewster, P.G. & G. Tarsouli: A STRING FIGURE SERIES FROM GREECE.
Scissors or Needles — Fig. 10

Buck, P.H.: SAMOAN MATERIAL CULTURE.
Duck's Feet, I & II — p. 558

Culin, S.: GAMES OF THE NORTH AMERICAN INDIANS.
Hogan — Fig. 1046
Crossbeam of Ladder — Fig. 1072

Cunnington, W.A.: STRING FIGURES AND TRICKS FROM CENTRAL AFRICA.
Wooden Spoon — #5

Davidson, D.S.: SOME STRING FIGURES OF THE VIRGINIA INDIANS.
Crow's Feet — Fig. 143

ABORIGINAL AUSTRALIAN STRING FIGURES.
Emu Foot — #14

Elffers, J. and M. Schuyt: CAT'S CRADLES AND OTHER STRING FIGURES.
Crow's Feet — p. 195

Gordon, G.B.: IN THE ALASKAN WILDERNESS.
Sent "Crow's Foot" to K, Haddon — p. 130

Gray, J.: SOME SCOTTISH STRING FIGURES.
Leashing of Lochiel's Dogs — Fig. 4 (4)

Griaule, M.: JEUX DOGONS.
Patte-d'oiseau — p. 78

Gryski, C.: MANY STARS AND MORE STRING GAMES.
Leashing of Lochiel's Dogs — p. 30-31

Haddon, A.C.: A FEW AMERICAN STRING FIGURES AND TRICKS.
Two Hogans — p. 217

Haddon, K.: CAT'S CRADLES FROM MANY LANDS.
Two Hogans — #32
Leashing of Lochiels's Dogs — #48

SOME AUSTRALIAN STRING FIGURES.
The Flying Foxes — #19

Haddon, K. & H.A. Treleaven: SOME NIGERIAN STRING FIGURES.
A Drum — #1

Hornell, J.: STRING FIGURES FROM FIJI AND WESTERN POLYNESIA.
Balawa (ScrewPine Tree) — Fig. 1
Vaepato (Duck's Feet) — Fig. 63
Morna Pag (Peacock's Footprint) — Fig. 77

STRING FIGURES FROM SIERRA LEONE, LIBERIA, AND ZANZIBAR.
Bili Gutu (Short Drum) -- one foot -- — Mende #4
Ndopo Vanggi (Little Drum) — Mende #5
Fanka (Drum) — Temne #3
Kadir (Rice Mortar) — Temne #4
Karump (Rice Pounder) — Temne #5, #6
Koide Gerto (Fowl's Feet) — Fula #1
Hiniki Pel-le (Sleeping Hammock) — Sherbro #4
Lompo (Pidgeon's Tail) — Sherbro #5

STRING FIGURES FROM GUJARAT AND KATHIAWAR.
Morna Pag — #12, #13

STRING FIGURES FROM THE ANGLO-EGYPTIAN SUDAN.
Riglel Naam (Ostrich's Feet) — #6, #7
Riglel Wizza (Goose's Foot) — #8

Jackson, M. & M. & K. Watkins: STRING GAMES.
Brolga's Feet — Fig. 62

Jayne, C.F.: STRING FIGURES.
Leashing of Lochiel's Dogs — Fig. 267
Two Hogans — Fig. 273

Koch-Grünberg, T.: ZWEI JAHRE UNTER DEN INDIAREN REISEN IN NORDWEST-BRASILIEN.
Paxiuba-Palme (Photo) — Vol. II, p. 130

Kraus, H.: LUFAMBO.
Gude-Tree Representation — #5

Leakey, M.D & L.S.B: SOME STRING FIGURES FROM NORTH-EAST ANGOLA.
Matemo (The Hoes) — p. 9

Leeming, J.: FUN WITH STRING.
Leashing of Lochiel's Dogs — pp. 150-151

Maude, H.C.: STRING FIGURES FROM NEW CALEDONIA AND THE LOYALTY ISLANDS.
Shuttle for Fishing Net — #5

STRING FIGURES FROM TONGA.
No name — #12

Maude, H.C. & C.H. Wedgwood: STRING FIGURES FROM NORTHERN NEW GUINEA.
Four Gourds — #2

Métraux, A.: QUELQUES JEUX DE FICELLE L'AMERIQUE DU SUD.
No name — Fig. 6

Noguchi, H.: STRING FIGURES IN JAPAN, II.
Clow's Feet — #26

STRING FIGURES IN JAPAN, IV.
Nihon Bouki (Two Blooms) — #95

Parkinson, J.: YORUBA STRING FIGURES.
Omori-odo (Fu-fu Stick) — #4

Paterson, T.T.: ESKIMO CAT'S CRADLES.
Flying Loon — Fig. 4B

Ransom, W.R.: PASTIMES WITH STRING AND PAPER.
Leashing of Lochiel's Dogs — P. 62

Roth, W.E.: CRATCH-CRADLE IN BRITISH GUIANA.
Emu — Plate 4, Fig. 5
Four Prong Spear — Plate 12, Fig. 1

Saito, T.: AYATORI ITOTORI, Vol. II.
Nihon Bouki (Two Brooms) — #1

Schwab, G.: TRIBES OF THE LIBERIAN HINTERLAND.
Photo — Fig. 100.C

Starr, F.: ETHNOGRAPHIC NOTES FROM THE CONGO FREE STATE.
Mbangba (Throw-Knife) — #32

Tessmann, G.: DIE KINDERSPIELE DER PANGWE.
Fusse der Ente (Feet of the Duck) — #10

Tessmann, G., E.M. von Hornbostel, & K. Haddon: CHAMA STRING GAMES.
Romuo Tapo (Floor for Drying Tobacco) — #8
Table of Distribution of Bird's Feet (9) — p. 167

Tracey, H.: STRING FIGURES (MADANDI) FOUND IN SOUTHERN RHODESIA.
Mashiri Mana (Four Birds) #9, pic. #11

Trapp, K.: BANTU-SPIELE.
Hen's Toes Fig. 1

ADDENDUM: CROWS FEET

We append three additional constructions of the Crows Feet which further serve to illustrate the text material. The first is "The Flying Foxes" [K. Haddon: "Some Australian String Figures", No. 19]:

O.A: M (L2n) # □ 2 | : H2↓ (5n) : H2↓ (1∞) : <2 (#) (□ 5n-string from 2) | H1 (o; 2∞) (□ former 1∞) : □ 5 : □ M# (H1)] (fingers pointing away).

If we turn the fingers directly (away and) down, and lay figure flat, we have the classic Crows Feet pattern -- a Method II.B construction. The symmetric bilaterally specific Method II.A procedure, e.g.

O.A: T (L2f) # □ 2 | : H2↓ (1f) : H2↓ (5∞) : >2 (#) (□ 1f-string from 2) | H5 (o; 2∞) (□ former 5∞) : □ 1 : □ T# (H5)]

is an extremely reasonable production of pseudo-Crows Feet by a Method II.A construction -- but is, apparently, unknown as a "source" figure.

The second construction is of the peripheral "one foot" variety (cf. pages 134-136), which we include here only because it provides a second example of the Method I.A-technique. The original figure is "Bili Gutu" (Short Drum) [J. Hornell: "String Figures From Sierra Leone, Liberia, and Zanzibar; Mende No. 4] and the working is on R and two (!) toes; we have given an equivalent working on R and L1, L2:

O.1: □ 5 : R1∞ → R12 :: L2 (R12d) # R12↓ (L2∞) : R1*2 (L1f): draw L1f back up thru L2∞, draw this string towards you over all strings, and hang it on L1 (where it becomes the uL1n-string): □ R1*2 :: N (L1) # □ R2] .

The result is pseudo-Crows Foot (L) -- on the left hand only.

The final construction, like those on the feet and knees (page 133) is in a class by itself, and definitely has a "different feel" from the other constructions presented in this section. It is called "Romuo Tapo", which means " a floor for drying tobacco" [G. Tessmann, E.M. von Hornbostel, and K. Haddon: Chama String Games]:

O. Hang string on R2, so that there is a R2∞ and a long h∞: HL2*3 (R2∞) □ R2, so that there is a small L2*3 dorsal loop: turn this loop down over the tips of L2 and L3:: Using extreme low point of Lh∞, form the structure exactly similar to the above on R2 and R3: 1. R1*2 (L3n):

draw this string towards you and hang it over L2 (where it becomes uL2n-string):□R1*2#| 2. ∾ 1. (↶R\L↷): 3. $\overleftarrow{\overline{R1*2}}$ (L2n-s): lift this over the tip of L2 and deposit it between L2 & L3:□R1*2 #| 4. ∾ 3. (↶R\L↷): 5. $\overleftarrow{\overline{R1*2}}$ (L2p): draw this out:□R1*2| (sharply): 6. ∾ 5. (↶R\L↷) I.

V. THE BEAR

For our next topic we shall consider the many ways that one particular animal, the Bear, comes to be represented in the string. The "source"-literature, for obvious reasons, is almost exclusively generated by researches into the ethnology of circumpolar peoples (Inuit -- or "Eskimo") and -- although the number of bibliographic entries is somewhat less than that of previous sections, -- the tradition is a particularly rich one, which abounds with remarkable designs. In the exposition, below, we shall endeavor to give the "spirit" of two aspects of this rich tradition: The first is typified by minor constructional variations on a "Hearth"-figure -- usually, a Bear -- which may produce wildly dissimilar final designs. Such groupings of string-figures are usually referred to as a "string-figure cycle.". The second aspect is typified by wildly dissimilar constructive techniques which produce the same, or very similar, final designs -- again, usually a Bear. Often these different constructions, themselves, form the "Hearth"-figure of a string-figure cycle. Finally, we remark that the Bear is to be thought of as an example -- albeit a preeminently distinguished one -- of (classes of) animals whose string-figure representations may (and, ultimately, will) be so discussed.

BEARS. I.

The first aggregate of figures we shall discuss is the Brown Bear cycle [D. Jenness: Eskimo String Figures (Canadian Arctic Expedition), No's. 1-14]. It will prove useful, in this and many other Inuit constructions, to introduce the movement "Katilluik":

K ----- Katilluik.

In the Calculus,

K ≡ $\underleftarrow{R1}$↑(L1∞):□L1# $\underrightarrow{L1}$↑(R1∞$^{(2)}$) #|

Of rare occurrence -- but frequent enough to warrant its own symbology -- is the symmetric move

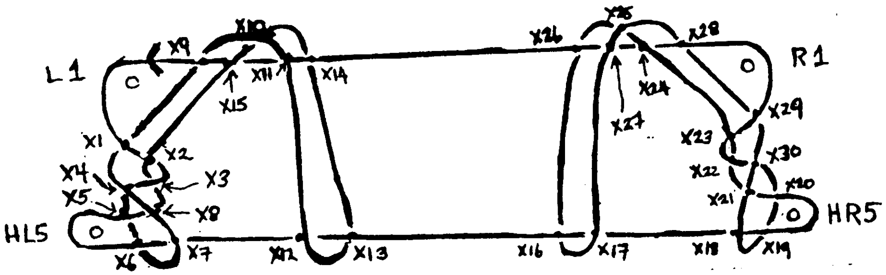
K(L) ≡ $\underrightarrow{L1}$↑(R1∞):□R1# $\underleftarrow{R1}$↑(L1∞$^{(2)}$) #|

Note that these two moves represent a new "interchange"-of-loops manipulation (cf. discussion, pages 26-28); the latter, K(L), is read "Katilluik Left". With these few preliminaries, we are ready to proceed with the "Hearth"-figure.

<u>Brown</u> <u>Bears</u>: <u>O</u>.A:2ꝏ→1: 1 <5ꝏ→2|:: 5↑(ℓ1ꝏ):5↑(2ꝏ):H5(2n)#(H5) (□ℓ1f from 5)|: H2(1n-s): <2(#)(□former 2ꝏ): N1|: 2 K: 1(<u>2n</u>): (ℓ1ꝏ(2))N1#(H5)|:□2:1ꝏ→2]:1(<u>o</u>;lower outside arm of c-◊)#(H5):1(<u>2n</u>)#(H5):N1:□2]

At the final extension, two Brown Bears separate from the central tangle of strings, and lumber off towards their respective hands.

<u>Note</u> on the construction: the "geometric" referent "lower outside arm of c-◊" uniquely specifies the "o;p-5f, whose continuation(s) form c-X". This matter will be further explicated in the subsequent constructional analysis of the figure.



<u>Fig. 66</u>: The Two Brown Bears; Schema.

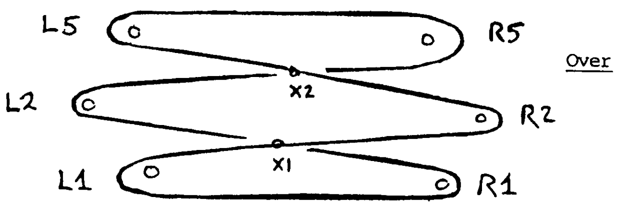
The associated linear sequence (whose starting point and orientation is demarked by a red arrowhead) is given by

⊨ L1: x1(∅): x2(U): x3(∅): x4(U): x5(U): x6(U): x7(∅): x8(∅): x4(∅):x1(U): x9(U): x10(∅): x11(∅): x12(∅): x13(U): x14(U): x10(U): x15(∅): x2(∅): x3(U): x8(U): x5(∅): HL5: x6(∅): x7(U): x12(U): x13(∅): x16(∅): x17(U): x18(U): x19(∅): HR5: x20(∅): x21(U): x22(U): x23(∅): x24(∅): x25(U): x26(U): x16(U): x17(∅): x27(∅): x25(∅): x28(U): x29(U): x30(∅): X21(∅): x18(∅): x19(U): x20(U): x30(U): x22(∅): x23(U): x29(∅): R1: x28(∅): x24(U): x27(U): x26(∅): x14(∅): x11(U): x15(U): x9(∅) ■

One should remark the (inverted) positions of fingers 1, 5 in the schema of Fig. 66; this is because the 5-finger is in "Hooked" position. Generally speaking, Hooked fingers appear to the near side of 1 in the canonical schema-representation, in inverted order. Also to be noted is the "viewpoint" represented by the schema -- <u>i.e.</u> that of the performer. Here, for the first time, we have a figure with no ("horizontal") midline symmetry -- and our previous perspective ("frame owner peeks

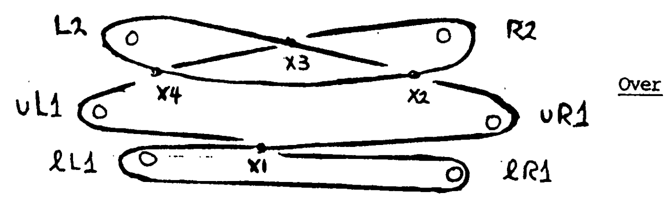
over tips of <u>fingers and looks directly down on string-position</u>") would entail an "upside-down" presentation of Fig. 66. And while this would surely (continue to) be entirely consistent, it would not constitute the best utilization of the schemata as visualization/learning tools with respect to the construction and its analysis. We shall denote these two perspectives as "<u>Over</u>" (as with the previous schema) and "<u>Under</u>" (as with Fig. 66); the latter viewpoint (and the ability to <u>change</u> perspective) is a Hallmark of Inuit string-figures.

We now proceed to an abbreviated constructional analysis of the Brown Bears again recording the crossing <u>alias</u> transformations as we go:



<u>Fig. 67.I</u>: Brown Bears, <u>O</u>.A.

⊨ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■



<u>Fig. 67.II</u>: Brown Bears, <u>O</u>.A:2ꝏ→1: <5ꝏ→2|.

⊨ ℓL1: x1(∅): uR1: x2(U): x3(∅): L2: x4(∅): x2(∅): R2: x3(U): x4(U): uL1: x1(U): ℓR1 ■

<u>Fig. 67.I</u>		<u>Fig. 67.II</u>
x1	⟶	x1
x2	⟶	x3

The crossings x2,x4 of Fig. 67.II are created by the movements "2ꝏ→1: <5ꝏ→2|" applied to Fig. 67.I.

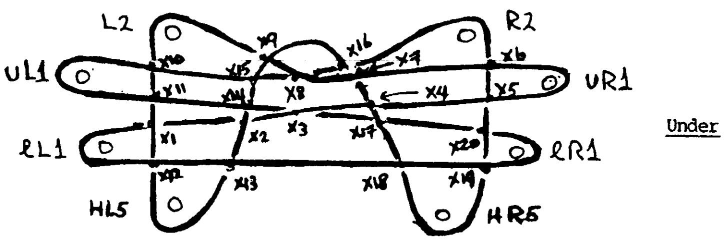


Fig. 67.III: Brown Bears, O.A: $\overleftarrow{2}\infty \rightarrow 1:\langle\overleftarrow{5}\infty\rightarrow 2\,|\,\underset{\leftarrow}{5}\uparrow(\ell 1\infty):\underset{\rightarrow}{5}\uparrow(2\infty):$ $\overleftarrow{H}5\,(\overline{2n})\,\#\,(H5)\,|$.

|⇒ ℓL1: x1(U): x2(U): x3(Ø): x4(Ø): x5(Ø): uR1: x6(Ø): x7(Ø): x8(Ø): x9(U): L2: x10(U): x11(U): x1(Ø): x12(U): HL5: x13(U): x2(Ø): x14(U): x15(U): x9(Ø): x16(Ø): x7(U): x4(U): x17(Ø): x18(U): HR5: x19(U): x20(Ø): x5(U): x6(U): R2: x16(U): x8(U): x15(Ø): x10(Ø): uL1: x11(Ø): x14(Ø); x3(U): x17(U): x20(U): ℓR1: x19(Ø): x18(Ø): x13(Ø): x12(Ø) ■

Fig. 67.II		Fig. 67.III
x1	→	x3
x2	→	x6
x3	→	x8
x4	→	x10

The crossings x1, x2, x4, x5, x7, x9, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20 of Fig. 67.III are created by the move "$\underset{\leftarrow}{5}\uparrow(\ell 1\infty):\underset{\rightarrow}{5}\uparrow(2\infty):\overleftarrow{H}5(\overline{2n})\#(H5)\,|$" applied to Fig. 67.II.

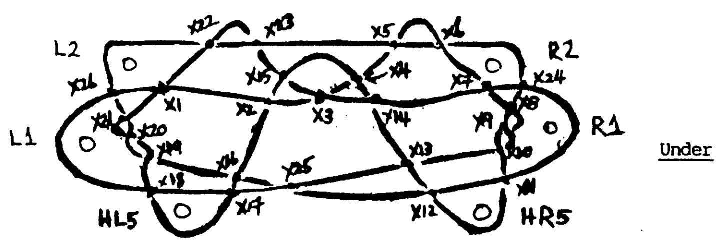


Fig. 67.IV: Brown Bears, O.A: $\overleftarrow{2}\infty \rightarrow 1:\langle\overleftarrow{5}\infty\rightarrow 2\,|\,\underset{\leftarrow}{5}\uparrow(\ell 1\infty):\underset{\rightarrow}{5}\uparrow(2\infty):\overleftarrow{H}5\,(\overline{2n})$ $\#(H5)\,|::\overleftarrow{H}2\,(\overline{1n-s}):\langle 2(\#):\ N1\,|$.

Fig. 67.III		Fig. 67.IV	Fig. 67.III		Fig. 67.IV
x1	→	x19	x11	→	x18
x2	→	x16	x12	→	x22
x3	→	x25	x13	→	x23
x4	→	x12	x14	→	x17
x5	→	x11	x15	→	x2
x6	→	x24	x16	→	x4
x7	→	x14	x17	→	x13
x8	→	x3	x18	→	x5
x9	→	x15	x19	→	x6
x10	→	x26	x20	→	x10

The crossings x1, x7, x8, x9, x20, x21 of Fig. 67.IV are created by the move "$\overleftarrow{H}2(\overline{1n-s}):\langle 2(\#):\ N1\,|$" applied to Fig. 67.IV. Note that {x19, x20, x21} of Fig. 67.IV form an Osage Diamond $\mathbf{X}_7$-type of complex crossing, while {x8, x9, x10} form an $\mathbf{X}_8$-type complex crossing; this is structural. For the associated linear sequence we have

|⇒ L1: x26(Ø): x1(Ø): x2(Ø): x3(U): x4(U): x5(U): x6(Ø): x7(U): x8(Ø): x9(U): x10(Ø): x11(U): HR5: x12(U): x13(Ø): x14(U): x4(Ø): x15(Ø): x2(U): x16(Ø): x17(U): HL5: x18(U): x19(Ø): x20(U): x21(Ø): x1(U): x22(Ø): x23(U): x15(U): x3(Ø): x14(Ø): x7(Ø): x24(Ø): R1: x11(Ø): x12(Ø): x25(U): x16(U): x19(U): x20(Ø): x21(U): x26(U): L2: x22(U): x23(Ø): x5(Ø): x6(U): R2: x24(U): x8(U): x9(Ø): x10(U): x13(U): x25(Ø): x17(Ø): x18(Ø) ■

At this point we interpose a schematic simplification before proceeding to the next manipulation in the construction of the Brown Bears. The previous schema, Fig. 67.IV, is a representation of string-position-so-far which is very useful for the accompanying crossing-analysis; but a simpler representation is to be obtained by rotating both hands slightly towards you (i.e. in the "<" direction) until the 1∞'s "clear" the central configuration. Of course, the resulting schema (Fig. 67.IV', below) must be "equivalent" to the previous schema (Fig. 67.IV) -- as both are representations of the same string-position. Thus, we find

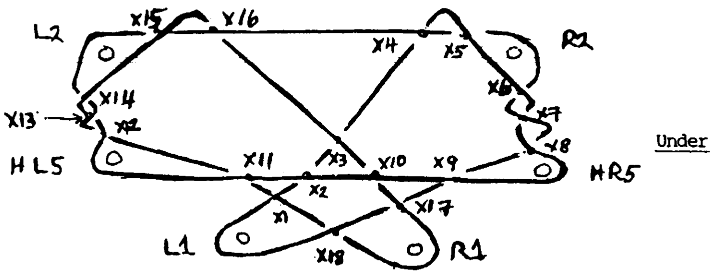


Fig. 67. IV': Simplification of Fig. 67.IV.

|⇒ L1: x1(∅): x2(U): x3(U): x4(U): x5(∅): x6(∅): x7(U): x8(∅): HR5: x9(∅): x10(∅): x2(∅): x11(∅): HL5: x12(∅): x13(U): x14(∅): x15(∅): x16(U): x3(∅): x10(U): x17(∅): R1: x18(U): x1(U): x11(U): x12(U): x13(∅): x14(U): L2: x15(U): x16(∅): x4(∅): x5(U): R2: x6(U): x7(∅): x8(U): x9(U): x17(U): x18(∅) ∎

For the "crossing-analysis" between these two equivalent representations, we find

	Fig. 67.IV		Fig. 67.IV'		Fig. 67.IV		Fig. 67.IV'
C.	x1	→	∅		x14	→	x10
	x2	→	x2		x15	→	x1
	x3	→	x3		x16	→	x11
	x4	→	x17	C.	x17	→	∅
	x5	→	x4	C.	x18	→	∅
	x6	→	x5		x19	→	x12
C.	x7	→	∅		x20	→	x13
	x8	→	x6		x21	→	x14
	x9	→	x7		x22	→	x15
	x10	→	x8		x23	→	x16
C.	x11	→	∅	C.	x24	→	∅
C.	x12	→	∅		x25	→	x18
	x13	→	x9	C.	x26	→	∅

Note that the crossings x1, x7, x11, x12, x17, x18, x24, x26 of Fig. 67.IV cancel pairwise -- {x1, x17} , {x7, x12}, {x11, x24}, {x18, x26} -- in transition to Fig. 67.IV', after (several applications, each, of) Lemma 2.B; this, as the 1∞'s slide over, and to the near side of, the central configuration of strings. Further, the crossings {x15, x16} and {x4, x5} are complex, of types $\mathbf{X}_5$ and $\mathbf{X}_6$, respectively -- and are structural.

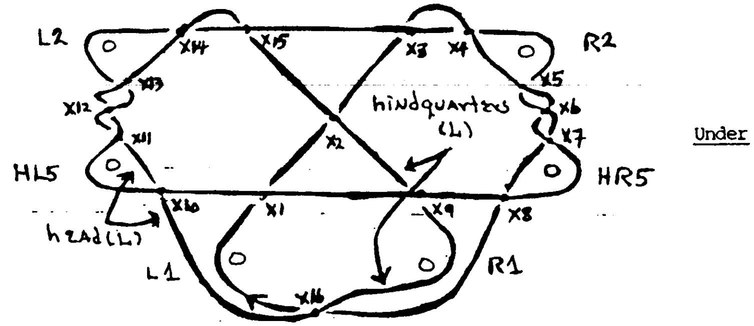


Fig. 67.V: Brown Bears, O.A: 2∞→1:<5∞→2|: 5↑(ℓ1∞):5↑(2∞):H5(2n) #(H5)|::H2(1n-s):<2(#): N1|: K.

This is a [(g)]Schema (because of the multiple 1-∞'s), on which has been marked those strings ("arcs") which are, ultimately, to form the Left-Bear's head and hindquarters, respectively. That is, the double 1∞'s are to eventually disentangle themselves and separate to their respective sides, where the Left Bears's constituent "parts" will be formed from the arcs as indicated in the above figure. The linear [(g)]Sequence associated to Fig. 67.V is

|⇒ L1: x1(U): x2(U): x3(U): x4(∅): x5(∅): x6(U): x7(∅): HR5: x8(∅): x9(∅): x1(∅): x10(∅): HL5: x11(∅): x12(U): x13(∅): x14(∅): x15(U): x2(∅): x9(U): R1: x16(∅): L1: x10(U): x11(U): x12(∅): x13(U): L2: x14(U): x15(∅): x3(∅): x4(U): R2: x5(U): x6(∅): x7(U): x8(U): R1: x16(U) ∎

We remark that the existence of two 1∞'s in this [(g)]Schema implies that each of L1, R1 should appear exactly twice in the associated linear sequence -- as, indeed, they do. The crossing-analysis is simple:

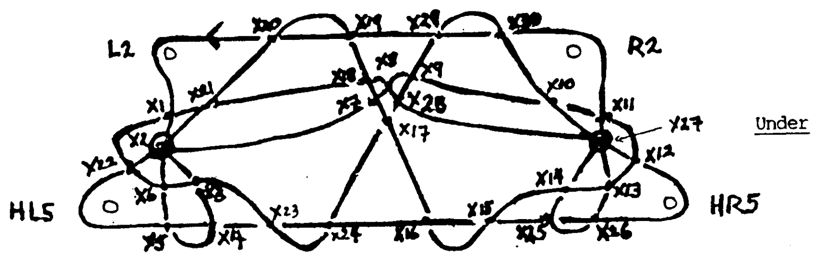
	Fig. 67.IV'		Fig. 67.V	Fig. 67.IV'		Fig. 67.V		Fig. 67.IV'		Fig. 67.V
C.	x1	→	∅	x7	→	x6		x13	→	x12
	x2	→	x1	x8	→	x7		x14	→	x13
	x3	→	x2	x9	→	x8		x15	→	x14
	x4	→	x3	x10	→	x9		x16	→	x15
	x5	→	x4	x11	→	x10	C.	x17	→	∅
	x6	→	x5	x12	→	x11		x18	→	x16

Obviously, the crossings {x1, x17, x18} of Fig. 67.IV' coalesce to crossing x16 of Fig. 67.V under the stated consolidation of the 1∞'s (under "K"). As usual, this is justified by Lemma 2.B.

The next stage of the construction is the (intermediate) string <u>figure</u> resulting from the manipulational sequence

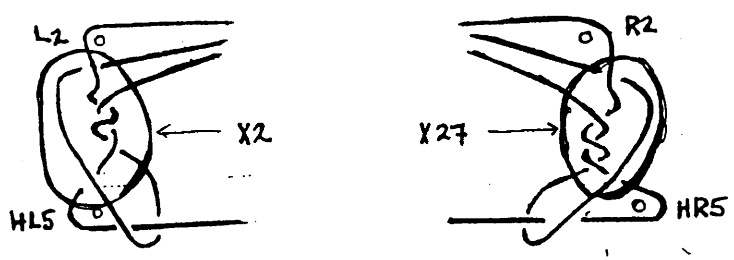
$$\underline{O}.A: 2\overleftarrow{\infty}\rightarrow 1 : <5\overleftarrow{\infty}\rightarrow 2 \mid : \underleftarrow{5}\uparrow(\ell 1\infty) : \underrightarrow{5}\uparrow(2\infty) : \overleftarrow{H5}(\overline{2n})\#(H5) \mid :: \overleftarrow{H2}(\overline{1n-s}) : <2(\#) : N1 \mid : K : \overrightarrow{1}(\underline{2n}) : (\ell 1\infty^{(2)}) N1\#(H5) : \square 2 : \overrightarrow{1\infty}\rightarrow 2] .$$

The gross-schema for this string-figure is



<u>Fig. 67.VI.A</u>: Gross-schema, Brown Bears intermediate figure.

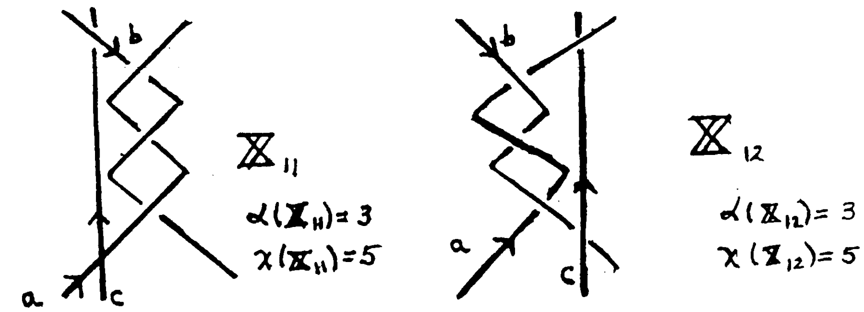
The fine-schema of the left- and right-hand complex-crossings, x2,x27 respectively, is given by



<u>Fig. 67.VI.B</u>: Brown Bears, complex structural crossings.

<u>Be sure to note</u> that the loop formed by the lower lateral arm of the central diamond -- and its continuation -- about the palmar string(s) has been omitted from Fig. 67.VI.B; its crossings with the respective palmar strings are constructional and will disappear in the subsequent move of the construction.

To represent the complex-crossings of Fig. 67.VI.B in forms suitable for inclusion in the $\mathbb{X}$-Dictionary, we have



<u>Fig. 68</u>: $\mathbb{X}$-Dictionary (continued)

which naturally complement the crossings $\mathbb{X}_9$ and $\mathbb{X}_{10}$ introduced in the Crows Feet (see page 113); as with these earlier crossings, the Osage Diamonds complex-crossings $\mathbb{X}_7$, $\mathbb{X}_8$ (page 69) form the "heart" of $\mathbb{X}_{11}$, $\mathbb{X}_{12}$ -- a third string, "c", being interlaced with -- and juxtaposed to -- these new entries, as before. The four crossings $\mathbb{X}_9$, $\mathbb{X}_{10}$, $\mathbb{X}_{11}$, $\mathbb{X}_{12}$ represent all the ways in which a third string may be juxtaposed to the Osage Diamonds complexes in an "elementary" fashion -- and so the introduction of $\mathbb{X}_{11}$, $\mathbb{X}_{12}$ may be viewed as the completion of a thought begun some 45 pages ago. We can now give the (complex) linear sequence associated to the string-figure whose schema is given in Fig. 67.VI.A:

$\models\!\Rightarrow$ L2: x1(∅): x2($\underline{\underline{11}}$,b+): x3(U): x4(∅): x5(U): x6(U): x2($\underline{\underline{11}}$,c+): x7(U): x8(∅): x9(U): x10(U): x11(U): x12(∅): x13(∅): x14(∅): x15(∅): x16(U): x17(∅): x7(∅): x18(∅): x19(U): x20(∅): x21(∅): x2($\underline{\underline{11}}$,a-): x22(U): HL5: x5(∅): x4(U): x23(U): x24(∅): x16(∅): x15(U): x25(U): x26(∅): HR5: x12(U): x27($\underline{\underline{12}}$,b-): x28(U): x8(U): x18(U): x21(U): x1(U): x22(∅): x6(∅): x3(∅): x23(∅): x24(U): x17(U): x28(∅): x9(∅): x29(U): x30(∅): x10(∅): x27($\underline{\underline{12}}$,c-): x13(U): x26(U): x25(∅): x14(U): x27($\underline{\underline{12}}$,a+): x11(∅): R2: x30(U): x29(∅): x19(∅): x20(U) ■

For the crossing-analysis, we find

<u>Fig. 67.V</u>		<u>Fig. 67.VI.A</u>
x1	→	x24
x2	→	x17
x3	→	x29
x4	→	x30
x5, x6, x7	→	x27
x8	→	x26

<u>Fig. 67.V</u>		<u>Fig. 67.VI.A</u>
x9	→	x16
x10	→	x5
x11, x12, x13	→	x2
x14	→	x20
x15	→	x19
x16	→	x8

The crossings x1, x3, x4, x6, x7, x9, x10, x11, x12, x13, x14, x15, x18, x21, x22, x23, x25, x28 of Fig. 67.VI.A being created by the complex movement

$\vec{1}(\underline{2n}): (\ell 1\infty^{(2)})$ N1#(H5) | □2:$\overrightarrow{1\infty}$→2]

applied to Fig. 67.V.

The final (complex) move of this construction -- "$\underset{\Rightarrow}{1}$(o; lower outside arm of c-◊)#(H5): $\vec{1}$(2n)#(H5): N1:□2]", applied to Fig. 67.VI.A -- begins with the 1-pickup

{ $\underset{\Rightarrow}{L1}$(o;x3-x23) / $\underset{\Rightarrow}{R1}$(o;x14-x15) } ,

which oblique strings form the lower lateral arms of the two central diamond configurations; hence, the above encoding. The subsequent "Navaho"-move passes this string over the (respective) 2∞ to the far side of the figure. Thus the crossing-pairs {x1, x3} and {x11, x12} will transfer to the top transverse string of the figure (L2f-s) where, upon extension, they will pass through one another -- left over right -- to the opposite sides of the figure [as remarked earlier (page 156, middle)]. We shall study this pretty phenomenon in some detail, adopting (1) the alibi approach to the crossing-transformations, and (2) the schema perspective "Over" -- to better visualize this complicated passage.

Applying the final (complex) movement of this construction to the string-position of Fig. 67.VI.A -- without the ultimate final extension, "]", -- results in a string-position with (Over) schema given by

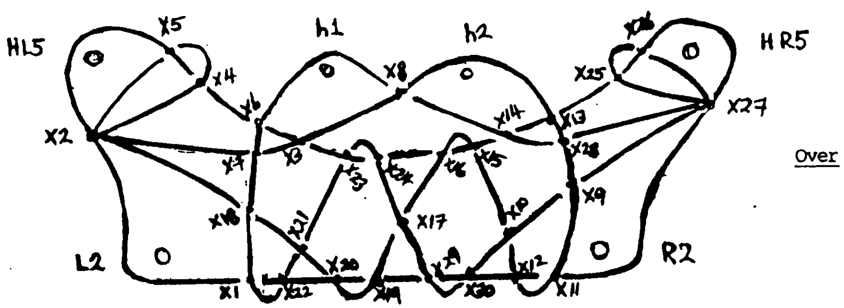


Fig. 67.VII.A: Brown Bears, preparatory to] .

Here, the crossings have maintained their labels, from Fig. 67.VI.A, in their new relative positions resulting from the given manipulation -- hence the name "Alibi" for this view of the crossing-transformation. Note the introduction of

the two weak fingers h1 and h2 for the two hanging loops in the above representation.

Now, an examination of the above schema reveals that, under slight extension, the three simple crossings x7, x8, x28 of the strings of the hoo$^{(2)}$ will coalesce to a single simple crossing -- x7 [since {x8, x28} → ∅ by the Lemma 2.B]. This will result in the simplified schema

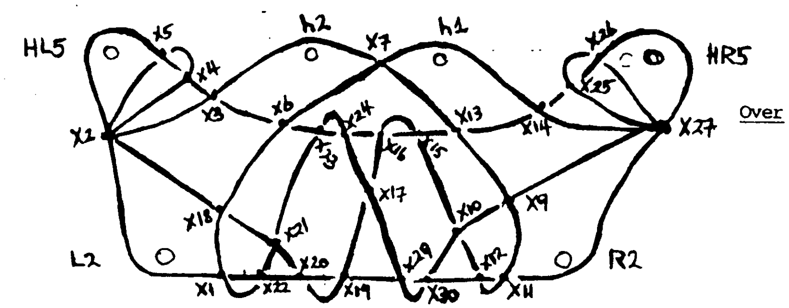


Fig. 67.VII.B: Brown Bears, during] (1).

Next we identify two distinguished central-lateral loops ("concatenation of arc"). The first, on the left-central side in Fig. 67.VII.B is composed of the sequence of arcs joining the crossings ⟨x7, x6, x18, x1, x22, x21, x23⟩ -- in order -- the first four of which are "over" crossings along this path, the last three being "under". The second, on the right-central side of this figure, is composed of the arcs joining ⟨x7, x13, x9, x11, x12, x10, x15⟩ -- the first being "under", the next three "over", the last three "under". Thus both of these loops will pass to the center of the design over the central configuration (between 2n-s and H5f-s); the right one will pass through (i.e. inside) the left (because of x7's crossing type); and they will continue to the opposite sides over the central configuration -- to assume each other's original place relative to the original string-position. Effecting this movement in the string (on the figure of Fig. 67.VII.B) results in

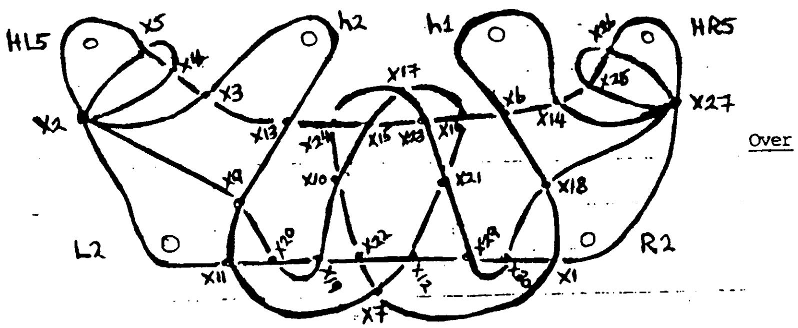


Fig. 67.VII.C: Brown Bears, during I (2).

The string-position represented by the schema of Fig. 67.VII.C has several obvious extension-cancellable crossings; the pairs {x3, x13}, {x6, x14} will cancel by Lemma 2.B -- as will x9 and x18, upon rearrangement of the complex crossings x2 and x27, respectively -- as the left- and right-hoo's pass over them and disappear under I. Thus, we proceed with an additional slight extension of the figure, to produce the design

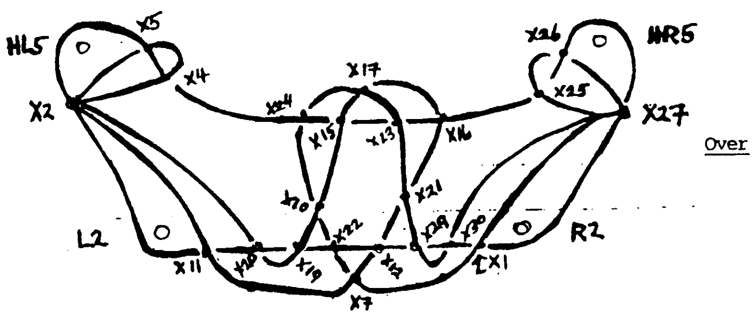


Fig. 67.VII.D: Brown Bears, during I (3).

Here, analogous to the situation in Fig. 67.VII.B, we identify two distinguished central-lateral loops: The first is composed of the sequence of arcs joining the crossings ⟨x7, x12, x21, x16, x17, x15, x10, x19⟩ -- in order -- the first of which is "over", the next four "under", and the last three "over" -- on this path. The second is composed of the arcs joining ⟨x7, x22, x10, x24, x17, x23, x21, x29⟩ -- the first four of which are "under", the last four of which are

"over" -- on this path. Thus both loops will pass over the central configuration (between 2n-s and H5f-s) in either direction; further, the parities of x7, x10, x17, x21 insure that the first loop will pass to the left through (i.e. "inside") the second loop -- as this loop passes to the right. That is, these two loops will completely disentangle, one from the other, and pass to the side of the figure opposite to that now occupied-- right through left. During this process, the crossings {x17, x21} → ∅ -- and, subsequently {x7, x10} → ∅ -- by (several applications of) Lemma 2.B. Thus, effecting this movement in the string (on the figure of Fig. 67.VII.D) results in

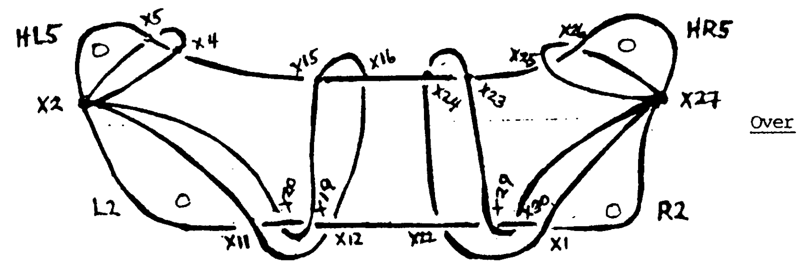


Fig. 67.VII.E: Brown Bears, during I (4).

which lacks the final, complete "tug" of I -- and being turned-over -- of being Fig. 66 (the two Brown Bears). Note that (1) the crossings x10, x25 of Fig. 66 occur because, in that presentation of the final design, the "hindquarter" loops are turned-over (on H5f-s), in comparison with their presentation in Fig. 67.VII.E (above). And (2) the aggregates of crossings {x1, x2, x3, x4, x5, x8} and {x20, x21, x22, x23, x29, x30} of Fig. 66 represent alternate presentations of complex-crossing-types $\mathbb{X}_{11}$ and $\mathbb{X}_{12}$, respectively (i.e. they are identifiable with complex crossings x2 and x27, respectively, of Fig. 67.VII.D). As the χ-number of the Fig. 66-representation is

$$6>5 = \chi(\mathbb{X}_{11}) = \chi(\mathbb{X}_{12}),$$

the Fig. 67.VII.D (and 𝕏-Dictionary) representation is preferred, as per our earlier convention.

The usual "Alias"crossing-transformation analysis between Fig. 67.VI.A and Fig. 66, under the complex move "1(o; lower outside arm of c-◊)#(H5): I⃗ (2n)#(H5): N1:□2I" applied to Fig. 67.VI.A, may now be determined by a comparison of the names for the corresponding crossings in the equivalent schemata of Fig. 67.VII.E and Fig. 66; that is, the "Alias" transform is easily derivable from the "Alibi" **analysis. We find, directly**

	Fig. 67.VI.A		Fig. 66		Fig. 67.VI.A		Fig. 66		Fig. 67.VI.A		Fig. 66
	x1	→	x28		x11	→	x9	C.	x21	→	ϕ
	x2	→	{x1, x2, x3, x4, x5, x8		x12	→	x11		x22	→	x27
C.	x3	→	ϕ	C.	x13	→	ϕ		x23	→	x16
	x4	→	x6	C.	x14	→	ϕ		x24	→	x17
	x5	→	x7		x15	→	x13		x25	→	x19
C.	x6	→	ϕ		x16	→	x12		x26	→	x18
C.	x7	→	ϕ	C.	x17	→	ϕ		x27	→	{x20, x21, x22, x23, x29, x30
C.	x8	→	ϕ	C.	x18	→	ϕ	C.	x28	→	ϕ
C.	x9	→	ϕ		x19	→	x14		x29	→	x26
C.	x10	→	ϕ		x20	→	x15		x30	→	x24

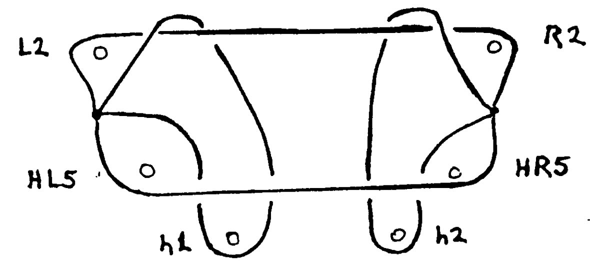
Note that the crossings x3, x6, x7, x8, x9, x10, x13, x14, x17, x18, x21, x28 of Fig. 67.VI.A which cancel in transition to Fig. 66 have all been accounted for in the gradated "extension-sequence" of Fig's. 67.VII. A-E; the "new" crossings of Fig. 66 (not in the range of the alias transformation) are precisely x10, x25 -- which arise as a consequence of the particular final presentation (see Note (1), page 162). The "Alibi"-derivation of the "Alias"-crossing transformation between Fig. 67.VI.A and Fig. 66 has, thus, yielded complete information.

This completes the crossing-specific constructional analysis for the string-figure Two Brown Bears. Perhaps its most remarkable feature is the complicated final disentanglement and separation of the left and right halves (Bears) of the figure. For this is independent of the internal parameters of the string being worked, and takes place with equal fluidity in all. This is in severe contra-distinction to future string-figures to be investigated, many of which require "special-property" strings for their successful (intermediate and) final extensions. In general, string-figures which exhibit this kind of string-variational independence prove to be well-springs of information, well worth a detailed analysis -- regardless of how complicated that proves to be.

Before proceeding to the "Heart"-sequence for the Brown Bears, we make two rather lengthy observations about the previous construction; the first of which will prove very valuable in that discussion.

First, from the final double-disentanglement of the two principals of

the Brown Bears' construction, we might be led to hypothesize that this could be achieved -- directly -- at an earlier stage of the manipulations. And a minimum of experimentation with the intermediate schemata therein confirms these suspicions. For example, dropping the l∞'s in Fig. 67.IV' -- and allowing them to separate to their respective opposite sides of the figure (as they will do, naturally) -- so that they become h∞'s, we obtain the figure whose schema is



Under

Fig. 67.IV": Separation of l∞'s in Brown Bears.

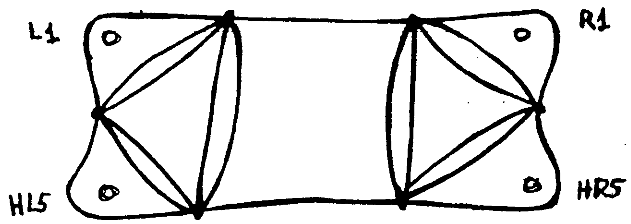
This is a complex schema whose fine structure, after the previous detailed analysis, is completely understood. [The simple crossings x1, x3, x17, x18 of Fig. 67.IV' have cancelled, in pairs, in the passage to Fig. 67.IV" via "□1".]
Now perform the manipulational sequence

$\underset{\rightarrow}{1}\uparrow$(h∞)#(H5):$\vec{1}$(2n)#(H 5):N1:□ 2]

on Fig. 67.IV"; the result is the Two Brown Bears!
Note that the h∞'s, in reality -- under gravity -- lie well below the central strings; our schema for these "brings them up to the near side of HL5∞, in the plane of the figure"; thus the move "$\underset{\rightarrow}{1}\uparrow$(h∞)" in the physical situation of the string-on-the-hands, must mean "pass 1 away below the h∞, and thence back up into them, towards you." This should be well-understood from our previous discussions of the schematic conventions. In fact, the manipulation

$\vec{1}\downarrow$(h∞):twist (each) 1 to the inside, and up (#):
$\vec{1}$(2n)#(H5):N1:□2]

-- also discovered while experimenting with Fig. 67.IV" -- produces the figure "Two Caves" [D. Jenness, No. 4], whose gross-schema is



Under

Fig. 69: Two Caves.

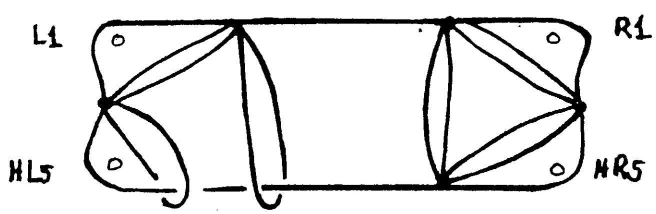
This figure can also be made via

Two Caves: O.A':<2∞—>1:∾1. & 2. of Brown Bears].

Note: This figure begins with O.A' (see page 36); if we (mistakenly) begin with O.A, instead, the two lateral "Caves" will not disentangle and separate one from another at the final extension. [The figure may, however, be made from O.A, as above, if we replace the move "K" in the construction by "K(L)"]. Finally, applying the manipulation

{ L1↑(Lh∞) / R1↓(Rh∞):Twist R1 inside, and up# } :1 (2n)#(H5):N1:□2]

directly to the string-position of Fig. 67.IV", produces the very appealing gross-schema [D. Jenness, No. 3]



Under

Fig. 69': Brown Bear and Cave.

of a left-facing Bear to the left in the string, and a cave to the right. And we hypothesize that, if the central loops will pass through one another at the stage of the final extension, we may produce a really fine representation of a Bear issuing from his Cave by the manipulational string

O.A:{ <L2∞—>L1 / R2∞—>R1 } :∾1. & 2. of Brown Bears].

We perform these movements in the string, and Fig. 69' results; the final extension is magnificent! Of course, we may construct the symmetric figure with Cave to the left, Bear to the right from

O.A':{ L2∞—>L1 / <R2∞—>R1 } :∾1. & 2. of Brown Bears]

et cetera.

Our second, and last, remark before proceeding to the "Heart"-sequence for the Brown Bears concerns a "minimal-variational" construction for this figure, which is contemporary.

Brown Bears': O.A:<5∞—>2:: H345(ℓ2∞):3↑(1∞):3↑(u2∞):H3(u2n)#(H3,45)|::□45:H5↓(H3∞):□3#(H5)| H2(1n):<2(#) (□former u2∞):□1|:1(ℓ2n)#(H5)|:∾2. of Brown Bears].

Note that the manipulations of the Movement 2. of Brown Bears are applied to a different string-position (i.e. not that of Fig. 67.IV') in the present case as, for example, now there are two distinct 2∞'s. The resulting final designs are, nevertheless, identical (Fig. 66).

The corresponding "Heart"-sequence for the Two Brown Bears -- with its "Osage-Diamonds" -- based complex crossings X_{11}, X_{12} -- must contain some loop-specific maneuver which produces X_7, X_8; two of whose constructions were previously analyzed. In fact, here the relevant manipulation is (essentially) the "alternate" one (pages 66-67), involving the simultaneous passage (or "lacing") of two loops, one through the other. We now introduce a notation specific to this complex loop-manipulation: Begin with the string-position

O.A:□1|5∞—>4#|

so that there is now a 2∞ and a 4∞ on each hand. We define

{ 2∞↓(4∞):2∞—>5 / 4∞↑(2∞):4∞—>1 } ≡ pass 2f away and down through 4∞ and, drawing this string away from you, hang it -- as a loop -- on 5: pass 4n towards you and up through the 2∞ and, continuing to draw this string towards you, hang it-- as a loop -- on 1:□2 & 4#|

The result is complex-crossing X_7 on Lp, and X_8 on Rp. The similarly-defined manipulation

$\left\{\begin{matrix} \underrightarrow{2\infty}\uparrow(4\infty):\overrightarrow{2\infty}\rightarrow 5 \\ \overleftarrow{4\infty}\downarrow(2\infty):\underleftarrow{4\infty}\rightarrow 1 \end{matrix}\right\} \equiv$ pass 2f away and up through 4∞, continue drawing this string away, and hang it -- as a loop -- on 5: pass 4n towards you and down through the 2∞, continue passing this string towards you, and hang it -- as a loop -- on 1:□2 & 4#|

This time the result is complex-crossing $\mathbf{X}_8$ on Lp, and $\mathbf{X}_7$ on Rp.

Recall that our view of a "Heart"-sequence has only to do with loops' relative movements in relation to one another, and that which particular frame-elements are involved in the representation/realization of these relative motions is immaterial from that viewpoint; that is, the gedanken experimenter envisions "pure loops in space, mutually interacting" (cf. page 35). He has, however, to present these machinations in a particular realization thereof -- and so do we! And we choose to do this relative to the "frame" already introduced, where the appropriate language has been developed. But we emphasize that, after these complex loop-manipulations are well-understood, the particular realization which is chosen for the representation -- i.e. the frame-specific details -- should be de-emphasized, finally to disappear altogether from our thinking thereof. With these few qualifying preliminary remarks, we proceed to the Brown Bear's "Heart"-sequence.

Initially, we form the "starting configuration"

$$\underline{O}.A:\overrightarrow{2\infty}\rightarrow 3:\overrightarrow{1\infty}\rightarrow 2\#|$$

on the hands. The first several movements of the Brown Bears"Heart"-sequence -- on the above starting configuration -- are

$$\overleftarrow{3\infty}\downarrow(2\infty):\underleftarrow{3\infty}\rightarrow 1::>\overrightarrow{2\infty}\rightarrow 3:<\overleftarrow{5\infty}\rightarrow 4.$$

Then follows the "Osage Diamonds-alternate" type lacing move

$$\left\{\begin{matrix} \overrightarrow{3\infty}\downarrow(4\infty):\underrightarrow{3\infty}\rightarrow 5 \\ \underrightarrow{4\infty}\uparrow(3\infty):\overleftarrow{4\infty}\rightarrow 2 \end{matrix}\right\}$$

as previously discussed; note the complex-crossings $\mathbf{X}_7$ on Lp, $\mathbf{X}_8$ on Rp. At this point, the string-position on the hands is essentially that of Fig. 67.IV', except that the frame elements 2 and H5, here, are being retained on 5(unHooked) and 2 -- respectively -- and the schema orientation is "Over", here (as opposed to "Under", in Fig. 67.IV'); these are matters of no consequence to the gedanken-viewpoint of the "Heart"-sequence.

Now, following the preliminary observations of page 164, we "separate" **the 1∞'s --passing them to the opposite sides of the figure. Note that the** 1∞'s orientation with respect to the frame does not change during this movement, and that the L1∞ passed down through (i.e. "inside" of) the R1∞ in the process; we stipulate that each thumb-loop comes to rest on the opposite thumb at the termination of this manipulation. Thus we view the entire movement as an "interchange" of 1∞'s -- passing L through R; this is an "inter-hand" transfer (as opposed to the "cross-hand exchange" of pages 27-28), entirely reminiscent of the 2∞-motion occurring in the "Heart"-sequence of the Crows Feet. And by analogy with the encoding adopted for this latter figure, we write

$$\left\{\begin{matrix} \overrightarrow{\overrightarrow{L1\infty}}\downarrow(R1\infty):\underrightarrow{\underline{L1\infty}}\rightarrow R1 \\ \underleftarrow{\underline{R1\infty}}\rightarrow L1 \end{matrix}\right\}$$

for this complex, simultaneous maneuver.

The final manipulations of the "Heart"-sequence are standard, and well-anticipated after the previous discussion:

$$\overleftarrow{5\infty}\downarrow(1\infty):\underleftarrow{5\infty}(1n):\overrightarrow{5\infty}\rightarrow 5::\square 1 \mathrm{I}.$$

The resulting design is the Two Brown Bears, with ↶2⟍5↑ (5 not Hooked) in the "Over" perspective.

Putting the above remarks together, the "Heart"-sequence for the Brown Bears is

$$\overleftarrow{3\infty}\downarrow(2\infty):\underleftarrow{3\infty}\rightarrow 1::>\overrightarrow{2\infty}\rightarrow 3:<\overleftarrow{5\infty}\rightarrow 4::$$
$$\left\{\begin{matrix} \overrightarrow{3\infty}\downarrow(4\infty):\underrightarrow{3\infty}\rightarrow 5 \\ \underrightarrow{4\infty}\uparrow(3\infty):\overleftarrow{4\infty}\rightarrow 2 \end{matrix}\right\}:\left\{\begin{matrix} \overrightarrow{\overrightarrow{L1\infty}}\downarrow(R1\infty):\underrightarrow{\underline{L1\infty}}\rightarrow R1 \\ \underleftarrow{\underline{R1\infty}}\rightarrow L1 \end{matrix}\right\}::$$
$$\overleftarrow{5\infty}\downarrow(1\infty):\underleftarrow{5\infty}(1n):\overrightarrow{5\infty}\rightarrow 5::\square 1 \mathrm{I}$$

applied to the string-position resulting from

$$\underline{O}.A:\overrightarrow{2\infty}\rightarrow 3:\overrightarrow{1\infty}\rightarrow 2\#|;$$

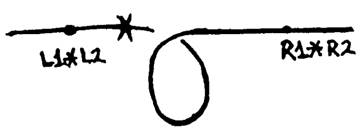
i.e. "Opening A on the wrong fingers."

Note ① It may be objected that the above fails to be the "Heart"-sequence for this figure as it elides the intermediate 1∞-wrap-up and subsequent disentanglement at "I" for this construction. And, technically speaking, the above is the "Heart"-sequence for the alternate construction of page 164. In fact, we view the two constructions as "equivalent", at this level of the analysis, and view the above symbol-string as the "Heart"-sequence of both. (2). An examination of intermediate string-position Fig. 67.IV reveals that the Osage Diamonds-type complex crossings -- $\mathbf{X}_7$ on Lp, $\mathbf{X}_8$ on Rp -- occur here for the first time in the construction. To see specifically how this lacing occurs in the present instance, form the starting-position O.A:□5| in the string; then -- as in the figure -- proceed.

5↑(1∞):5(2∞):H5(2n)#(H5)|:: H2(1n-s):<2(#):□1|

and examine the complex-crossings on the respective palmar strings -- they are the same. This is a "Hooked 5" version of the "Osage-Diamonds-alternate"-type lacing of loops delineated in the above "Heart"-sequence [cf. the Calculus string appearing in the caption of Fig. 67.IV.] It should be added to our repertoire of techniques to effect this distinguished interlacing of loops, as it will be met with again in subsequent figures; it is not exclusive to Inuit string-figures although, perhaps, it finds its most common occurrence in that setting.

We now adopt a viewpoint even more "primitive" than that of the "Heart"-sequence, and consider the Bear as a "string-design", or Knot. We begin by grasping the closed loop of string between 1*2 of either hand, about 8" apart; and throwing a small, inverted loop in this short string, passing right over left.



Under

Fig. 70.I: KnotBear (Right); Step 1.

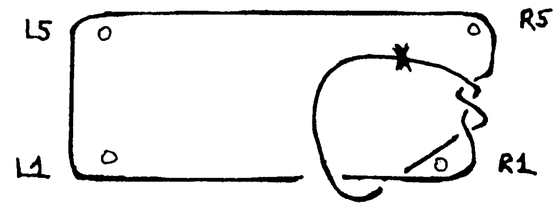
Now, seizing the left-side string issuing from this loop at the point marked X in Fig. 70.I, bring this towards you and to the near side of the small loop, then push it directly away through the loop, and back up to the left side of the figure; this gives*

Under

Fig. 70.II: KnotBear (Right); Step 2.

*Note the formation of the Osage Diamond-type complex-crossing at this point in the procedure.

Now, releasing R(1*2), pass R5 directly away at the point marked R5 in the above figure, and -- releasing L(1*2) -- pass R1 directly away at the point marked R1 [as in O.1(R)] Pick up the far end of the string (the long h∞) as in O.1(L), so that the 1n and 5f strings are straight (i.e. uncrossed). Now #, and observe the schema



Over

Fig. 70.III: KnotBear (Right); Step 3.

[All schema-orientations will be "Over", for the remainder of the KnotBear discussion. We shall henceforth omit this caption-tag.] Note: the central loop should be fairly large for the subsequent working; and this is the time to adjust its size, if that is needed.

Now throw a small upright loop on the far string of this central loop -- at the point marked X in the Fig. 70.III -- passing left over right. This gives

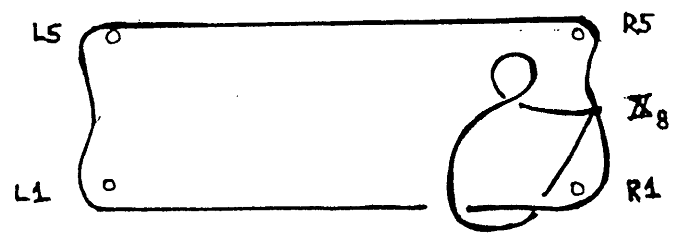


Fig. 70.IV: KnotBear (Right); Step 4.

Note: We have gone to a complex-schema, giving the complex-crossing type, $\mathbf{X}_8$, for the node on Rp. Next, pass the R5∞ towards you, up through this small upright loop, and directly back to R5. This gives

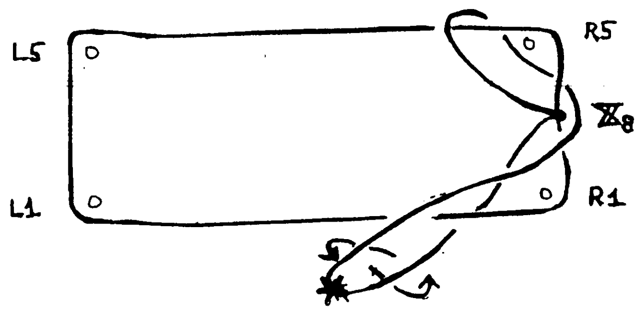


Fig. 70.V: KnotBear (Right); Step 5.

Now, seizing up the near loop depending from ln-s, at ✕, throw a small, upright loop thereon, passing L over R, and -- turning this loop directly down, to the right -- thread it over each of the R1∞ & R5∞, in turn, to the far side of the figure [Note: R1 and R5 must temporarily relinquish their loop to accomplish this passage.] The result is

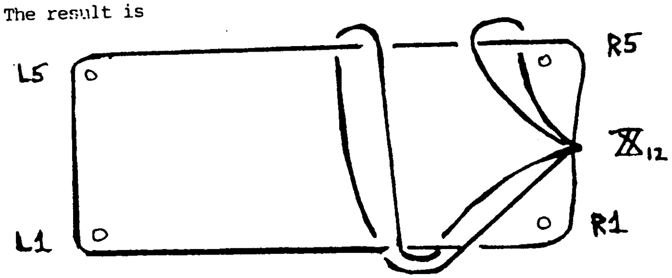


Fig. 70.VI: KnotBear (Right); Inverted.

Note: We have "upgraded" the complex-crossing type of the Rp-node to X_{12} (cf. Fig. 66). Turn hands with fingers pointed away ("under" perspective) to display the KnotBear (Right) figure.

Interchanging R and L in the above construction will produce the Two Brown Bears' final design (except that 5 is not in the Hooked position). And minimal variations of this construction will produce a variety of beautiful Bear figures, the foremost among these being that of the gross-schema (1)

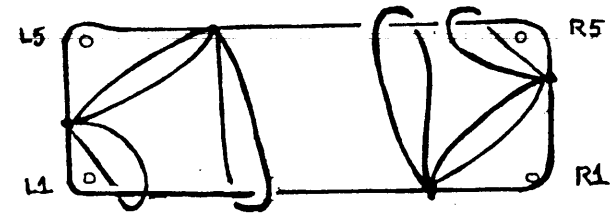


Fig. 71: KnotBear invention

which has a sublime "alternate" extension* (after >$\overrightarrow{R15\infty}$→ R15; i.e. turn R loops over away from you)

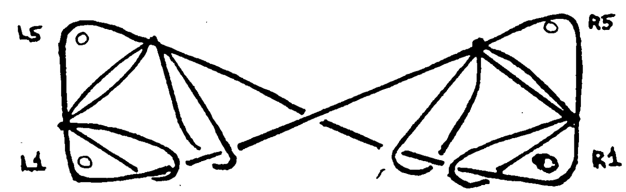


Fig. 71': KnotBear invention (alternate I).

We now turn to a complex-crossing analysis of the Brown Bears (Fig. 66), and the required augmentation of our burgeoning X-Dictionary.

The complex-crossing analysis for the Brown Bears entails several, interesting, "fine points" for discussion. The first concerns the actual presentation of the final figure, itself. And, while the representation of Fig. 66 is a presentation of the figure -- being an arrangement of the final design which actually occurs in the string upon I -- it is not the "simplest" such expression (in terms of the number of simple crossings involved). Note that any of the four loops about H5f may be "turned over" on that string, resulting in more (or fewer) simple crossings elsewhere in the figure. Further -- as remarked earlier -- the simple crossings x9, x10, x11, x14, x15 of Fig. 66 (for example) may present themselves in several ways, with different cardinalities associated to each. Thus it behooves us to find a "canonical" presentation of the ultimate string-design Brown Bears; one which entails the fewest number of

---

* cf, P.E. Victor: "Jeux d'Enfants et d'Adultes chez les Eskimo d'Angmagssalik", No. 6, "Pādi".

simple crossings. We do this by grouping simple crossings into the complex-crossings already entered in the $\mathbb{X}$-Dictionary (which were chosen on the basis of minimal χ-number) whenever possible and, for the remaining simple crossings, grouping them into complex-crossings (to be added to the $\mathbb{X}$-Dictionary, as "generated" by the current figure) in their optimal presentation. In the case of Brown Bears -- with respect to $\mathbb{X}$-Dictionary-so-far -- this turns out to be

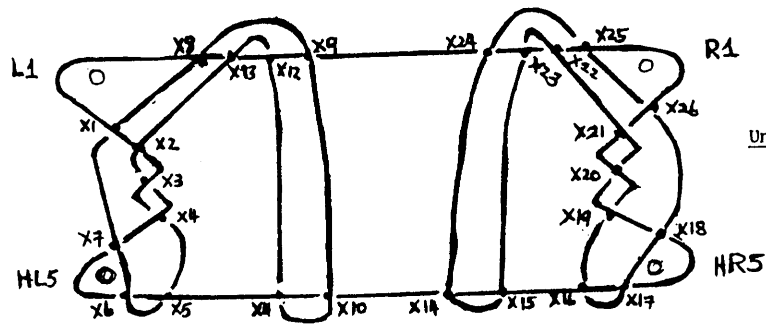


Fig. 72: Brown Bears, (Canonical)

which shows 26 simple crossings, total, as compared to the 30 of Fig. 66. Further, in the above presentation, we recognize well-known groupings of various subsets of the simple crossings into $\mathbb{X}$-types; in particular

Crossing-subset	$\mathbb{X}$-type
x1, x2, x3, x4, x7	$\mathbb{X}_{11}$
x5, x6	$\mathbb{X}_{6}$
x10, x11	$\mathbb{X}_{5}$
x14, x15	$\mathbb{X}_{6}$
x16, x17	$\mathbb{X}_{5}$
x18, x19, x20, x21, x26	$\mathbb{X}_{12}$

This list omits the 8 crossings x8, x9, x12, x13, x22, x23, x24, x25 of Fig. 72, which obviously group themselves into two (new) complex-crossings — composed of four of the above simple crossings each, namely

x8, x9, x12, x13 and x22, x23, x24, x25.

Thus, in terms of complex-crossings, the canonical Brown Bears exhibits exactly eight such -- with no simple crossings unaccounted for; those six of the previous

list (with each of the types $\mathbb{X}_5$, $\mathbb{X}_6$ occurring twice therein), plus the two new $\mathbb{X}$-types

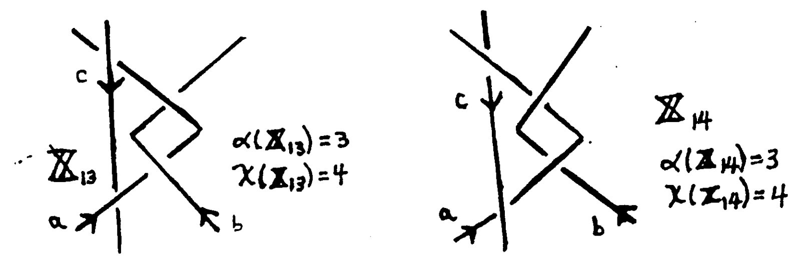


Fig. 73: $\mathbb{X}$-Dictionary (continued)

which exactly cover the eight omitted simple crossings of the previous list. The gross-schema for Brown Bears is, thus

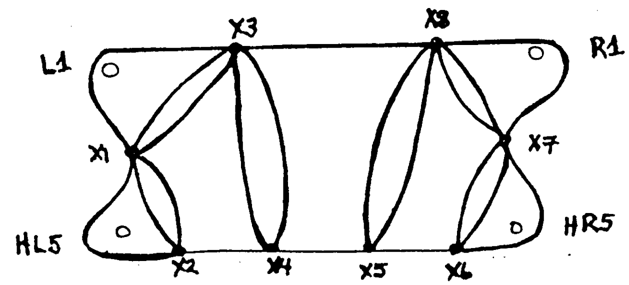


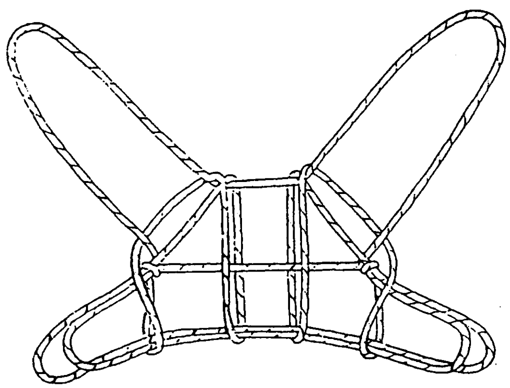
Fig. 74: Gross Schema, Brown Bears (Canonical)

showing exactly eight (complex) crossings, as promised. The associated complex linear sequence is

$\models$ L1: x1(11,b+): x2(6,b-): x1(11,c+): x3(13,c-): x4(5,b+): x3(13,b-): x1(11,a-): HL5: x2(6,a+): x4(5,a+): x5(6,a+): x6(5,a+): HR5: x7(12,b-): x8(14,b-): x5(6,b-): x8(14,a-): x7(12,c-): x6(5,b+): x7(12,a+): R1: x8(14,a-): x3(13,a-) ■

Before leaving the topic of complex-crossings, in the context of the Brown Bears figure, we remark that a comparison of the crossing types $\mathbb{X}_{13}$, $\mathbb{X}_{14}$ with the previously introduced $\mathbb{X}_9$, $\mathbb{X}_{10}$ (page 113) gives the distinct impression that the former -- entailing a "third" string, c, in elementary juxtaposition (interlace) with crossings $\mathbf{X}_5$(a-,b), $\mathbf{X}_6$(a-,b-), respectively* -- are "evolutionarily prior" to the latter; and should be so committed to the **X**-Dictionary. And, indeed, if the entries of the **X**-Dictionary were (partially-)ordered on the basis of some definition of "crossing complexity", this would be the case. In the present development, however, our focus is on the string-figures themselves -- rather than any such subconstruct thereof -- and it is the order of their introduction (whose basis is entirely subjective) that induces the ordering of the **X**-Dictionary. At a later point in the development of the subject as a whole, we are free to reorder the entries of the Dictionary along more geometric lines.

We now proceed to a representative sample of the other figures in the Brown Bear cycle. The first of these, "Brown Bears joined at the rectum" [G. Mary-Rousselière: <u>Les Jeux de Ficelle des Arviligjuarmiut</u>, No. 8], which -- although an extremely simple variant of the parent figure -- is included here because of the sheer beauty of its final design.

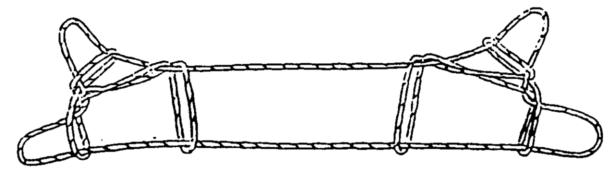


<u>Fig. 75</u>: Brown Bears joined at the rectum

<u>O</u>.A:2⃖∞→1: <5⃖∞→2::□R2: seize L2n and pass it away across L2p to the far side of L2, then back between L2 & L3, then towards you across L2d to the near side of L2 (thus forming a loop about L2):R⃖2(<u>2n</u>$^{(2)}$)ǂ#|<u>5</u>↑(ℓ1∞):<u>5</u>↑(2∞$^{(2)}$):H5(2̄n$'^{(2)}$)#(H5)|: H⃖2(1̄n̄-̄s̄):<2(#):N1|:∾ <u>2</u>. of Brown Bears ⌶.

* <u>cf</u>. page 114

The next two figures address distinct "multiplicative aspects" of the Brown Bears construction. The first of these concerns the matter of his "ears" [D. Jenness: No. 4]:



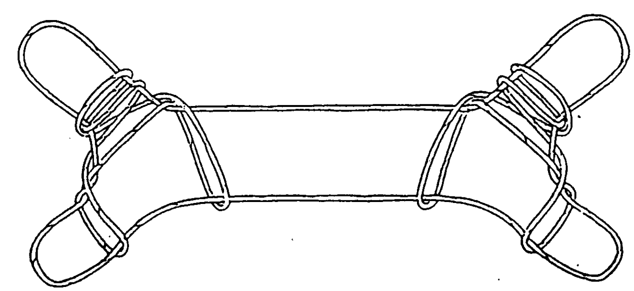
<u>Fig. 76.A</u>: Two Mountain Sheep.

Make Brown Bears: <u>3</u>. 1⃗∞→ 2: pass 1 away, under, and pick up from below the lower of the two strings which comprise the Bear's hind leg:∾ <u>2</u>. of Brown Bears ⌶.

This puts "ears" on the Brown Bears [<u>cf</u>. G. Mary-Rousseliere: No. 11]. Repetition of this procedure, <u>viz</u>.

Make Two Mountain Sheep:∾ <u>3</u>. of Two Mountain Sheep ⌶.

adds one to the number of "ears" <u>per</u> Bear -- as does each subsequent repetition of the procedure, length of string permitting -- and produces a design known as the "Two Caribou with their Horns". The figure is illustrated, below.



<u>Fig. 76.B</u>: Two Caribou with their Horns.

The second of the "multiplicative aspects" of Brown Bears concerns the matter of their "cubs".

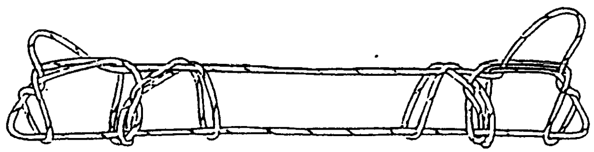


Fig. 77: Brown Bears and their Cubs.

Make Brown Bears: straighten 5 in its loop # O.A (with 1f-string): $\overleftarrow{2}$∞—>1: ∾ 1. of Brown Bears:☐1: $\underrightarrow{1}$(o;H5∞, after it passes under H5f) #(H5): $\underrightarrow{1}$(2n)#(H5): N1:☐2 I.

At I, two Bears and a following cub (each) separate from the central tangle of strings and lumber off to their respective opposite hands. As with the earlier "ear" construction, the "cubs" construction is multiplicative, and may be repeated to produce as many trailing "cubs" as length of string allows. As a final remark in this direction, we note that careful experimentation as to "which string to pick up, and where" will ultimately result in a viable mixture of these two distinct multiplicative aspects of Brown Bears, producing various admixtures of "Bears" plus "ears" plus "cubs"; as the several final string-designs are not very satisfying -- and lack "source" confirmation-- we shall content ourselves with the above presentation, and pursue these matters no further in these notes.

We now give four representatives of the many string-figures in the Brown Bears cycle. The first two of these are, themselves, closely related constructionally.

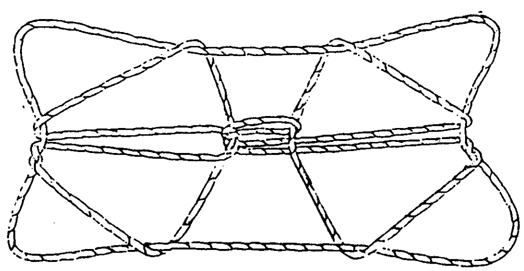


Fig. 78: NiɣciliɣyuK.

O.A:>$\overleftarrow{2}$∞—>1: <$\overleftarrow{5}$∞—>2:: $\underleftarrow{5}$↑($1\infty^{(2)}$):$\underrightarrow{5}$↑(2∞):$\overleftarrow{H5}$($\overline{2n}$)#(H5) | $\overleftarrow{H2}$($\overline{1n-s}$): <2(#):N1 |: K(L):: $\overrightarrow{1}$ (2n)#(H5):(ℓ$1\infty^{(2)}$)N1#(H5):☐2 I.

This represents a "Sky Spirit" with his "Hook", [D. Jenness, No. 11].

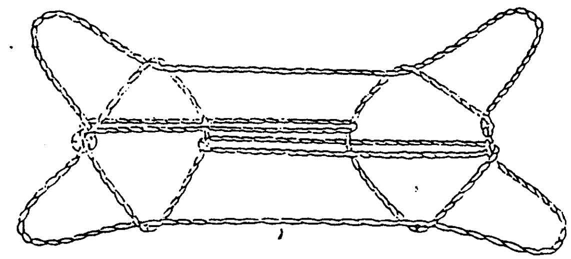


Fig. 79: Two Men Hauling on a Sled [D. Jenness, No. 12].

O.A:>$\overrightarrow{1}$∞—>2ǂ: >$\overleftarrow{2}$∞—>1: <$\overleftarrow{5}$∞—>2:: $\underleftarrow{5}$↑($1\infty^{(2)}$):$\underrightarrow{5}$↑(2∞):$\overleftarrow{H5}$($\overline{2n}$)#(H5)|: $\overleftarrow{H2}$($\overline{1n-s}$):<2(#):☐u1∞ [the 1n-s string]: K(L):$\overrightarrow{1}$(2n)#(H5): (ℓ$1\infty^{(2)}$)N1:☐2 I.

An entirely similar figure -- but with central diamonds composed of double strings, rather than single strings -- occurs on Nauru Island. By this is meant that the final designs are "identical in spirit"; the constructions are unrelated, at the present level of analysis. Our view of this phenomenon is that certain final designs in the string represent "resonances" among the possible patterns to be found therein. And that various string-figure traditions discover these resonances independently, within their own individual systemology. That is, the present example-pair is viewed more as an instance of "parallel evolution" than of "cultural diffusion". A consequence of this viewpoint is the deemphasis of similarity (or, indeed, identity) of final designs between two figures from disparate cultures, and an accent on the constructions thereof. Of course -- as in the case of the Physical Sciences -- the identification of the "resonances" among string-designs would be very useful in its own right. And we believe that the designs Brochos, Crows Feet, Osage Diamonds, and Brown Bears must be numbered among these. In fact, it is the author's own personal view of this phenomenon which forms the subjective basis of the order of the introduction of the string-figures in these notes — as alluded to earlier (page 175, top).

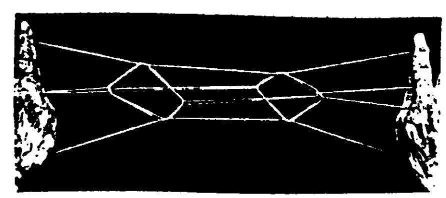


Fig. 80: Dagabe and Demadang

["Dagabe and Demadang", H.E. Maude: The String Figures of Nauru Island, No. 73]

O.A:: 1↓(2ꝏ) : 1 (5n) # 5↑(2ꝏ) : 5 (1f) #□2 | 1. HR2 (Lp$^{(2)}$) # (HR2) : Grasp all L strings between R1*3, hold taut: HL2 (L5n$^{(2)}$) : HL2 (L1n$^{(2)}$) : <L2(#) :: L1 (L5f$^{(2)}$) # (L1) :□L5 : >L2ꝏ$^{(2)}$→L5# (L) |□L strings on R1*R3 :□HR2# | : ∾1. (↓L R↗) :□u5ꝏ [whose strings run to center of figure; it must not be tangled with 5f] :□u1ꝏ [running to center; it must not be tangled with 1n] # | >1ꝏ→2 : 1↑(5ꝏ) # 2ꝏ→1 IP.

Dagabe and Demadang were Nauruan martial artists, specializing in the long staff, or "pole".

The penultimate figure we have chosen for a representative of the Brown Bear cycle is the "Rattlesnake and the Boy" [C.F. Jayne: String Figures and How to Make Them, page 109].

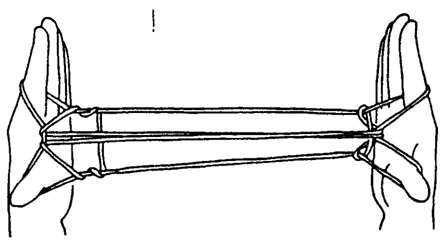


Fig. 81.A: A Rattlesnake and a Boy.

O.A : <5ꝏ→2 :: H345 (ℓ2ꝏ) : 3↑(1ꝏ) : 3↑(u2ꝏ) : H3 (u2n) # (H3,45) | : □45 : H5↓(H3ꝏ) :□3# (H5) | H2 (1n) : <2(#) (□former u2ꝏ) :□1 | : 1↑(ℓ2ꝏ) : K : 1 (u2n) # (ℓ1ꝏ$^{(2)}$) N1 :□2 | : >1ꝏ→2 | :: 1↑(c-◊) : 1 (c-s$^{(2)}$) # (H5) ::□5# | H5 (1n$^{(2)}$) # (H5) :□1 | :: 1↑(H5ꝏ$^{(2)}$) : 1 raises H5n-string that runs to H5f$^{(2)}$ and, drawing this towards the palm, 1 picks up remaining H5n-string from above #(1) :□H5# | :: M (L1n) :□L1 : L2ꝏ→L1 :: L5↑(R2ꝏ) :□R2#R1ꝏ→R5 : R1↑(Mꝏ) #□M].

This produces a representation of a "Rattlesnake" to the left in the string, and a "Little Boy" to the right. The "snake" can be made to "run up and bite the boy" by □L5 and "seesawing" the hands in a quick (and "snappy")]. This is illustrated, below.

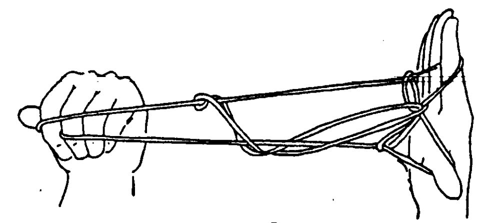


Fig. 81.B: Rattlesnake Bites Boy

Notes on the construction: (1). The initial segment of the construction is identical to that of Brown Bears' (page 166); (2). The final "loop-transfer" movement of the inital figure is an echo of that first encountered in "Tutae Takahuri 2" (Fig. 56.IV, page 131, top); (3). The final extension of the second ("action"-) figure is greatly facilitated by HL2345↓(L1ꝏ), as shown in Fig. 81.B, above. (4). The figure also admits a "classic" Brown Bears construction (through the string-position of Fig. 67.IV.A), but there is no "source" confirmation for this method. (5). Current ethnographic collections of Inuit string-figures, curiously enough, make no reference to this interesting, appealing figure.

The final figure in the Brown Bears cycle which we shall discuss is the contemporary figure "Snoop" -- the name of a particular dog who loved to play with rocks [cf. D. Jenness, "The Dog and its Ordure", No. 9].

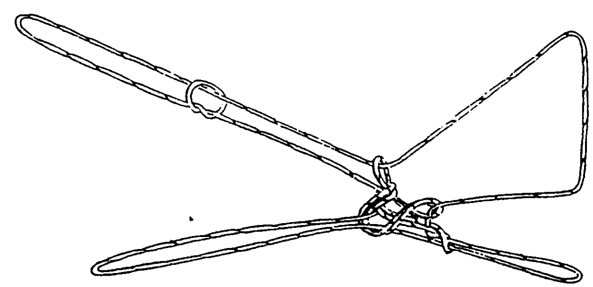


Fig. 82: Snoop.

O.1 :: R2 (Lp) # L2 (R2∞) : L2 (Rp) # | : 2∞ → 1 : < 5∞ → 2 # | :: 5 ↑ (1∞$^{(2)}$) : 5 ↑ (2∞) :
H5 (2n) # (H5) | H2 (1n-s) : < 2 (#) : N1 | : K : 1 (2n) # (H5) : (ℓ 1∞$^{(2)}$) N1 : □ 2 :
1∞ → 2] :: R1 ↑ (R2∞) : HR1 (o$^{(2)}$; R2∞) : > R1 (#) : L1 ↑ (R1∞$^{(2)}$) # (H5) : 1 (2n)
(H5) : (ℓ 1∞$^{(2)}$) N1 : □ 2 | : < B: lay flat (do not □ B) :: HR2 (u-o$^{(2)}$; Lp-Rp) :
R2 ↓ (R1∞) : HR2 (5f-s; between 5f-∞'s) # (R2) [□ former R2∞$^{(2)}$] :: □ L5 :
R2∞ ↓ (R1∞) : R2∞ → L5 [direct] : > B # (H5) | : work h∞ up thru & to L
of "Dog"].

This gives Fig. 82; for the release, "dissolve" L "rock" by

L2 ↓ (L1∞; to R of "rock") : □ L1 # (H5) : < L2∞ → L1].

This gives the "Dog". Continue with

2 (H5f; between 5f∞'s) : □ 5 # > 2∞ → 5]

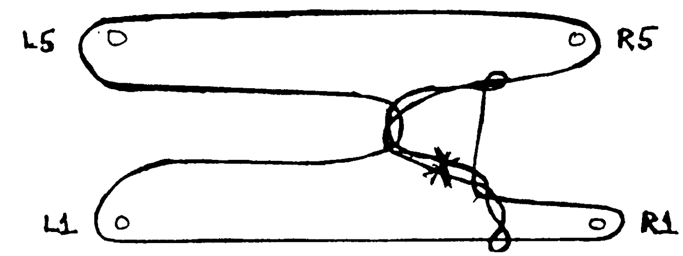
to produce "Dog's Stomach". Now

2 (base of c-◁$^{(2)}$) : H2 (5f) : > 2 (#) : □ 5 : 2∞ → 5].

This gives the "Rock He Swallowed", near the base of R1. Continue with □ R1] to completely dissolve figure.

Notes on the construction: (1). The opening is a modification of O.A, in which L2 reaches across (rather than down into) R2∞ to pick up Rp. (2). After the figure is laid flat, we observe two sets of double oblique strings, Lp-Rp; one set lies beneath the central tangle of the design, the other (set) lies right on top of it. It is this latter pair to which the symbology "u-o$^{(2)}$;Lp-Rp" refers. Thus R2 Hooks these two strings towards you, and down into the R1∞; then R2 proceeds away, under, to the 5f-string between the two loops lying on this string (ultimately, these become the "legs" of the "Dog"). Now R2 Hooks up this 5f-string, from above, and returns to its normal upright position, retaining this string only. Finally, after □ L5, R2 makes a direct (cross-hand) transfer of its loop to L5, passing it down through R1∞ and under the central design to do so. (2) [There should be a H5f-s string at the completion of this complex(!) move.] (3). The h∞ referred to immediately prior to] passes loosely through the body of the "Dog", to limply encircle L1∞. The M may be used to seize this loose string and pull it free of the "Dog" figure [if you've seized the wrong string, it won't give at all when you pull with the M; try another, nearby string. After two or three successful constructions, this string will reveal itself every time.] (4). At the stage of the "release" procedure called "Dog's Stomach", one meets

with the string position



Over

Fig. 82.A: Dog's Stomach (sketch).

Here, (both) 2 are to pass away under the double strings at ✱ in the above figure and thence away over all intermediate strings to Hook-up 5f from above. (5). The final figure "Rock He Swallowed" is identical to that of Fig. 70.III [KnotBear (Right); Step 3], whose construction may be carried out at this time in lieu of final dissolution. (6). No known "source" validation exists for this entire "dissolution" procedure.

This completes the discussion of the Brown Bears cycle (Bears, I).

Bears.II

In this subsection we shall sample some of the diverse construction-types which produce Bears. Although several of the figures below live in "string-figure cycles", we shall suppress this aspect of the subject here -- a rather extensive example having just been discussed in the previous subsection -- only producing the "Hearth"-figure of the cycle; and, when that is not a Bear, showing how the Bear is related thereto.

The first figure to be discussed is the "Fish Net Torn by Two Polar Bears" [D. Jenness, No. 47]:

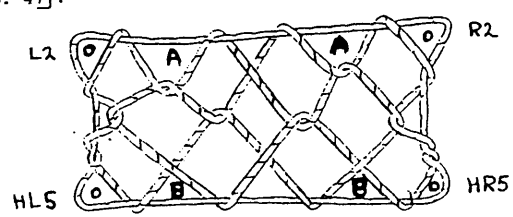


Fig. 83.A: Fish Net.

O.A: $\overrightarrow{1}$ (2n) : 1→ (5n) # $\overleftarrow{3}$ (1f) # □1 | : $\overrightarrow{1}$ (2n) : 1→ (5f) # □2 | $\overrightarrow{3\infty}$ → 5 : $\overleftarrow{5\infty}^{(2)}$ → 2 | :
←H5 ($\overline{1n}$) # (H5) : □1 | :: 1→ ↑ (H5∞) : $\overleftarrow{1}$ ↑ (u2∞) : $\overrightarrow{H1}$ ($\overline{u2f}$) # (H1,5) : 1→ (H5f) # (1)
[□ u2f from 1] : □5 # | : ←H5 ($\overline{1n}$) # (H5) : □1 | : $\overleftarrow{2\infty}^{(2)}$ → 1 :: $\overleftarrow{2}$ ↓ (1∞$^{(2)}$) :
←2 (1n-o) # (H5) : $\overleftarrow{H2}$ ($\overline{1n-s}$) : < 2 (#) [□ 1n-o from 2] : □1].

This gives the beautiful Fig. 83.A, "Fish Net". We recognize both complex-crossing types X_6 (above) and X_7 (below) on Lp, for example.

Continue

$\overleftarrow{2\infty}$ → 1 | .

This gives Fig. 83.A, with 2,H5 replaced by H5,1 (in the Over perspective), respectively. Continue

Pass $\overrightarrow{2}$↓ into central design at A: bring ←2↑ (back) into central design at B: $\overleftarrow{H2}$ ($\overline{1n-s}$) : < 2 (#) : □1 # (H5)].

This gives the "Two Polar Bears", illustrated below.

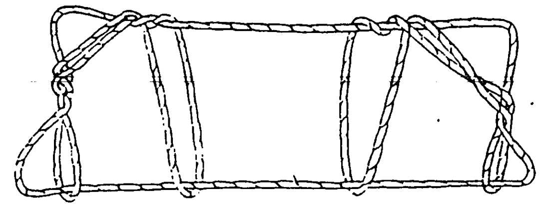


Fig. 83.B: Two Polar Bears.

Notes on the construction: (1). The (only) "□2" move in the manipulation sequence -- near the beginning of the construction -- is, indeed, startling to most (even very accomplished) string-figure practitioners upon first encounter with this figure; it just feels "wrong", somehow. This tends to make the figure difficult to commit to memory for these same people (author included); the feeling of "wrongness" never abates, as there is little reinforcement for this curious "release move" elsewhere in the string-figure literature. (2). The two "double" loops which occur in this construction -- first on 5; then moving, as a pair to 2; and ultimately coming to rest on 1 -- are to be maintained as distinct throughout the working of the figure. Again, such two-loop interdigit transfers occur infrequently among the string-figures of the world. (3). The complex passage of 2 through the "Fishnet" design in passage to the "Two Polar Bears" figure is the third (and final!) odd movement of rare occurrence found in this serial figure. (4). During the final extension, "]", of the "Two Polar Bears", the Bears separate centrally, one from another, in a manner entirely reminiscent of the final disentanglement of the Brown Bears (Fig's. 67.A-E). Here, in the present figure, the relevant ultimate separation is greatly facilitated by repeatedly seizing up, raising, and letting-go of the o$^{(2)}$;2∞ with the backs of the thumbs; this being done several times during the course of the "tug" of the "]".

Let us spend a minute to discuss how a Polar Bear compares to a Brown Bear, using the "String-design"-- or "Knot" -- approach. To construct the Polar KnotBear, proceed as in the Brown KnotBear through the position of Fig. 70.II (page 169). Now, referring to that figure, pass R1 directly towards you (from the far side of the configuration) into the design at the point marked R5 and, similarly, pass R5 towards you at the point marked R1 therein [as in O.1(R)]. Pick up the h∞, as before, to form O.1(L) -- so that 1n and 5f are straight (i.e. uncrossed). Now #, and observe the schema

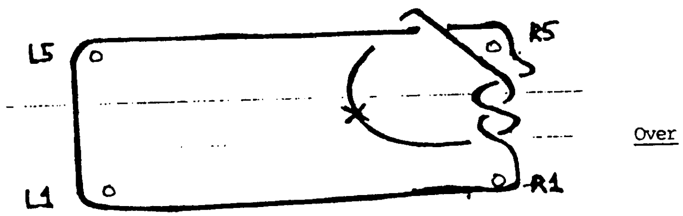


Fig. 84.A: Polar KnotBear, III.

Note, Polar KnotBear I and II are identical to Brown KnotBear I and II (Fig's. 70.I and II, respectively). Now -- enlarging the central loop, if necessary -- throw a small upright loop thereon, passing L over R, at the point marked X in the Fig. 84.A. This gives

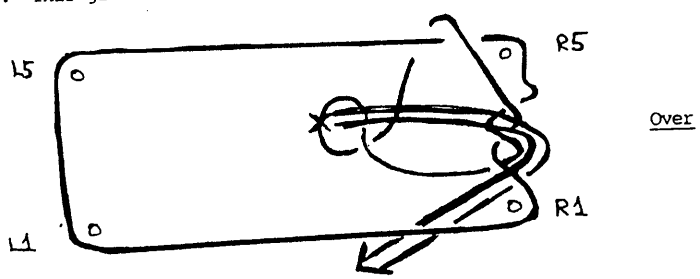


Fig. 84.B: Polar KnotBear, IV.

Now, seizing this small, upright loop at the point marked X in Fig. 84.B, thread it (like a "ribbon" -- i.e. no twists) to the R over Rp; then turn it directly down over this string and draw it towards you -- under the figure -- to the near side of R1n. This gives

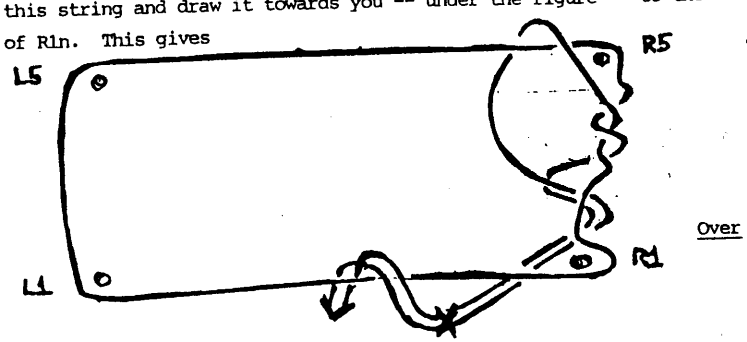


Fig. 84.C: Polar KnotBear, V.

Continue threading this loop (still "like a ribbon") over, away, and back up under R1n again. Finally, turning this loop directly to the right and down, thread it over each of R1∞ & R5∞ -- in turn -- to the far side of the figure (this is also the last "threading" move of Brown KnotBear, page 171). The result is Polar KnotBear (Right), the rightmost Bear of Fig. 83.B, above. Its anatomical dissimilarity to Brown KnotBear (Right) is egregious.

The second figure we shall consider is the "Siberian House", nearby to which lurk Bears. There are two "Siberian Houses", with the same basic construction [G.B. Gordon: "Notes on the Western Eskimo", Fig. 22]. The plate, below, is from K. Haddon: Cat's Cradles From Many Lands, Fig. 36.

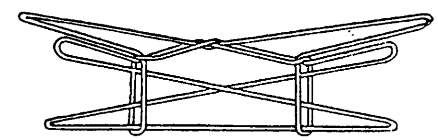


Fig. 85.A: Siberian House, I.

O.A: 2345 (1n) #□1 | : 1 (2345n) : 1 (2345f) [below 5f] #□ 2345∞ I ,
This gives Fig. 85.A. Continue

□ 2 I

and the "House" breaks; two "Little Boys" run away to either hand.

The elaborated "big" Siberian House, below, comes from K. Haddon: Artists in String, Fig. 6.

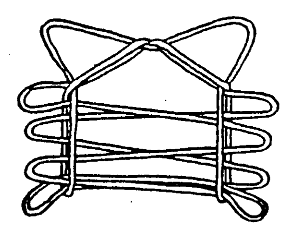


Fig. 85.B: Siberian House, II.

O.1: Raise the palmar string on each side, and drop it behind 3 (of the same side) :: ⇐R2 (L2p) # ⇒L2↓(R2∞) : L2⇒ (R2p) # | :: ⇐R4 (L4p) # ⇒L4↓(R4∞) : L4⇒ (R4p) # | : ⇐2345 (1n) # □1 | : →1 (2345n) : 1→ (2345f) [below 5f] # □2345].

This gives Fig. 85.B; essentially, it is Fig. 85.A with two extra finger-loops. Continue

□2] ---- "Break him",
□3] ---- "Fix him",
□4] ---- "Break him";

and two "Little Boys" run away, as before. Note that the "Opening" of this figure (through the second occurrence of "|" in the construction) is a five-loop version of the Opening A; at this point, you should have a loop on every finger. The remainder of the construction is that of Siberian House, I -- up to the final "release"-sequence.

The "Two Bears" earlier alluded to live in two "caves" below a "mountain", near to Siberian House, I [C. Gryski: Many Stars and More String Games, pages 34-35.]

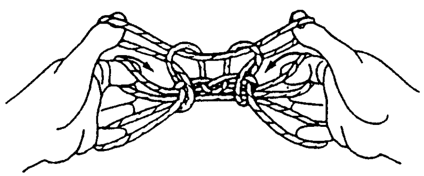


Fig. 86.A: Two Caves Below the Mountain.

O.A: ⇐2345 (1n) # □1 | : →1 (2345n) : 1→ (5f) [above 2345f] # □1 → h∞ (gently): →1↓(h∞) : 1→ (2345f) # | : □2345∞] (Fingers pointed away, and slightly down).

This gives Fig. 86.A. Continue

□2]

and "Two Bears" issue forth from the "caves" -- at either side of the central design. The "Two Bears" are illustrated below.

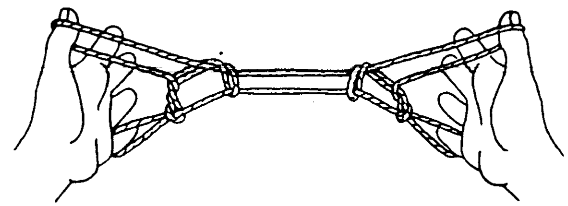


Fig. 86.B: Two Bears

And a similar pair of "Bears" -- 'though the final figure is not so identified, for obvious reasons -- lurks nearby the Hawaiian "house" figure (which is, similarly "broken" and "restored") "Ku e Hoopi'o ka La" [L. Dickey: String Figures From Hawaii, Fig's. 2 & 3].

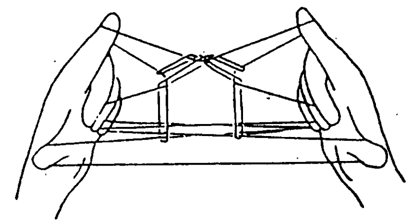


Fig. 87.A: Ku e Hoopi'o ka La

O.1: ⇐HR2 (Lp) # (HR2) : ⇒L2↓(HR2∞) : HL2⇒ (Rp) # (H2) | [This is O.A with H2's] : <2→↑(5∞) : □5 ǂ # | : ←H5 (2n^(2)) # (H5) : →1 (2f-s) # (H5) □2 (2f-s) | : ←R2∞ → R1 : ⇐R2↓(L2∞) : □L2 # (R2) : ⇒L2↓(R2∞) : ←L2↓(uR1∞) : □uR1∞ # (L2) | : ←4 (2f) : ←4↑(2∞) : ←H4 (2n) # (H4,5) | : 1→↑(2∞) | : ←H3 (all strings) [just to hold things]: X (u1∞) (R) : □3 # (H4,5) : 1→↑(2∞) : □2 | :: ←2↓(u1∞^(2)) : ←H2 (u1n-s) : <2 (#) | □ (u1∞^(3))].

After minimal arrangement of the strings of the central design, this gives Fig.87.A Continue,

→1 (2f) [after it is looped by a small ring on 2∞] : □2 : u1∞ → 2] (gently, so central loops catch).

This gives "Kona", illustrated below.

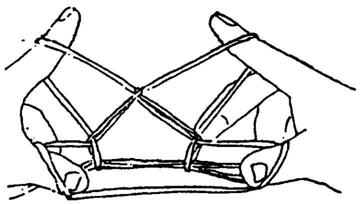


Fig. 87.B: Kona.

Continue,

I (firmly, so central loops pass).

This gives "Kau", illustrated below.

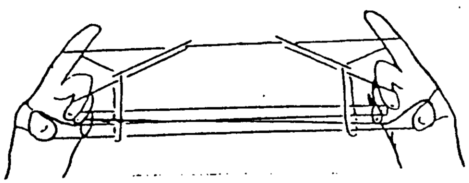


Fig. 87.C: Kau.

Continue,

□4 I.

This gives "Puna", illustrated below.

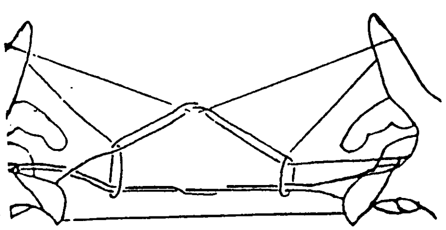


Fig. 87.D: Puna.

Continue,

2ꝏ ⟵ →1: ⟵2 ↓(u1ꝏ): ⟵H2(χ1n-s):<2(#):□1 I.

This gives "Hilo", illustrated below.

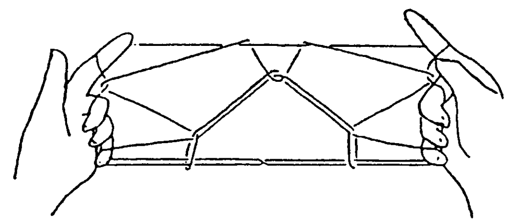


Fig. 87.E: Hilo.

Continue,

Pass 1 and to the center, and insert 1 into the small "rings" which encircle H5f(2) #(H5) I.

This gives "Hamakua", illustrated below.

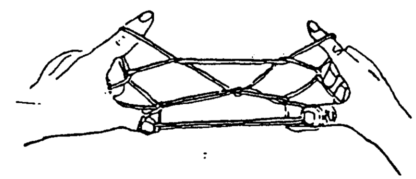


Fig. 87.F: Hamakua.

Finally, continue to the final design

□5: <<<1 I [palms away, 1 below 2]

extended as in the illustration, below.

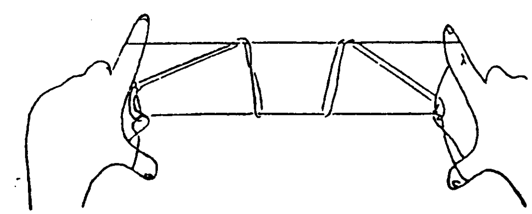


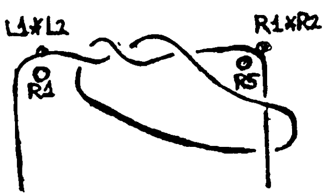
Fig. 87.G: Kohala.

This gives "Kohala—where there are no Bears."

Notes on the construction: (1). The Opening is "O.A with index fingers turned down" — mirabile dictu! (2). The symbology "□2(2f-s)" should be read "release the 2f-s string (only) from 2." (3). The "4-move" in the construction, i.e.

"$\underleftarrow{4}$(2f): $\overleftarrow{4}\uparrow$(2∞): $\overleftarrow{\overline{H4}}$($\overline{2n}$) #(H4,5)|" requires much of a ring-finger unused to such independent operation. The H4∞, at "|", is to be drawn well <u>behind</u> (!) the two strings on the Hooks of 5 -- and held close to the palm -- until its ultimate release, in Fig. 87.D. (4). The penultimate (complex) 2-move finds four (!) 1∞'s -- and, consequently four 1n-strings; the lower two of these are "straight" -- passing directly from thumb to thumb. Of these two 1n-s strings, one is much looser than the remaining strings of the design and, consequently, hangs well below the other 1n-s string, which is at standard current tension, <u>i.e</u>. taut. And it is this latter string which is Hooked-up by 2 during this movement -- this explains the Calculus used to encode the manipulation. (5). In our extension of the final design, "The Kohala Bears", we have added a full twist (<u>i.e</u>. <<1) to the display of this figure -- as opposed to Dickey's instructions -- the better to exhibit the analogy with the Bears of the Siberian House cycle. There is no "source" confirmation for this extension; it is "poetic license", pure and simple -- although we <u>do</u> believe in Koala Bears!

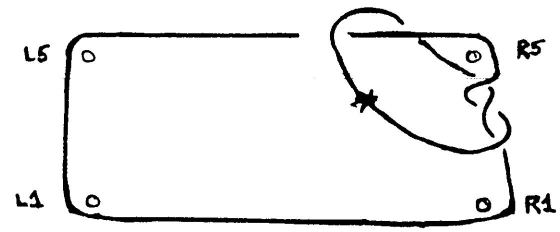
Let us again spend a minute to discuss the current Bear under consideration, the "Siberian"Bear (Fig. 86.B), as a Knot. To produce the Siberian KnotBear, proceed as in the first two steps of the Brown KnotBear construction (page 169), interchanging "Left" and "Right", throughout. This gives



Under

<u>Fig. 88.I</u>: Siberian KnotBear.

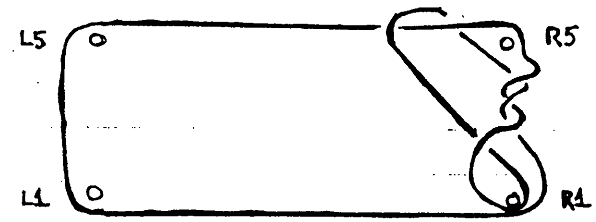
Now, take this configuration onto the hands (analogous to <u>O</u>.1) exactly as in Step 3 of the Brown KnotBear (page 170, top). This gives



Over

<u>Fig. 88.II</u>: Siberian KnotBear (continued).

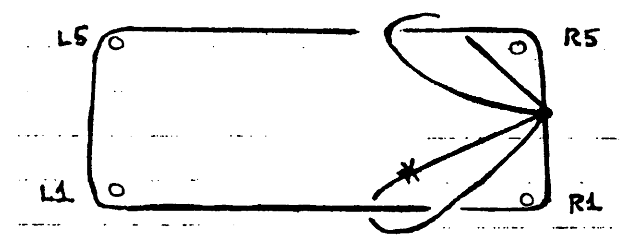
which is analogous to Fig. 70.III (<u>and</u> Fig. 84.A). Now, seizing the central loop at the point marked ✳, throw a small upright loop thereon, passing L over R, and -- turning this small upright loop directly toward R1 and down -- place it on R1 (as an uR1∞). This gives (the (g)Schema)



Over

<u>Fig. 88.III</u>: Siberian KnotBear (continued).

Now, $\overrightarrow{\text{R1∞}}$→uR1 (over), and then N(R1) -- this whole movement has been dubbed "double-Navaho" R1. This gives the (complex) schema



Over

<u>Fig. 88.IV</u>: Siberian KnotBear (continued).

Now, seizing the near central loop (about R1n) at the point marked ✳ in the figure, throw a small upright loop thereon -- passing R over L -- and turning this small loop directly down to the right, thread it over each of R1∞ & R5∞ -- in turn -- to the far side of the figure (this is the same last "threading" move as in each of the two previous KnotBears). The result is Siberian KnotBear (Right), the rightmost Bear of Fig. 86.B.

There is precisely one more "Bear" figure we wish to analyze as a "KnotBear", and it arises from the following figure. At the completion of this discussion we feel that a sufficient number of these micro-analyses has been carried out on "Bear"-types to give a good representative sample of the variety of such figures (which all look more-or-less alike, at some gross level of cognition). The following figure is from J. Averkieva: Kwakiutl String Games, No. 75, "A Bear" [all Averkieva plates are by Mark Sherman (personal communication).]

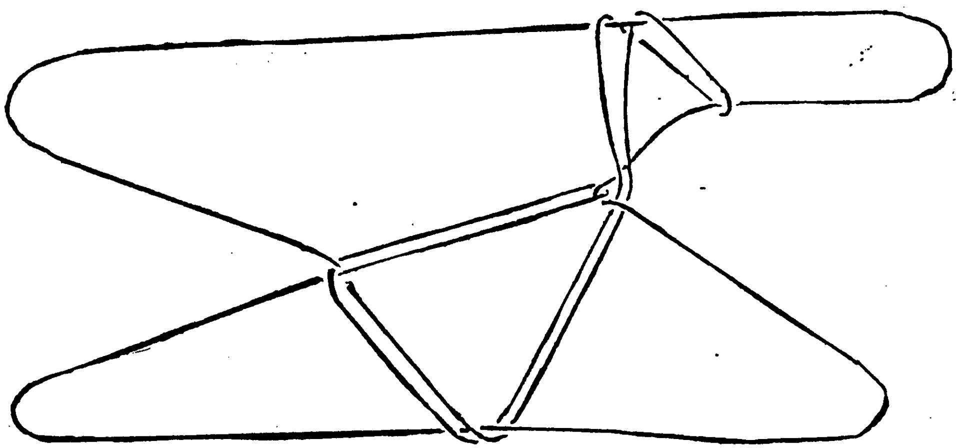


Fig. 89.A: A Goose.

Hang loop on L12, with the long hoo on the palmar aspect of L: Pass R1 right, and to the near side of L: R1(L1n & L12d) # L2∞ → L1|: 5↑(1∞$^{(2)}$) #| (widely): □5(5f-s), then H5 over this just-released string # (H5)|:: 2↓(1∞$^{(2)}$): 2(1n-o) #(2): H2(1n-s): < 2(#) [□ former 2∞] = □1] "Two Diamonds". L1↑(L2∞): Draw L1 to center, and pick up both upper lateral and lower central arm of (left) central diamond: #(L1): R1↑(L1∞$^{(2)}$) #(H5)|: 1↑(2∞): (ℓ1∞$^{(2)}$}N1: #(H5): □2|: There are now three s;Rp-Lp -- two lying above the central configura-

tion, the other below: Pass L2 these three strings (near Lp) to far side of figure: HL2(L5f) #(L2) :: □ R5: L2∞↓(L1∞): L2∞ → R5 [direct] #(H5)].

This gives Fig. 89.A. Continue

□ HR5: HR2345↓(R1∞)].

This gives "Bear" (Left), illustrated below.

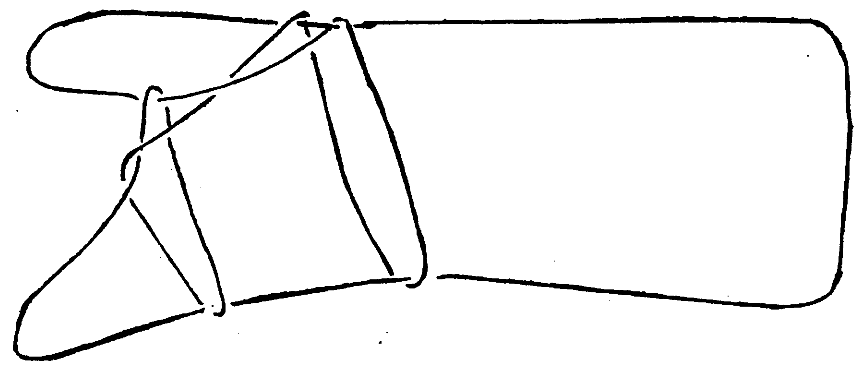
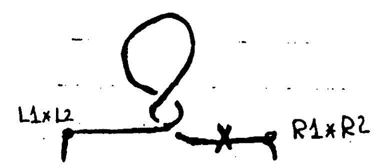


Fig. 89.B: A Bear

Note the unusual opening, and the similarity of the final complex L2-move (of "Goose") to the final complex R2-move of "Snoop" (Fig. 82). We shall meet with a variation of both below, before the end of the current section.

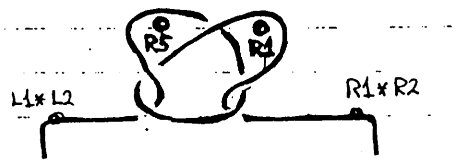
If we examine the left palmar string somewhat closely, now, we find no "Osage Diamond"-type crossing ($\mathbb{X}_7$, $\mathbb{X}_8$) anywhere to be seen! Surely this figure is a prime candidate for a KnotBear analysis. We begin by throwing a small upright loop in the string -- passing left over right -- and then give this loop an additional (180°-) twist. This gives



Under

Fig. 90.A: Kwakiutl KnotBear, I.

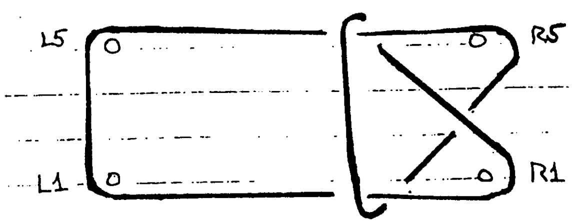
Now, seizing the right-side string issuing from this loop at the point marked X in Fig. 90.A, draw this away from you and to the far side of the small loop, then push it directly towards you through the loop, and back up to the right side of the figure; this gives



Under

Fig. 90.B: Kwakiutl KnotBear, II.

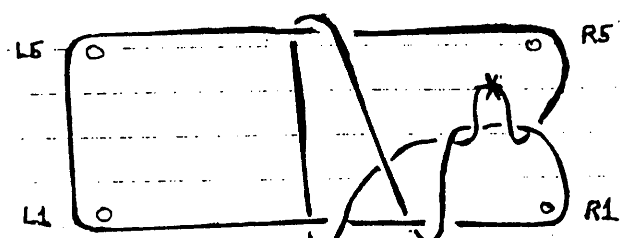
Now, releasing R1*R2, pass R to far side of figure; pass R1 and R5 directly towards you into the figure at the points marked R1 and R5, respectively [as in O.1(R).] Pick up the far end of the string (the long h∞) as in O.1(L), so that the 1n and 5f strings are straight, and uncrossed. Now #, and observe the schema



Over

Fig. 90.C: Kwakiutl KnotBear, III.

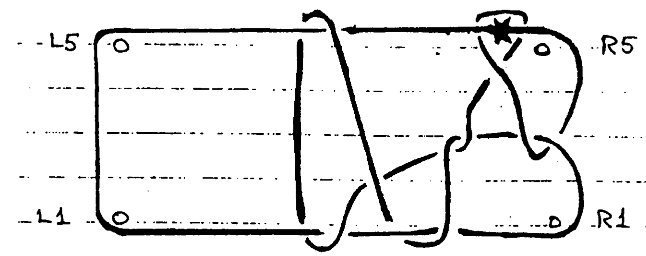
Now >>R1 and, reaching down through this (R1) loop, seize R5n and draw it back towards you, and up through the R1∞. This gives



Over

Fig. 90.D: Kwakiutl KnotBear, IV.

Still holding this (former) R5n-string -- at the point marked ✱ in Fig. 90.D -- throw a small, upright loop thereon (passing right over left) and continue drawing this string away from you, under, until the small, closed loop passes beneath the R5f-string. You should now observe the schema



Over

Fig. 90.E: Kwakiutl KnotBear, V.

Now, □R5 gently, and pass R5 up into the small loop lying beneath R5f -- passing to the near side of this string -- and with the back of this finger push R5f (at the point marked ✱ in Fig. 90.E) away and down through this loop, thus picking up this string on the back of R5; #]. This produces Kwakiutl KnotBear (Right)-- for comparison purposes with previous KnotBears -- which is made, directly, by ↓L̲R↑ in the construction of Fig. 89, "Goose/Bear". The Bears remaining to be discovered in the present section fall into two categories: Those whose gross-schemas are (essentially) that of Brown Bears -- here the "Knot-analysis" is, at worst, a minor variant of one already met with; and those whose gross-schema bears little or no relation to that of Brown Bears. For these reasons, this type of analysis will not be pursued further at this time, the above four samples being taken as "representative" of the whole.

Next we examine two examples of the young of the species, the Bear Cubs. The first figure, Brown Bear Cub [D. Jenness, No. 49], is one of those whose gross-schema bears little resemblance to Fig. 66. This is not so surprising, since the "Cub" is depicted "rolling on his back".

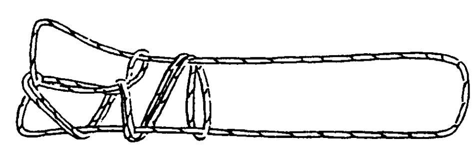


Fig. 91.A: Brown Bear Cub.

O.A: X2(L) :: <L2oo→L1: L5oo→L1# |: L5↑(ℓL1oo$^{(2)}$): HL5(uL1f) # (HL5) |: L2↓(uL1oo$^{(2)}$): HL2(ℓL1n): <L2(#): □L1 |: L1↑(L2oo): Pass L1 over o$^{(2)}$; L2oo and Hook up, on the palm of L1 -- from below -- the R2n-string: #(L1): L1↑(L2oo) |□ R1 |: □R5 :: Thread R2oo, as a ribbon, directly to L and down through the two loops jointly held by L1 & L2, continue down and under all central strings, back to its original position on R2 #(HL5): □L1: <<R2 I.

This gives Fig. 91.A. Continue

□R: seize L5f in R, just to the left of L5f's rightmost loop: draw this out I.

This gives the "Polar Bear Cub", illustrated below.

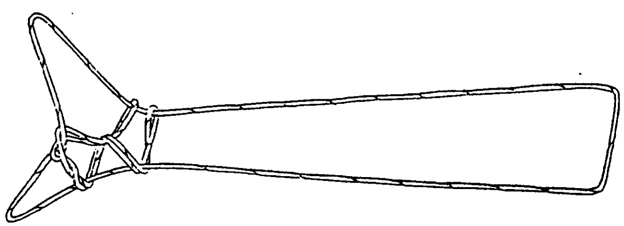


Fig. 91.B: Polar Bear Cub.

Continue

□ R: seize L2n in R, just to the left of L2n's rightmost loop: draw this out I.

This gives the "Young Beaver", illustrated below.

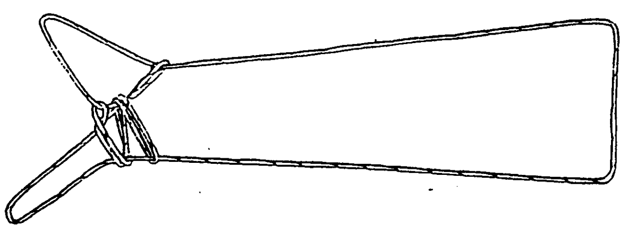


Fig. 91.C: The Young Beaver.

Continue

□ R: seize L5f in R, just to the left of L5f's rightmost loop: draw this out I.

This gives the "Young Man", illustrated below.

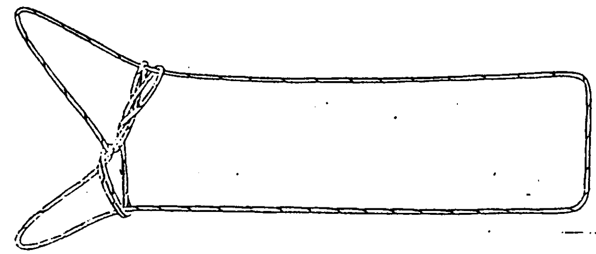


Fig. 91.D: The Young Man.

Continue

□ R: seize L5f in R near the base of L5: draw this out I.

This gives "His Small Rope", illustrated below.

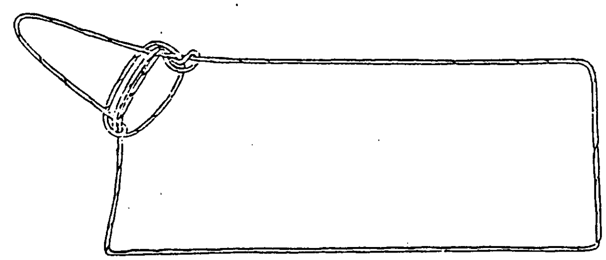


Fig. 91.E: His Small Rope.

And the serial figure ends. Notes on the construction: Some practice with the intermediate string tensions is necessary to get the full, beautiful final extension that this figure enjoys. Some tips: (1). After the "X2(L)"-move, pull the central "crosses" closer to R; (2). At the move "□R1", a long loop, coming from the left to wrap round the Rp-string, will be observed. The final extension is facilitated by keeping this loop as long as possible; (3). When threading the R2oo to right, down through L1oo$^{(2)}$, and back under to R2, make sure the loop identified in (2)., above, follows the R2oo in its passage. In particular, this loop should end up in complete extension to the right, at the ~~completion of the move.~~ (4). Just before I --at the completion of "<<R2", the R345 fingers may join R2 in its loop, to widen it for a more perfect extension.

The second "Cub" figure [J. Averkieva, No. 78], is the only "source"-identified "Grizzly Bear" in the collection. Its working is unusual.

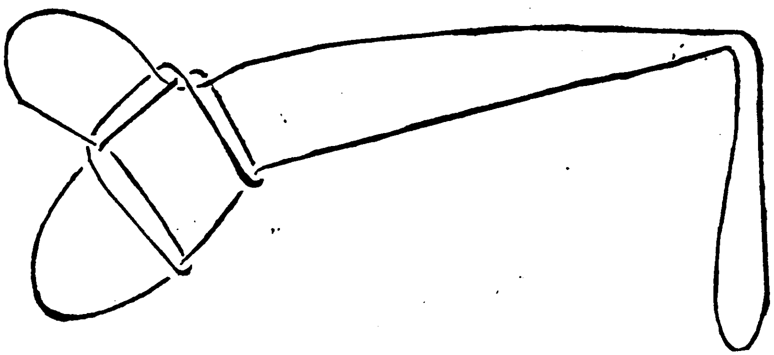


Fig. 92: Small Grizzly Bear.

Hang loop on L12, with the long hꝏ on the palmar aspect of L: Pass R away and grasp (both strings of) hꝏ about 12" from L: HR2 (L12d): draw this string to right, away over both R-strings, and back up [#(2)] to near side |: HL345 (L1n) # (HL345) :□ L1 | : L1↑(R2ꝏ) :□ R2 # (L1) | : L1 (L2f) # (L1) : N(L1) :□ L2 I.

This figure provides insight into what a Bear-in-string "really is"; both the construction and the hand position (especially, Right) are quite remarkable! And, in that same vein, we present another Kwakiutl Bear-figure which is sui generis [J. Averkieva, No. 79, "A Bear"].

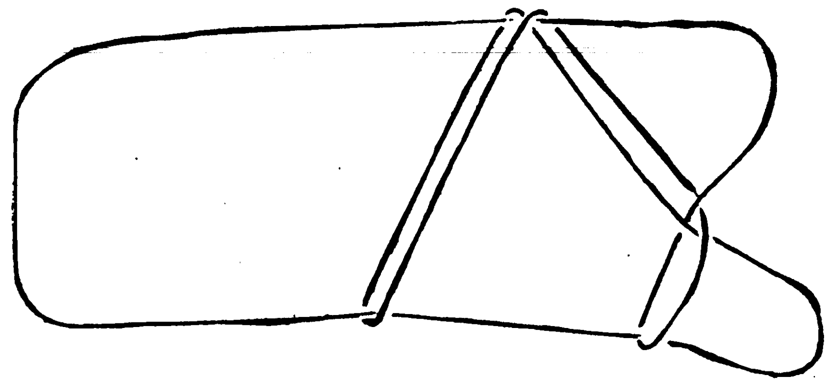


Fig. 93.A: A Bear.

Hang string over L arm so that there is a nhꝏ and a fhꝏ of equal size: from near side, with R, seize lower string of nhꝏ: Pass R, pushing upper string of nhꝏ before it, on its back -- then away from you past fhꝏ -- and take up, from the far side on the back of R1, the upper string of fhꝏ: #(HR2345) [using "archer's motion -- thus □ upper string of nhꝏ from the back of R] ::□ R2: R2↑(R1ꝏ): HL345 (uLWn) # (H345) | R1*2 (uLWd): □ LWꝏ(2) # (H345) I.

Notes on the construction: The opening position is

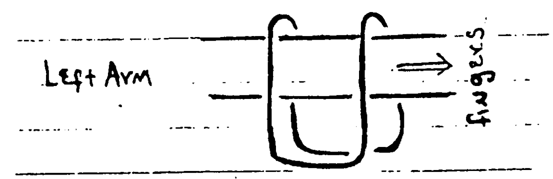


Fig. 93.B: A Bear, Opening position.

At I, the string held in R1*2 usually slips down over R1 to become an R1ꝏ.

Now, we offer another variation of the ever-popular "Bear emerging from his Den (Cave)"-theme -- only, this time, the stationary "Cave", itself, transforms into the emergent "Bear"; another of the truly remarkable, distinctive Kwakiutl figures [J.Averkieva, No. 6, "The Bear and His Den"].

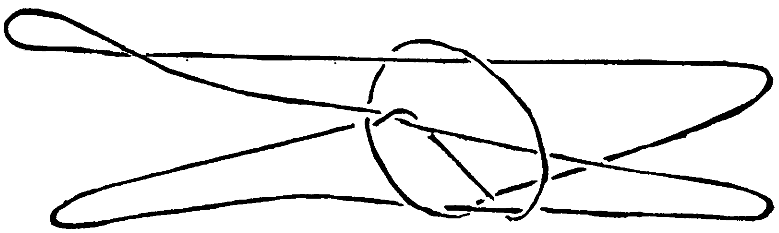


Fig. 94.A: The Bear's Den (Cave).

O.1:: $\overleftarrow{\overleftarrow{HR2}}$($\overline{Lp}$): <R2# $\overleftarrow{L5}$∞—>L2:□R5 | : H345 ($\overline{2n}$)#(H345) | :$\overrightarrow{\overline{L2}}$↓(R2∞):
L2(R2f)#(L2) [□ former L2∞] | : $\overrightarrow{\overline{L1}}$(L2n)#(H345): N(L1) | :□R2:
$\overleftarrow{\overleftarrow{R2}}$↓(L2∞): R2(L2f)#(R2) | :□L2 | :□R1 I.

This gives Fig. 94.A. To witness the "Bear" emerging from this cave, continue

□R2 I

and the strings comprising the "Cave" momentarily sag, then reform themselves into a "Bear", who proceeds off to the left hand, in the figure. He is illustrated, below.

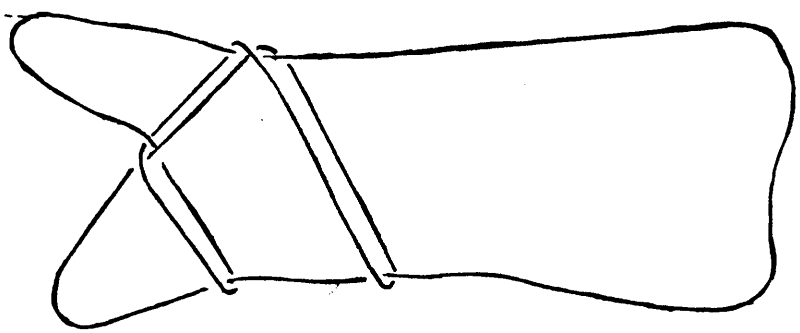


Fig. 94.B: A Bear.

We remark that if, during I, the R and L hands are held at nearly the same height, the "Bear" erupts from his "Cave" and glides off leftward swift, and silent as the night; if R be held somewhat lower than L during this I, the "Bear" can be made to clamber out of his "Cave" and stomp off — uphill — to the left hand.

Another pair of "Bears" is to be found among the stars. The following serial figure is from J. Averkieva, No.51:

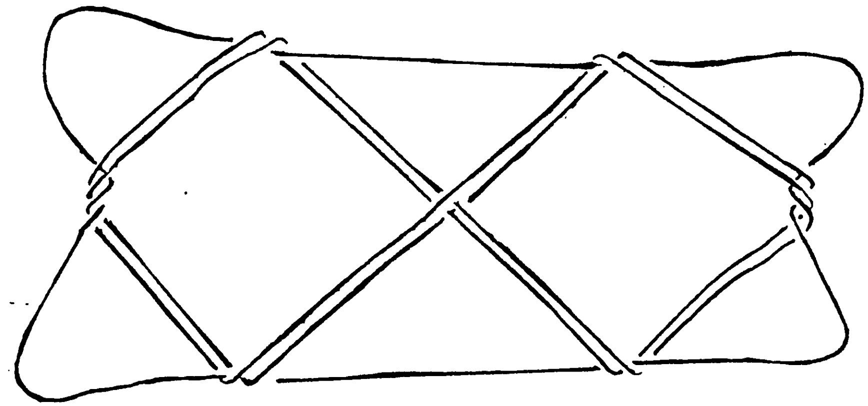


Fig. 95.A: The Burbot.

O.A: $\overleftarrow{2}$∞—>1:$\overleftarrow{5}$∞—>1::5↑(ℓ1∞$^{(2)}$): $\overleftarrow{H5}$($\overline{u1f}$)#(H5) | :: $\overleftarrow{2}$↓(u1∞$^{(2)}$):
H2(ℓ1n):<2(#):□1 | .

This gives Fig. 95.A. It is also known as "Navaho Two Stars" [C.F. Jayne: String Figures and How to Make Them, pages 129-131], and, contemporarily, as "Twin Stars". Before proceeding to extricate the "Bears" from these stars, we must examine the string-position of Fig. 95.A more closely. The geometric center of this figure is a "cross", each of whose arms consists of a pair of strings; and, where these four strings cross, a tiny four-sided figure -- or "quadrangle" -- is defined between them. Arrange the strings of the c-X so that the upper, lateral strings of the quadrangle are those that proceed to the upper transverse string of the figure (i.e. 2n-s), whose continuations loop singly about the respective palmar string [Note: the other pair forms an Osage-Diamond-type complex-crossing with the respective palmar string]. Having thus arranged the strings of the central quadrangle, continue

1↑(c-quadrangle):>$\overrightarrow{2}$∞—>1#(1) | (gently): $\overrightarrow{2}$ (H5f; between the two loops on H5f): raise H5f-s slightly, then □ this string from 2, dip 2 towards center of figure and pick up, from below, both strings of the central H5f-loop):#(H5) | : $\overleftarrow{H2}$($\overline{1n-s}$;center): <2(#) [□ former 2∞$^{(2)}$] #(H5):□1 I.

This gives "Two Bears", illustrated below.

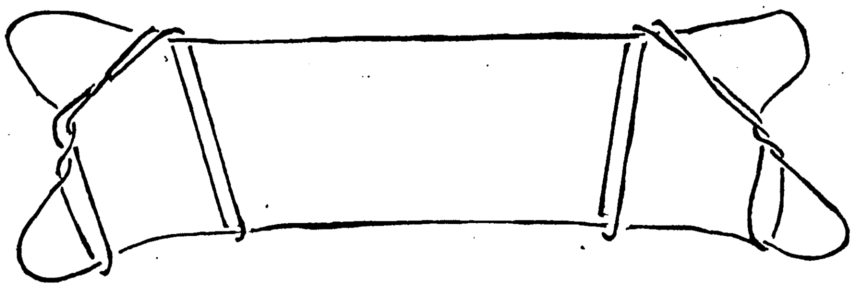


Fig. 95.B: Two Bears.

Note on the construction: At the move "| (gently)", the two halves of the figure will separate; it will be found that the subsequent "2-pickup" will be easier if this separation is not allowed to become too extreme [i.e. if the loops to be picked-up by 2 maintain some central contact -- the meaning of "(gently)" should be so construed.]. The Knot-analysis of these "StarBears" is a synthesis of the Kwakiutl and Siberian KnotBears, discussed earlier (pages 194-196 and 191-193, respectively) and should be considered a "minor variation" thereof.(3)

The penultimate figure of the present sub-section of Bears, II is the appealing "Brown Bear's Px" [D. Jenness, No. 117].

Loop string over backs of 1 -- held about 6" apart -- so that long hꝏ depends from palmar side :: H̄2(1̄n):<2(#)::R1↑(L1ꝏ)#(R1):L1↑(R1ꝏ)#|::

5↑(ℓ1ꝏ): { H̄L5(L̄2f) / H̄R5(uR̄1f) } #(H5)|: Keeping 1ꝏ$^{(2)}$ taut, bend 1 away and allow a single 1ꝏ to drop off: #(1)|1⃗(2n):#(1):N1:□2].

This gives the intermediate stage, illustrated below.

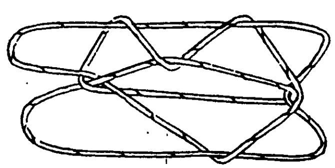


Fig. 96.A: Brown Bear's Pack; intermediate stage.

To get the "Brown Bear and his Pack", continue

1ꝏ→2: L1(o;HL5ꝏ)#(H5): R1↑(L1ꝏ)|: 1⃗(2n)#(H5): N1:□2].

This gives "His Blubber Poke", illustrated below.

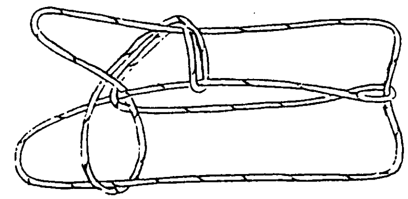


Fig. 96.B: His Blubber Poke.

Continue,

L̄2↓(HL5ꝏ): H̄L2(ō$^{(2)}$;Lp) [These strings form a Lpꝏ]:>L2(#)::
H̄L2(L̄1n):<L2(#):□L1|:□L5: H̄L345↓(L2ꝏ)].

This gives "The Brown Bear", proceeding off to the right with the "Pack" on his back, illustrated below.

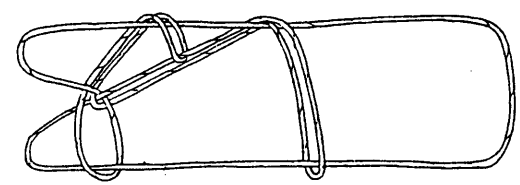


Fig. 96.C: Brown Bear and his Pack.

Continue,

R̄2 (1n; just to left of "Pack"): #(R2):□R1].

And the "Bear" continues off, right, without its "Pack".(4)

Just as the dog "Snoop" (Fig. 82) is a close neighbor of the (original) Brown Bear (Fig. 66) in its cycle, so too a "Little Dog with Big Ears" lives close to the present Brown Bear (with "Pack"), in its cycle [D. Jenness, No. 123]. Initially, construct the "intermediate stage" of the "Brown Bear's Pack" figure, Fig. 96.A -- the "Dog's Face" -- and turn figure to the "Over" perspective. Continue

HL2(o(2);Lp) [the Lp∞] : HL2↓(L1∞) : HL2(L5f) # (L1) | :□ R5 :: L2∞↓(L1∞) : L2∞ ⇒ R5 [direct] # (H5) | : HL2(Lp; between the two loops on Lp]: □L1 & L5 :: L1↑(HL2∞) :□ L2: HL5(L1f) # (H5)].

This gives the "Little Dog with Big Ears" -- illustrated below -- who can be made to "walk" off to the right by pulling gently with either hand. [The plate is from C. Gryski: Super String Games, page 80].

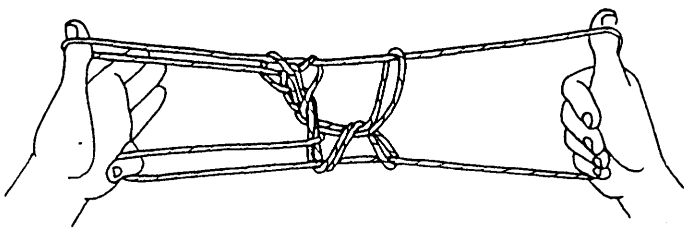


Fig. 97: Little Dog with Big Ears.

To dissolve figure, □ R and seize up the loop which comprises the Little Dog's left "ear"; pull this out, and he vanishes from the string.

The final string-figure of the present subsection of Bears, II -- designs with gross-schema related to Brown Bears (Fig. 66) -- is the Kwakiutl serial figure "Two Sitting on the Roof/Two Bears" [J. Averkieva, No. 30]. The Opening is the same as that of Fig. 94.A.

O.1 :: HR2(Lp) : <R2 # L5∞ → L2:□R5 | : Pass 2 -- with its loop -- towards you & down (deeply) into 1∞: >2(#) :□1 | : 1(ℓ2n) : 1(ℓ2f) # | : 3↑(1∞) : 3↑(u2∞) : H3(u2n) # (H3) : H45↓(H3∞) | :: pass L2 to far side of R2 and HL2(uR2f) :□ uR2∞ :: pass R2 to far side of HL2 and HR2(uHL2f(2)) : separate 2's (to enlarge their Hooked loop): (H)2(1n) # (2) [□H2∞(2)] | :□1: N2# (H345)].

This gives the "Two Sitting on the Roof", illustrated below.

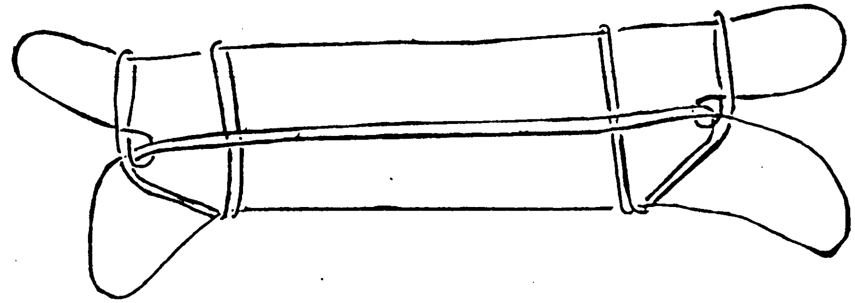


Fig. 98.A: Two Sitting on the Roof.

Continue,

H1(H5f-s, center) :□ H345 # (H1)] [fingers pointing away].

This gives the "Two Bears (as seen by those on the Roof)", illustrated below.

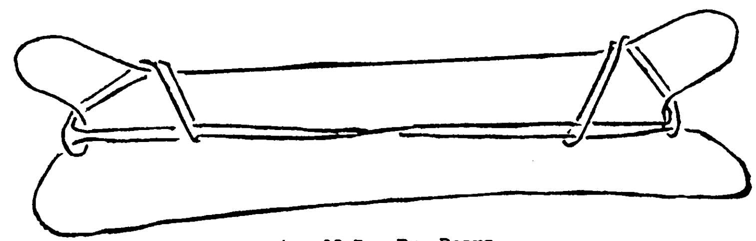


Fig. 98.B: Two Bears.

This figure is essentially "Two Rats on a Log" [D. Jenness, No. 131]; the two openings involved are distinct, but equivalent.

This final subsection of Bears, II concerns itself with a class of Bear-figures whose gross schemata are far more elaborate than that of the Brown Bears; all the present designs are of a single Bear. As contemporary figures, these string-figures are known under the various names Polar Bear, Ghost Bear, Spirit Bear, North Wind, and Buffalo. We shall present them, as usual, with their "source"-designations.

The first figure is the "Polar Bear" [D. Jenness, No. 137].

O.1 :: R1(L5f) : L1(L5f) : L1(R5f) #□5 | : 5↑(1∞(2)) # | (widely) :□ 5(5f-s), then H5 over this just-released string # (H5) | :: 2↓(1∞(2)) : 2(1n-o) # (2) : H2(1n-s) : <2(#) [□ former 2∞] ::□1].

This gives the intermediate figure "Two Diamonds", illustrated below.

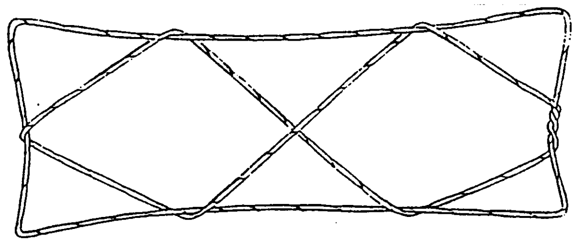


Fig. 99.A: Two Diamonds,

Continue,

1. ⇀1 (o;2ꝏ) : K : ⇀1 (2n) # (ℓ1ꝏ$^{(2)}$)N1:□2|.

This gives the schema

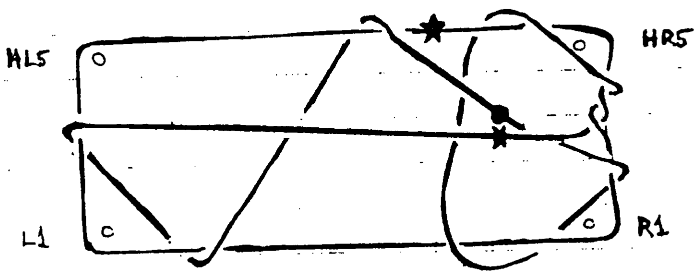


Fig. 99.B: Polar Bear, intermediate figure,

Continue,

2. ←HR2 over both strings marked X,O above: (H)←R2↓(R1ꝏ): ←HR2(‾5fs, at ✱)#(R2)::□L5:←R2ꝏ↓(R1ꝏ):⇐R2ꝏ→L5[direct] #(H5):<IB].

This gives the "Polar Bear" [plate from G. Mary-Rousslière: Les Jeux de Ficelle des Arviligjuarmiut, No. 6, "Nanorjuk"], illustrated below.

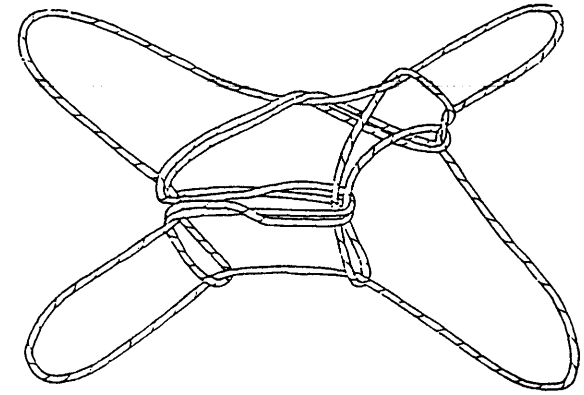


Fig. 99.C: The Polar Bear.

Notes on the construction: (1). The opening, and the construction of the initial figure in the series, "Two Diamonds", is strongly reminiscent of the similar initial figure in the series "Goose/Bear" (Fig's. 89 A,B). The two "Two Diamonds" figures are not identical, however, as a cursory examination of the complex-crossings on the respective palmar strings graphically reveals. (2). The complex R2-move of Step 2 in the construction is "physiologically" identical to the corresponding move in the construction of "Snoop" (Fig. 82) -- analyzed in Note (3) at the section's end. It should be remarked, however, that the string-position of Fig. 99.B, above, bears little similarity to the corresponding position in "Snoop", whence the results of this manipulation are predictably dissimilar, also

There are two variants of this figure to be discussed. The first is a minimal variation in which the first R2-pickup move in Step 2 of the construction is replaced by

2. ←HR2 over the string marked X, above.

That is, R2 does not, in addition, hook up the string marked O; this is the sole deviation from the parent construction. The result is illustrated below, from the "audience" viewpoint [G. Mary-Rousslière, Fig. 6'].

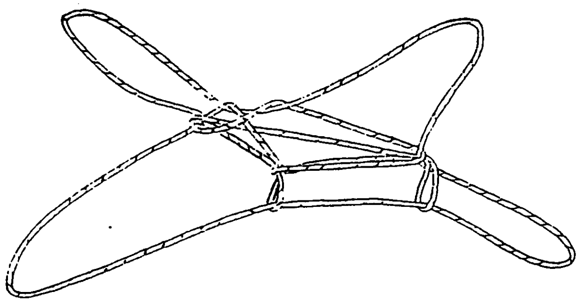


Fig. 99.D: Polar Bear (variant #1).

The second variation is more substantial. In this case, the Step 1 of the parent construction is replaced by

1'. 1→(o; lower central arm of its respective diamond):K:
1→(2n)#(ℓ1∞(2))N1:□2|.

This produces an intermediate position whose gross schema is

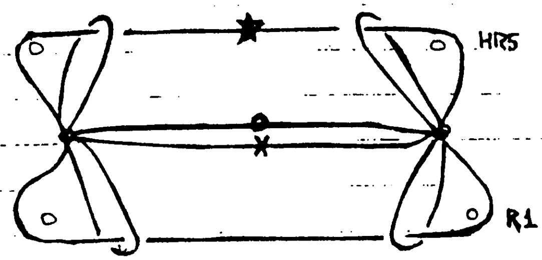


Fig. 99.E: Polar Bear-second variant; intermediate.

Over

Now, continuation of Step 2 of the original construction on the position of Fig. 99.E, above, gives the variation below [E. Holtved: "Contributions to Polar Eskimo Ethnography", No. 11, "Nanorsuk"].

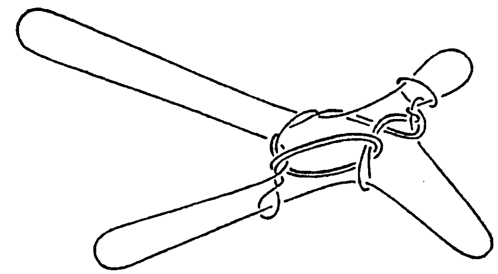


Fig. 99.F: Polar Bear (Variant #2).

A final variation of this figure -- not illustrated here -- results from the replacement of Step 1 in the construction by Step 1', and then performing Step 2 with √L\R↑ (i.e. performing the indicated movement with L2, instead of R2.)

This completes our discussion of the "Hearth"-figure Polar Bear.

The final figure of the section is the hauntingly beautiful "Spirit Bear" [T.T. Paterson: Eskimo String Figures and Their Origin, no. 108].

O.1:5∞→2::5(1n)#H5(2f)[□ 1n from 5]#(H5)|:R1 to dorsal side of L:R1(L12d):□L1::L1 to dorsal side of R:L1(R12d):N(R1) #(H5)|:L1(HL5f)#(H5)[□ former L1∞]:□L5|:HL5(L1n)#(H5):□L1|:: L1↑(L2∞):L1(o;L2∞):L1(HR5f)#(H5):□R5|:R5↑(L1∞):HR5(L1n): □L1#(H5):□R2].

This gives the "Mouth", whose schema is given below.

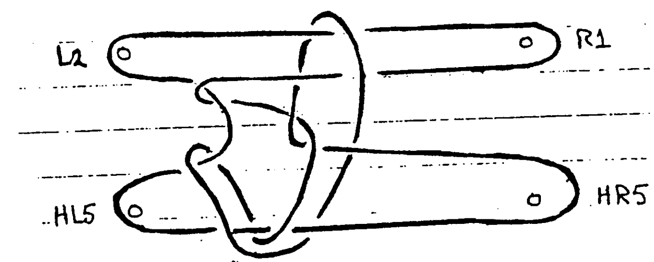


Fig. 100.A: The Mouth.

Under

Continue,

R2↓(R5∞):HR2(R1n):<R2(#):□R1|:R1(R2n;just to R of central loop):R1(HR5f)#(H5)[□2n-s from R1]:□R5|:L1(HL5f) (#HL5): □L5#|:>2:H5↓(H2∞):□2#(H5)].

This gives the "Spirit Bear", illustrated below.

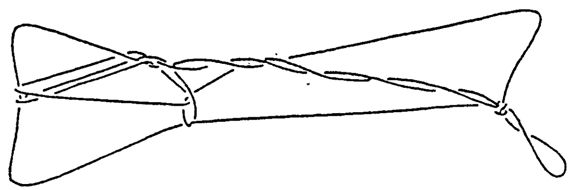


Fig. 100.B: The Spirit Bear.

An extra turn of the 1n-strings about each thumb -- accompanied by a "push" towards the center of the small loop about HR5∞ -- results in a "stockier" Bear in the final extension than that depicted in Fig. 100.B, above.

------- End Bears Discussion ------

BEARS NOTES

(1). (page 171): KnotBear Invention

Here we supply the missing details in the construction of the (invented) Fig. 71. First construct KnotBear (Right). Now -- keeping R1∞ and R5∞ secure on their respective fingers -- seize up a short (~8") section of L5f between 1*2 of either hand, releasing the previously held left-hand strings [O.1(L)] as you do so. Now repeat Steps 1 and 2 of KnotBear (Right), √R L. This produces

Fig. 70.II': KnotBear invention (Step 2).

Now, releasing L1*L2, pass L1 towards you, from the far side of the figure, into it at the point marked L1 in the above and, simultaneously (□R1*R2), pass L5 towards you at the point marked L5 [as in O.1(L)]. Now, returning to #, there is a 1n-s and a 5f-s running directly from hand to hand.

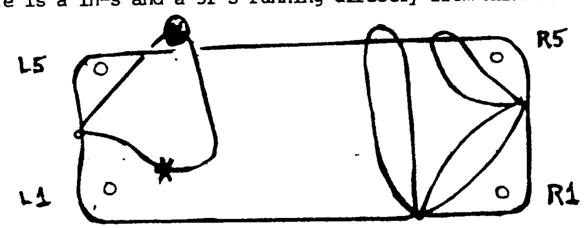
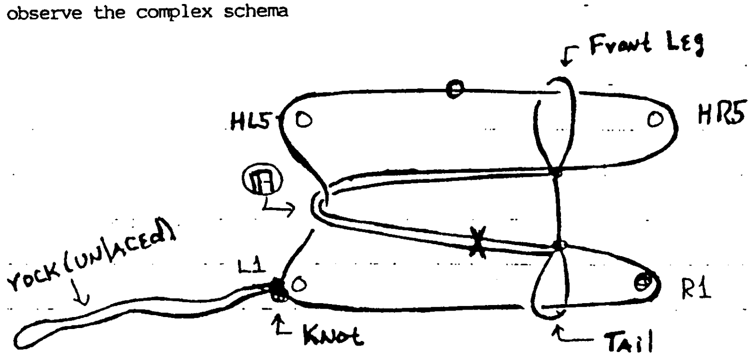


Fig. 70.III': KnotBear invention (Step 3).

Now -- as before, in KnotBear (right) -- throw a small upright loop at ✱ in Fig. 70.III', passing L over R; then pass L1∞ away from you, up through this small upright loop, and directly back to L1. Similarly, seize up the far loop depending from 5f-s, at (✱), throw a small upright loop thereon -- passing L over R -- and, turning this loop directly down to the left, thread it over each of the L5∞ & L1∞, in turn, to the near side of the figure. The result is KnotBear invention, Fig. 71.

(2). (page 181) Here we take a closer look at the complex R2-move in the construction of "Snoop" (Fig. 82). At the point in the construction where we come to "lay flat" the figure (in the "Over" perspective) we find two loops about the H5f-string; the rightmost loop -- small, tight -- is etymologically the "Dog's front leg", the leftmost loop -- large, loose -- the "rock". Gently □ HL5, and then re-Hook up the L5f-string just to the right of this latter loop. The "rock" loop may now be pulled through the central design and out to the near left side of the figure, where it becomes "added" to the L1ꝏ -- lengthening it significantly. Since the central design cannot be made to absorb all this "loose" L1ꝏ, tie off the end of the L1ꝏ (any kind of knot) so that it retains some semblance of its former size. This simplication -- getting rid of the "rock", which ultimately happens at I in the original construction -- allows us to more easily study the complex R2-move's effect on the Dog's ultimate structure. Thus, we observe the complex schema



Over

Fig. 82.B: Snoop R2-move, I.

Now, the complex R2-move of the construction, viz.

$\overleftarrow{HR2}$ ($\overline{u\text{-}o}^{(2)}$; Lp-Rp) [at X] : $\overleftarrow{R2}$↓(R1ꝏ) : $\underrightarrow{HR2}$ ($\overline{5f\text{-}s}$) [at O] # (R2) [□ former 2ꝏ$^{(2)}$] ::□ L5 : $\overleftarrow{R2ꝏ}$↓(R1ꝏ) : $\underleftarrow{R2ꝏ}$ ⇒ L5 [direct] # (H5) |

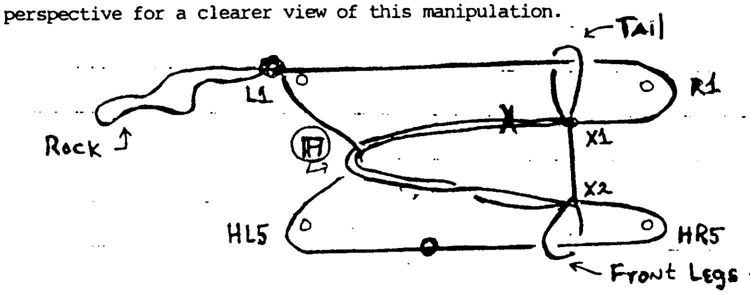
which on Fig. 82.B, above, produces the "Dog" plus the unlaced-rock/knot construct -- is seen to be equivalent to the move

Seize up the u-o$^{(2)}$;Lp-Rp at the point marked X, draw these two strings towards you (as a loop) and down through the R1ꝏ: pass them diagonally away under all strings of the figure to the base of HL5, then thread them up over the HL5ꝏ

(2). (Cont^{d}.)

[L5 must temporarily relinquish its loop to accomplish this passage] and release them as o$^{(2)}$;HL5ꝏ|

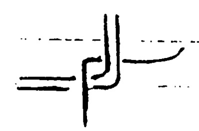
Clearly, this puts a double-string loop about the "Tail" (below the ln-s string) which continues to become a double-string loop about LH5ꝏ. The point marked (IH) in Fig. 82.B is a stationary point in this process. We now switch to the "Under" perspective for a clearer view of this manipulation.



Under

Fig. 82.C: "Under" view of Fig. 82.B.

Now, inspection of the complex-crossing x1 in Fig. 82.C, above reveals

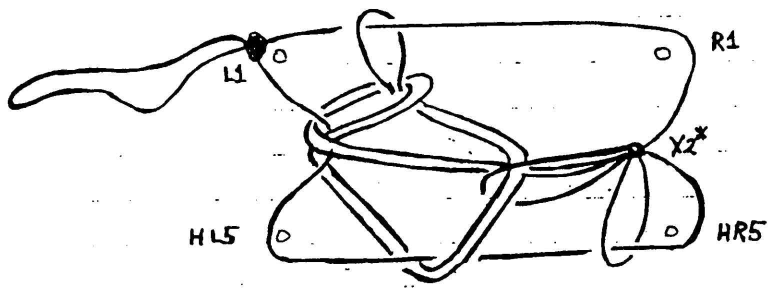


Under

Fig. 82.D: Crossing x1 of Fig. 82.C

whence the double-strings proceeding from x2 of Fig. 82.C to Lp, to x1, to ln (at "Tail"), back to x1, to Lp and, finally, returning to x2 -- comprise a single, central loop. And, from our previous analysis of the complex R2-move, after its completion this long, central loop will be pulled to the bottom of the figure (H5f-string), thence to proceed to the Lp-string, then on to loop around itself (just below ln-s), then back to the stationary point (IH), and -- finally -- to the "alibi"-transform of the crossing x2 in Fig. 82.C. That is, we should observe the resulting schema

(2). (Cont.^{d})



Under

Fig. 82.E: Dog-figure

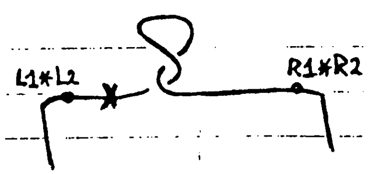
The alibi-transform of x2 has been denoted by "x2*"in Fig. 82.E, above. This is the final figure "Dog" -- after the "Rock" has been dissolved -- but he, too, like the previous, intermediary figures, will not easily accept the excess length of L1∞ arising from the "Rock's" dissolution; this accounts for the apparent distortion of the text figure, Fig. 82.

Note that as soon as we identified the long, central loop in Fig. 82.C -- essentially, by an examination of the fine-structure of complex-crossing x1 in that figure -- the alternate "moving double-string" view of the complex R2-move allowed us to "picture" the effect of this move directly from Fig. 82.C. This viewpoint is a "string-specific" analogue of the "Heart"-sequence (which is "loop-specific"), which may also be employed to great advantage in the analysis of complex manipulations inherent in specific constructions in the future.

(3). (page 203) Star KnotBear construction,

Here we supply a synopsized version of the missing details in the Knot-analysis of the Bear of Fig. 95.B.

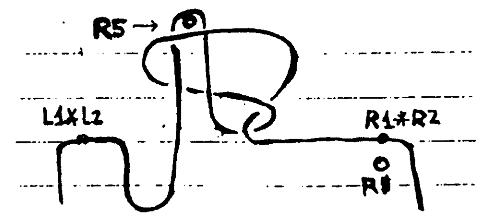
First, throw a small, upright loop in the string -- passing right over left -- and then give this loop an additional (180°-)twist. This gives



Under

Fig. 95.B.I: Star KnotBear, I.

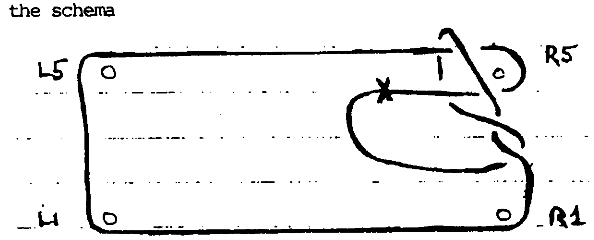
(3). (Cont.^{d})
Now, seizing the left-side string issuing from this loop at the point marked X in Fig. 95.B.I, draw this towards you and to the near side of the small loop, then push it directly away through the loop, and back up to the left side of the figure. This gives



Under

Fig. 95.B.II: Star KnotBear, II.

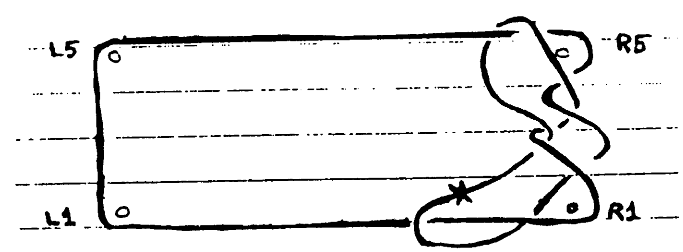
Now, from the far side of the figure, insert R1 and R5 into it, towards you -- as marked -- and pick it up [as in O.1(R)]. Pick up the long h∞ as in O.1(L), and observe the schema



Over

Fig. 95.B.III: Star KnotBear, III.

Now seize up the central loop at the point marked X, above and, turning this over towards you, thread it over the R1∞ [Note: R1 has to momentarily relinquish its loop to accomplish this passage]. This gives

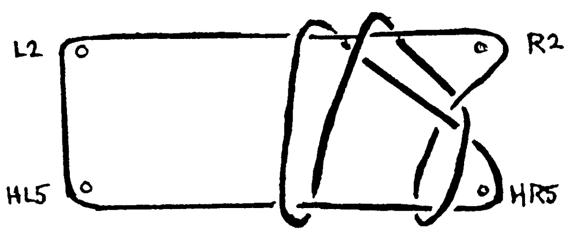


Over

Fig. 95.B.IV: Star KnotBear, IV.

③. (Contᵈ.)
Now, seize the loop about ln at the point marked X, above, and draw this under all strings to the near side of the figure, and up -- to form a small loop. Finally, turning this little loop directly to the right, and down, thread it over each of R1∞ and R5∞ -- in turn -- to the far side of the figure. This gives Star KnotBear (Right), the rightmost Bear of Fig. 95.B.

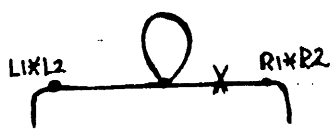
④. (page 204) The concluding figure of the series, "the Bear", has the classic gross-schema of Bear (Right), and is given below.



Under

Fig. 96.D: Brown Bear minus Pack.

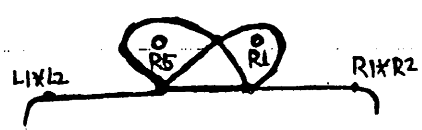
Note the extremely simple triple crossing near the right palm -- with no Osage Diamonds-type crossing in evidence. As a KnotBear, this figure combines only elements already met with in previous Knot-analyses. Let us briefly perform this analysis using only gross-schema -- as an exercise. ①. Throw a small upright loop in the string -- passing right over left -- and give this loop an additional (180°-) twist. This gives



Under

Fig. 96.E: Pack KnotBear, I.

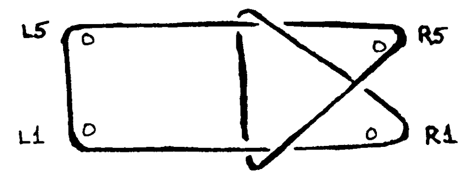
Now, ②. seizing the right-hand string at X, draw this towards you to the near side of the small loop, then push it directly away through the loop, and back up to the right side of the figure. This gives



Under

Fig. 96.F: Pack KnotBear, II.

③. (Contᵈ.)
③. Pass R1 and R5 towards you -- from far side of figure -- into the spaces marked R1 and R5, respectively, and take up the long h∞ as in O.1(L). This gives

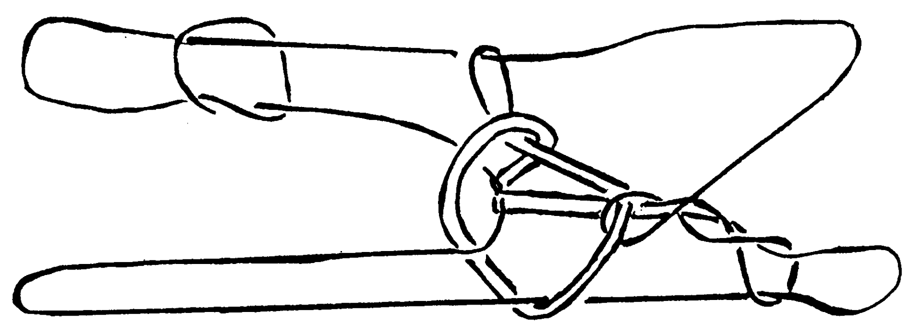


Over

Fig. 96.G: Pack KnotBear, III.

Now, ④. >>R1 and ⑤. seizing R1f (near the base of R1) throw a small, upright loop on this string, passing left over right. Now, ⑥. turning this small loop directly down to the right, thread it over the R5∞ -- to the far side of the central design [R5 will have to temporarily relinquish its loop to accomodate this passage]. The result, upon #], is the Pack KnotBear (Right); i.e. Fig. 96.D, in the Over perspective.

---- o ----



SNOOP

BIBLIOGRAPHY: BEARS

Averkieva, J.: KWAKIUTL STRING GAMES

The Bear	#3
The Bear and His Den	#6
Two Sitting on the Roof	#30
Two Bears	#52
A Bear	#75B
Small Grizzly	#78
A Bear	#79

Culin, S.: GAMES OF THE NORTH AMERICAN INDIANS

Polar Bear	Fig. 1051

Gordon, G.B.: NOTES ON THE WESTERN ESKIMO

Siberian House	Fig. 22

Gryski, C.: MANY STARS AND MORE STRING GAMES

Two Brown Bears and Their Caves	pp. 34-35
cf. Siberian House (CAT'S CRADLES, OWL'S EYES)	pp. 30-31

Haddon, K.: ARTISTS IN STRING

Fish Net Torn by Polar Bears	#4

Hansen, K.: STRING FIGURES FROM WEST GREENLAND

Nanorssuk (Polar Bear)	Fig. 24

Holtved, E.: CONTRIBUTIONS TO POLAR ESKIMO ETHNOGRAPHY

Nanorsuk	#11
Two Bears	#22
Bear and His Den	#23

Jenness, D.: ESKIMO STRING FIGURES (Journal of American Folklore)

Brown Bears	#1
Two Caves	#2
Bear and Cave	#3
Brown Bear	#4
Bear and Cubs	#6
Bear and Shaman	#7

Jenness, D.: ESKIMO STRING FIGURES (Journal of American Folklore, cont.)

Bear	#49
Bears and Caves	#61
Bearskin	#79
Children and Bear	#112
Bearskin	#137

ESKIMO STRING FIGURES (Canadian Arctic Expedition)

Brown Bear Cycle	#1-14
Two Brown Bears Issuing From Two Caves	#51
The Brant and the Black Bear	#102
Brown Bears Pack Cycle	#117-126
Polar Bear	#137
Fish Net Torn by Polar Bears	#47

Mary-Rousselière, G.: LES JEUX DE FICELLE DES ARVILIGJUARMIUT

Nanorjuk (Polar Bear)	#6
Akhlarjuk	#7
Two Brown Bears Joined at the Rectum	#8
Brown Bear Issuing From a Cave	#9
Brown Bears With Their Oreilles	#11
Brown Bears and Their Cubs	#10

Mathiassen, T.: MATERIAL CULTURE OF THE IGLULIK ESKIMOS

Bear	Fig. 180

Métraux, A: QUELQUES JEUX DE FICELLE L'AMERIQUE DU SUD

No name	Fig. 34

Paterson, T.T.: ESKIMO STRING FIGURES AND THEIR ORIGIN

Two Brown Bears	#8
Polar Bear	#16
A Bear Comes From His House	#24
The Black Bears and Their Cubs	#57
The Brown Bear Cub	#71
Fish Net Torn by Polar Bear	#106
The Spirit Bear	#108

Pearsall, M.: KLAMATH CHILDHOOD AND EDUCATION
Reference to bear p. 344

Rasmussen, K.: INTELLECTUAL CULTURE OF THE COPPER ESKIMOS
The Little Black Bears #14
The White Bear #28

Sherman, M: ILLUSTRATIONS for J. Averkieva: KWAKIUTL STRING GAMES. (personal communication).

SYMBOLS

- 𝔹 — Both hands
- 𝕃 — Left hand
- ℝ — Right hand
- M — Mouth
- W — Wrist
- T — Great toe
- # — Normal position (2)
- | — Extend hands (3)
- I — Final extension (121)
- ꝏ — Loop (5)
- x — Crossing (9)
- Ø/U — Over/Under (10)
- $\underline{+}$ — parity (11)
- $\rightarrow$, $\Rightarrow$ — Arrow(s) (15, 27)
- □ — Release (23)
- N — Navaho (24)
- |⇒, ■ — Associated Linear Sequence (6)
- :: — Grouping symbol (40)
- $⧗_n$ — Complex-crossing (46)
- α, χ — Crossing parameters (47)
- ǂ — Multiple loops need not be kept as distinct (122)
- * — pinch (123)
- GM — Gilbert Movement (131-132)
- I_G — Gilbert Extension (132)
- K — Katilluik (150)
- 1 — Thumb
- 2 — Index finger
- 3 — Middle finger
- 4 — Ring finger
- 5 — Little finger
- $\underline{O}$. — Opening (2)
- s — Straight string (4, 122)
- o — Oblique string (122, 126)
- n/f — Near/far string (3)
- d/p — Dorsal/palmar string (3)
- ℓ/u — lower/upper string (21)
- adj — Adjacent (11, 97)
- μ — Friction coefficient (12)
- $\emptyset$ — Empty functor (15)
- <,> — Twist (loops) (25)
- X — Exchange (loops) (27)
- ℙ — Pindiki (37)
- C/S — Crossing (41)
- xn $\rightarrow \emptyset$ — xn cancels (41)
- ∼ — Repeat (115)
- √R L↑ — Interchange "R" and "L" (115)
- hꝏ — hanging loop (123)
- H — Hooked (124)
- $⧗^{(g)}$ — Generalized crossing (139)

INDEX

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No. 16 (1988)

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String-Figures, II

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VI. STRING TRICKS

Perhaps the most universally known amusement with either a piece of string, or a closed loop of string, is the String Trick. Here performer seeks to amuse, amaze or confound the onlooker(s) with the "plausible impossible"; a knot that appears from nowhere, or dissolves when "blown" on; a hopeless tangle that miraculously "cuts" through the fingers, wrist, or head; a "game of chance" which onlooker invariably loses; a "simple catch" which onlooker cannot duplicate; or a "magic" loop that mysteriously travels from hand to hand. Henceforth we shall distinguish String-Figures from String Tricks by stipulating that the former must have some "representational" aspect to them, while the latter do not. Although, of course, we are aware that most -- if not all -- String-Figures have their amusing, amazing and confounding aspects, the distinguishing characteristic of "Tricks" is that it is all and only the string's interrelations to itself and to the frame which accounts for its novelty.

Although we consider String Tricks to be a separate subject from that of String-Figures, the relationship between them is, indeed, a close one. Every String-Figure expert knows -- and regularly performs -- several tricks in his traveling <u>repertoire</u>, and considers them important -- even indispensable -- thereto. And -- from personal observation in many cases -- the circumstances surrounding the learning of each trick are golden entries on the scroll of his memory; for example, the author can almost smell the mixture of old leather and Balkan Sobraine that permeated the den of the retired sea-captain on that rainy afternoon decades ago when he patiently "taught" me how (<u>not</u>) to tie the "Tom Fool's Knot" [No. I.A.2, below]. Thus, in the present section, we shall give an introduction to String Tricks as a delightful conjunct to the art of String-Figures.

For our purposes, it is convenient to classify String Tricks into six major categories:

<u>I</u>. Knots,
<u>II</u>. Plaits,
<u>III</u>. Do-As-I-Do,
<u>IV</u>. Swindles,
<u>V</u>. Releases,
<u>VI</u>. Illusions.

It should be remarked that several of the included tricks may claim legitimate membership in more than one of the above categories -- in particular, for example, anything that <u>can</u> be turned into a swindle <u>will</u> be, by someone, somewhere. Thus the above classification should be considered as but one of many such schema imposable upon the vast diversity of all "String Tricks".

In general, String Tricks tend to exhibit more bilateral specificity than do String-Figures; <u>i.e</u>. they often entail highly asymmetrical workings between the hands, with one hand being the principal agent in the construction. Of course, the symmetric construction "on the other hand" is always possible -- usually (but not invariably) obtained by the substitution √R|L↑ in the parent construction -- and the accomplished performer will be the master of both. In the discussion below, we shall present one of the "bilateral mates" for the construction of a particular String Trick, leaving it to the reader to deduce the symmetric working.

A consequence of this bilateral specificity is, often, a nonstandard (<u>i.e</u>. not #, <u>etc</u>.) positioning of the hands relative to one another during the manipulations of the strings. Thus, the descriptions encountered in this section may be expected to be more littoral than in previous sections -- 'though this is not necessary, strictly speaking. The added verbiage is intended to clarify the often unusual workings here to be encountered.

We begin with some examples of String Tricks from the category "Knots", and these may be differentiated into the subcategories "Appearing", and "Disappearin

I. KNOTS

A. Appearing. Here, performer fashions a knot in the string in a manner that excites cognitive dissonance in the observer. We have already encountered an example.

1. THE IMPOSSIBLE KNOT: See Osage Diamonds, Appendix A.2, pages 104-105. True, this example was presented for a single length of string (i.e. not a loop), as is often the case for String Tricks in this category; most may be accomplished equally well in a loop of string, doubled to approximate a single string. This one, in particular, lends itself well to this procedure.

2. THE TOM FOOL'S KNOT, I [C.W. Ashley: The Ashley Book of Knots, No. 2534.] This is tied as follows:

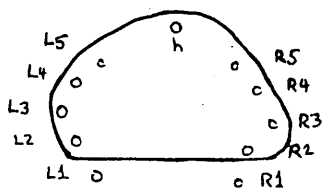


Fig. 101.A: Tom Fool's Knot, I.

I. With hands held palm up, fingers pointed away, hang a loop over the fingers of both hands (excluding the thumbs) so that a long hoo depends from their dorsal aspect.

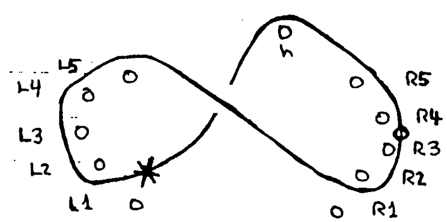


Fig. 101.B: Tom Fool's Knot, II.

II. $\overrightarrow{L1}$(L2345n)#(L1):□L2345: $\overleftarrow{L2345}$(L1n)#(L2345):□L1
This produces the string-position of the schema of Fig. 101.B -- with palms still turned up, fingers pointed away.

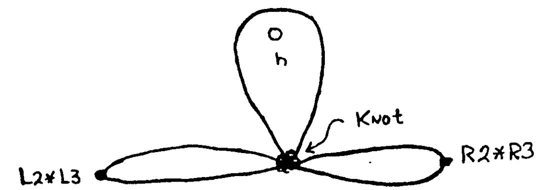


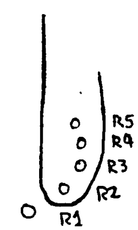
Fig. 101.C: Tom Fool's Knot, III.

III. Rotate hands (palms up) so that fingers point towards one another, and bring them together, passing ℝ over 𝕃: {L2*L3 (Rd; at O) / R2*R3 (Lp; at ✳)} # [□former L2345oo / □former R2345oo] I

This will produce the "bow-tie" knot of Fig. 101.C, with a long hoo dependent therefrom. To dissolve it, release 2 and 3 and take up the left string of the hoo (just below the knot) in 𝕃, the right string in ℝ, and separate hands widely.

With practice, this knot may be tied fluidly, "in a flash", in a manner that onlooker(s) invariably will be unable to duplicate, despite frequent repetition and performer's best efforts to "help" them. Note: Here is the classic example of a String Trick which the accomplished performer will master with equal facility on either hand, the better to be able to "aid" the unwary onlooker in his attempts to learn it.

3. THE ONE-HAND KNOT [Popular Mechanics: "Easy Tricks with String", pages 822-823.] This is a favorite "effect" of stage-Magicians, who often use a silk scarf for the working. As in No. 1, The Impossible Knot, the loop must be doubled to approximate a single string.



Over

Fig. 102.A: One-Hand Knot, I.

I. Hold ℝ with R1 up, fingers pointed away, and drape string between R1 and R2 so that 3/4 of its length depends from the palmar side of ℝ, 1/4 from the dorsal side [the end of the dorsal string(s) should be about 8" below ℝ].

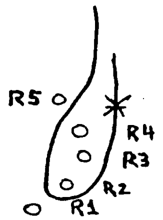
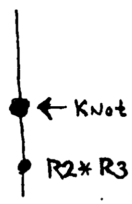


Fig. 102.B: One-Hand Knot, II.

II. $\overleftarrow{R5}$(Rp) = H$\overrightarrow{R23}$(Rp): continue passing R23 down, under R, and back to pinch Rd between them (at ✳).



Over

Fig. 102.C: One-Hand Knot, III.

III. Shake Rd off the back of R #□R5]

This produces Fig. 102.C, the One-Hand Knot. Again, if one wishes, practice will afford performer a speed and fluidity in the tying of this simple knot -- the individual manipulations blending into one "down-up" shake of R -- which render it all but impossible for the untutored onlooker(s) to follow.

4. THE STRING OF KNOTS [G. Budworth: The Knot Book, Fig. 96, "Overhand Knots Galore".]

This trick is a favorite of Sailors and Magicians, everywhere. Two characteristics distinguish it from the others of its genre here presented: (1). It requires some initial preparation, and (2). it is accompanied by a line* of "patter" which has apparently remained unchanged for centuries. These also being Hallmarks of the "close-up performer" in Magic, perhaps it is this performance discipline to which this String Trick rightfully belongs. Be that as it may, the trick is to tie a string of knots in a simple cord as rapidly as possible. With some initial "set-up", this may be accomplished with a celerity best described as "magical". Proceed as follows:

---

* "How I won the knot-tying championship of the world", et cetera.

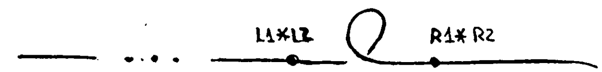


Fig. 103.A: String of Knots, I.

I. Seize up a short (8") section of a straight string between L1*L2 and R1*R2 about one foot from its rightmost terminus. II. Throw a small, upright loop therein, passing the right string towards you and to the left, (Fig. 103.A). Now, gently releasing L1*L2 for a moment, pass L1 from the near side away through this small, upright loop -- thus threading it onto L1 -- and regrasp the straight string between L1*L2.

This gives Fig. 103.B, below.

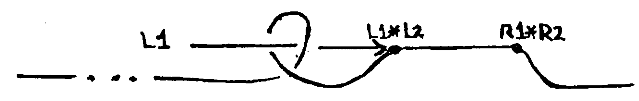


Fig. 103.B: String of Knots, II.

III. Repeat II.

This gives Fig. 103.C, below.

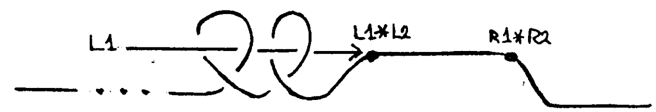


Fig. 103.C: String of Knots, III.

IV. Repeat II several more times, until the rightmost section of the string is all but exhausted -- only 2-3"" should remain.

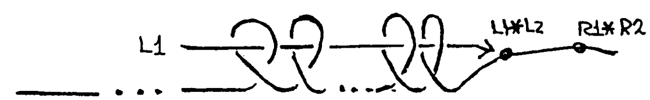


Fig. 103.D: String of Knots, IV.

V. Now, releasing L1 from all its loops, thread the rightmost end of the string down through all of these; regrasp it between R1*R2, and conceal the wonballed mass of loops in the closed L -- so that the string held by R1*R2 protrudes from the thumb-side of the left fist.

This completes the initial "set-up" for the String of Knots. When you wish to produce these, simply pull the R1*R2-string out of the left fist -- using the L-fingers to loosen any tangles that develop -- and a series of neatly-spaced knots will issue from the left fist; each will pass through L1*L2 with a satisfying "pop" as it goes.

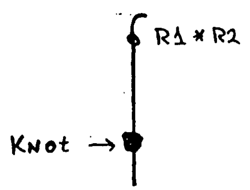
5. SHAKING A KNOT IN THE STRING [See J.C. Andersen: Maori String Figures, Trick No. 6, "Tying a Knot" or J. Ould: The Hindu Rope Book, page 9, "One-handed Knot".]

The last of the "appearing knots" we shall discuss is the most difficult to master, and requires hours of diligent practice for its successful performance. It is best learned in a single length of string (i.e. not a loop) but -- once mastered in that venue -- it may be equally well performed in a closed loop, doubled to approximate a single string.

Under

Fig. 104.A: Shaking a Knot, I.

I. Begin by grasping a straight string between R1*R2 about 10" from its upper terminus, and allow the remainder of the string to hang freely above the ground. Raise R slightly, and then drop it -- quickly -- to "poke" the string (with R1 and R2) about 10" from its nether end (at the point marked ✱, above).



Under

Fig. 104.B: Shaking a Knot, II.

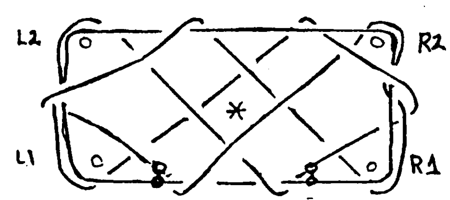
The above manipulations, done correctly, will result in the lower terminus of the string performing an upward-canted, circular arc which intercepts the traveling-wave downward on the bight to form a knot. Trust me! This is a feat of timing and coordination requiring -- on average -- 50 hours of practice to get to the point where you succeed (nearly) every time. After another 50 hours, or so, you will find that the "poke" of the string by R1*R2 during manipulation may be dispensed with; the knot will appear anyway. This is the point at which you are ready to essay this trick in your doubled String-Figure loop -- it will not be entirely easy, yet. Once mastered, however, this trick finds genuinely humorous application in the following context: Immediately upon dissolution of a complicated String-Figure, the string is given a "shake", as if to remove any remaining "kinks" therein -- performer is then "surprised" to find a knot in his string, and laboriously unties same. The effect may be repeated to the delight of onlooker(s), who believes that he knows something performer is unaware of.

We close this subsection of String Tricks (i.e. KNOTS, Appearing) with an example of a String-Figure knot, by way of comparison.

6. THE SUGAR BEET [See R.H. Compton: "String Figures From New Caledonia and the Loyalty Islands", No. 14, "The Sugar Cane".]

O.A:: 1→↑(5∞):>>1(#):□5| H345←(2̄n̄)#(H345)| H̄2←(1̄f̄$^{(2)}$):<2(#) [□former 2∞]:□H2345# |

This gives Fig. 105.A, below.



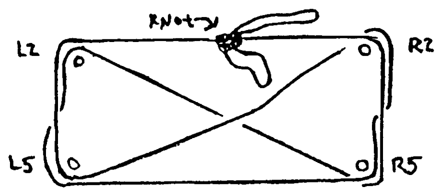
Over

Fig. 105.A: Sugar Beet, I.

Continue,

5←↑(c-◊;at✱):5←(lower strings$^{(2)}$; at 8)#□1 |:1→(2n-s)#□2 |: 2→↑(5∞$^{(2)}$)#□1 I (sharply; fingers pointed away)

This gives the Sugar Beet, illustrated below.



Under

Fig. 105.B: Sugar Beet, II.

The knot appearing centrally in the upper transverse string of this figure is a fair representation of the sugar beet for which it is named. Thus, from the present perspective, this design is properly a String-Figure, and not a trick.

We shall content ourselves with these few examples of "Appearing Knots", and now turn to the other subcategory of this genre of String Trick.

B. Disappearing. Here, obvious knots or tangles are made to dissolve in a simple, counter-intuitive manner.

1. THE TOM FOOL'S KNOT, II [~~W.R. Ransom: Pastimes with String and Paper, page 111, "The Surprise Snarl"~~.] J.Ould: Hindu Rope Book, "The Magic Shoe Laces", p. 10.]

The Tom Fool's Knot (I.A.2, page 234) exhibits both "appearing" and "disappearing" aspects, 'though the latter appears not to be so widely-known as the former. Begin with a single length of string, or with a String-Figure loop doubled to approximate one.

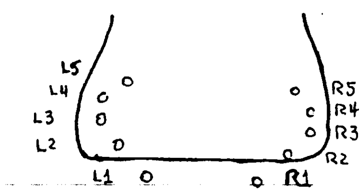


Fig. 106.A: Tom Fool's Knot, I.

I. With hands held palm up, fingers pointed away, hang the string over the fingers of both hands (excluding the thumbs) so that the free ends depend from their dorsal aspects on either side. [These free "hanging" ends should be slightly longer than the straight segment running from hand to hand, for smooth working of the trick.]

II. ∼ II and III of Tom Fool's Knot, I.
This gives Fig. 106.B, below.

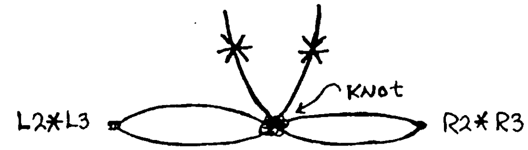


Fig. 106.B: Tom Fool's Knot, II.

Continue,

III. Pass each 1*2 away through their respective lateral loop of the "bow-tie" and seize up between them the hanging string (at ✱, above): pull these strings back up -- and entirely through -- this loop on either side: □3 [you should now be holding only the ends of the string in 1*2]

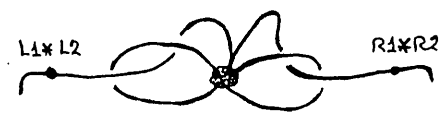


Fig. 106. C: Tom Fool's Knot, III.

This "knot" is routinely dissolved as follows: Separate and extend the hands, cautiously, until the lateral "bows" have collapsed into the central "knot" -- at which point (depending on the coefficient of friction of the string) the strings will "catch", slightly, and further movement will be impeded. Present this configuration to onlooker as a knot. Now "blow" on the central tangle and extend hands sharply; the "knot" vanishes, with a "pop"!

This simple trick is often extremely effective; most onlookers have experience with the knot canonically employed in the tying of shoelaces -- in which context the relacing of the free ends through the lateral loops of the "bow" would be, indeed, disasterous -- and readily confuse the two "bows". The surprise evinced by the ultimate "pop" of the final dissolution is usually manifestly genuine.

Let us examine this final dissolution as an example of the "extension-cancellation" analysis previously applied to such phenomena. Just prior to the final "tug" which ultimately dissolves the central tangle, we are met with a string-position whose schema is given in Fig. 106.D, below.

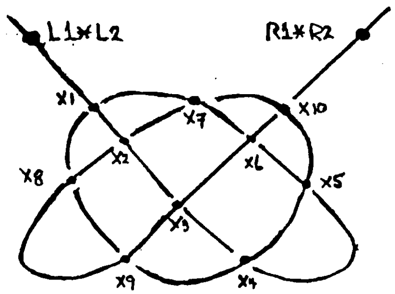


Fig. 106.D: Tom Fool's Knot, IV.

If we wish, we may associate a linear sequence to this schema in the usual way:

|⇒ L1*L2: x1(∅): x2(∅): x3(U): x4(U): x5(U): x6(U): x7(∅): x1(U): x8(U): x9(U): x4(∅): x5(∅): x10(∅): x7(U): x2(U): x8(∅): x9(∅): x3(∅): x6(∅): x10(U): R1*R2 ■

Now, the Lemma 2.B provides the sequence of cancellation pairs (of simple crossings)

1). {x4, x5} → ∅ ,
2). {x3, x6} → ∅ ,
3). {x2, x7} → ∅ ,

resulting in a string-position whose schema is given by

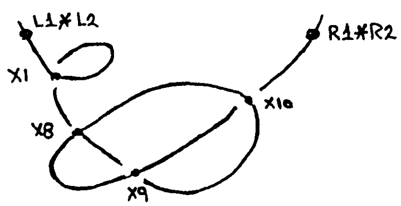


Fig. 106.E: Tom Fool's Knot, V.

And now, clearly, {x8, x9} → ∅ by Lemma 2.B, and then each of {x1} → ∅, {x10} → ∅ by Lemma 2.A; extension-cancellation is said to be complete in this case -- all central crossings have vanished. The corresponding linear sequence associated to the final string-configuration is, simply,

|⇒ L1*L2, R1*R2

exhibiting only frame-nodes.

We remark that if -- at Step III of the original construction -- the hanging strings are pulled up and through the opposite loop of the central "bow", then the resulting string-position is schematized by

Fig. 106.F: Tom Fool's Knot, VI.

This string-position admits no cancellation, i.e. it is a knot.

2. THE DISAPPEARING KNOT [F.J. Rigney: Cub Scout Magic, pages 75-76, "Another Disappearing Knot".]

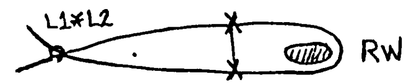


Fig. 107.A: Disappearing Knot, I.

I. Begin by grasping both ends of a straight length of string between L1*L2, the middle of the string being a simple loop about RW.

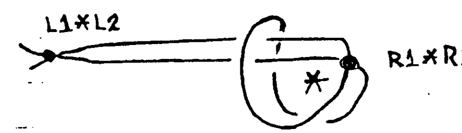


Fig. 107.B: Disappearing Knot, II.

II. Bend R down to the left and -- spreading R1, R2 widely apart -- pinch up both transverse strings (at ⨯) between them #:[]RW∞ |. [Fig. 107.B]

III. Now thread the strings held by L, from the near side, away from you through the central configuration (at ✱) and up to the far side of the figure:

Separate these strings and -- releasing all remaining strings from B -- seize them up between L1*L2 and R1*R2, respectively (order is not important, here).

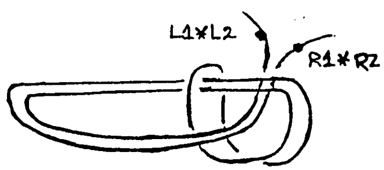


Fig. 107.C: Disappearing Knot, III.

This results in the "Disappearing Knot". It will dissolve in extension, i.e. under wide separation of the hands.

In fact, separating the strings, slightly, for clarity -- the ultimate figure just prior to the "tug" of the final dissolution is seen to have the schema

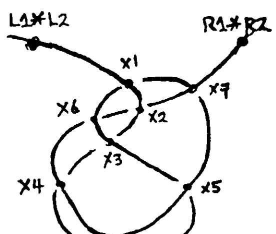


Fig. 107.D: Disappearing Knot, IV

whose associated linear sequence is given by

⊫ L1*L2: x1(∅): x2(∅): x3(U): x4(U): x5(U): x3(∅): x6(∅): x1(U): x7(U): x5(∅): x4(∅): x6(U): x2(U): x7(∅): R1*R2 ■

For variety, we demonstrate that extension-cancellation is complete in this case directly from the above associated linear sequence. First, by Lemma 2.B, we have {x4, x5} → ∅; whence the above linear sequence is extension-equivalent to

⊫ L1*L2: x1(∅): x2(∅): x3(U): x3(∅): x6(∅): x1(U): x7(U): x6(U): x2(U): x7(∅): R1*R2 ■

Here, Lemma 2.A implies {x3} → ∅; whence this sequence is extension-equivalent to

⊫ L1*L2: x1(∅): x2(∅): x6(∅): x1(U): x7(U): x6(U): x2(U): x7(∅): R1*R2 ■

At this point we recognize that {x2, x6} → ∅, by Lemma 2.B, and -- performing the indicated extension-cancellation -- we produce the equivalent sequence

⊫ L1*L2: x1(∅): x1(U): x7(U): x7(∅): R1*R2 ■

in which both {x1} → ∅, {x7} → ∅ by (separate applications of) Lemma 2.A. Again, performing the indicated cancellation results in the sequence

⊫ L1*L2: R1*R2 ■

which is thus seen to be extension-equivalent to the original; since only frame-nodes appear in this latter linear sequence, the extension-cancellation is seen to be complete.

The conclusion to be drawn from this simple, schema-independent, cancellation-analysis of the original associated linear sequence is that the string-position corresponding to this schema will completely resolve itself into the "empty" string as the hands are separated, i.e. under final extension.* We know this to be the case, directly, by performing the experiment; i.e. construction of the figure -- and now we know "why" this happens, analytically. It is instructive to mirror the sequence-analysis directly in terms of the schemata associated to each step thereof. It is, likewise, instructive to employ the sequence-analysis method to the complete dissolution of the Tom Fool's Knot, previously discussed.

3. THE ELUSIVE KNOT [W.R. Ransom: Pastimes with String and Paper, page 111, "The Surprise Snarl".]

In some sense, this is by far the most sophisticated of the String Tricks which we shall analyze in the present section. Its working is not at all difficult, but its effect is, indeed, counter-intuitive. We begin with a straight length of string, or a String-Figure loop doubled to approximate a single string.

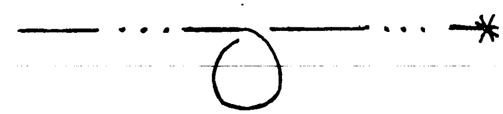


Fig. 108.A: Elusive Knot, I.

I. Begin by throwing a small, inverted loop in the rightmost third of the

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* But ...; cf. the caveat on page 12 of these notes.

string, passing left over right.

Fig. 108.B: Elusive Knot, II.

II. Now seize the right terminus of the string (at ✱) and, bringing it to the near side of the loop, thread it directly away through it, and back up to the right side of the figure.

Fig. 108.C: Elusive Knot, III.

III. Now bring both ends of the string to the top of the figure -- so as to form a "figure eight" -- and, crossing left over right, draw them out to the opposite side of the central design. [Fig. 108.C]

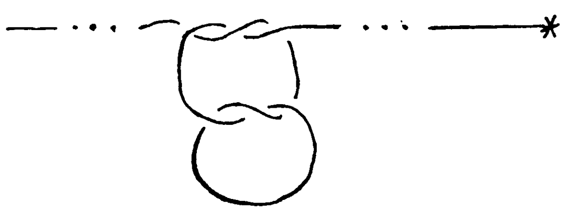

Fig. 108.D: Elusive Knot, IV.

IV. Seizing the left terminus of the string (at ✱), bring it to the near side of the upper loop of the central "figure eight", and thread it directly away through this loop, and back up to the left side of the figure.

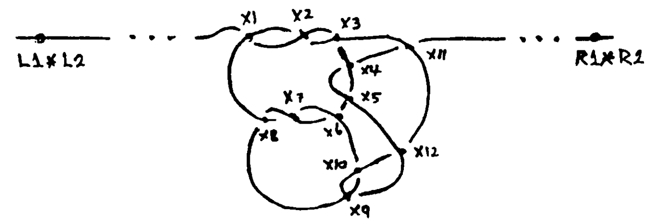

Fig. 108.E: Elusive Knot, V.

V. Seizing the right terminus of the string (at ✱), bring it to the near side of all intervening strings, and -- threading it directly away through the lower loop -- draw it back up under all strings to the right side of the figure: Again bring it to the near side of all strings and -- threading it directly away through the upper loop -- draw it back up under all strings to the rightmost side of the central design.

The result is the Elusive Knot (Fig. 108.E). As with the Disappearing Knot (Fig. 107.C) before it, it will dissolve upon seizure of either end between 1*2 and wide separation of the hands.

The linear sequence associated to the Elusive Knot -- in its presentation of Fig. 108.E -- is given by

|⇒ L1*L2: x1(U): x2(∅): x3(U): x4(∅): x5(U): x6(U): x7(∅): x8(U): x9(∅): x10(U): x6(∅): x7(U): x8(∅): x1(∅): x2(U): x3(∅): x11(∅): x12(U): x10(∅): x9(U): x12(∅): x5(∅): x4(U): x11(U): R1*R2 ■

And here -- no matter how diligently and cleverly we apply Lemma 2.A and B -- we cannot force cancellation of so much as a single crossing under "extension". And yet, all crossings do cancel in extension, as demonstrated by performing the experiment (i.e. construction of the design)! Here is the classic example of the need for a broader definition of "equivalence" between associated linear sequences* then mere "cancellation-equivalence". We may gain some insight into what this expanded definition must include by considering the following "alternate construction" of the Elusive Knot:

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* cf. page 44 of these notes.

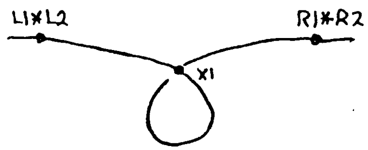


Fig. 108.F: Elusive Knot; Alternate Construction, I.

I. Seizing either end of the straight string between 1*2, throw a small, inverted loop therein, passing left over right. [Fig. 108.F].

Here, the crossing x1 -- created by the indicated string-manipulation -- is extension-cancellable by Lemma 2.A. In the following, we shall refer to any string-manipulation which produces a single crossing (in a given string-position) that is extension-cancellable by application of Lemma 2.A, as an $\emptyset_1$-operation (or "move") on that string-position; the inverse operation -- which cancels such a crossing -- will be referred to as an $\emptyset_1^{-1}$-operation. In like manner, we shall refer to any string-manipulation which produces a pair of crossings that are extension-cancellable by application of Lemma 2.B, as an $\emptyset_2$-operation on the parent string-position; the inverse operation -- canceling such a pair of crossings -- will be referred to as an $\emptyset_2^{-1}$-operation.

Definition: Two linear sequences (or schemata) are cancellation-equivalent if either may be obtained from the other by any finite sequence of the operations $\emptyset_1$, $\emptyset_2$ or their inverses.

The definition merely formalizes our previous standard practice. In the context of the present example, the initial "empty" string has associated linear sequence

|⇒ L1*L2: R1*R2 ■ ,

while the schema of Fig. 108.F has associated sequence

|⇒ L1*L2: x1(Ø): x1(U): R1*R2 ■

The two sequences are cancellation-equivalent, the second arising from an $\emptyset_1$ operation being applied to the first; or, alternatively, the first arises from an $\emptyset_1^{-1}$-operation being applied to the second. Either assertion is a justification for the stated equivalence.

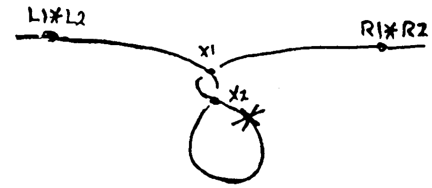


Fig. 108.G: Elusive Knot; Alternate Construction, II.

II. Put an extra (180°) twist in the small, inverted loop just created in Step I -- again passing left over right.

Here crossing x1 of Fig. 108.F goes to crossing x1 of Fig. 108.G; the crossing x2 of Fig. 108.G is created by an $\emptyset_1$-type move applied to the string-position of Fig. 108.F. The linear sequence associated to Fig. 108.G is given by

|⇒ L1*L2: x1(Ø): x2(U): x2(Ø): x1(U): R1*R2 ■

It is, of course, cancellation-equivalent to that of Fig. 108.F.

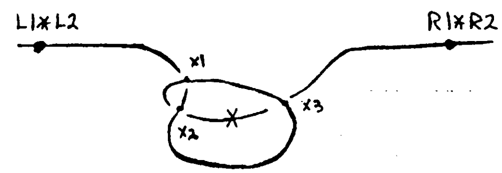


Fig. 108.H: Elusive Knot; Alternate Construction, III.

III. Seizing the top right side of the small, inverted loop (at ✶), draw this string up, over, and to the far side of crossing x1 (of Fig. 108.G).

Here, crossing x2 of Fig. 108.G slides up the leftmost string of that figure during the indicated manipulation, to become crossing x1 of Fig. 108.H, above; and crossing x1 of Fig. 108.G goes to x2 of Fig. 108.H The crossing x3 of the latter figure is created in the process. The linear sequence associated to the schema of Fig. 108.H is

|⇒ L1*L2: x1(U): x2(Ø): x3(Ø): x1(Ø): x2(U): x3(U): R1*R2 ■

As before, this linear sequence is cancellation-equivalent to its predecessor -- that associated to Fig. 108.G -- since both are easily checked to be cancellation-equivalent to that for the "empty" string, i.e.

|⇒ L1*L2: R1*R2 ■

and, hence, are cancellation-equivalent to each other. But something is missing from this method of verifying the cancellation-equivalence of these two linear sequences: Step III! True, beginning with the string-position of Fig. 108.G, we may cancel crossings by the moves $\emptyset_1^{-1}$ and $\emptyset_2^{-1}$ until we get the empty string -- and then, subsequently, build up the string-position of Fig. 108.H (in an entirely different manner) by employing moves $\emptyset_1$ and $\emptyset_2$ in the correct order; but these two string-positions are related, directly, by the much simpler Step III of the construction. This observation leads to the suspicion that there are closely-related string-positions which admit complete cancellation, whose associated linear sequences broach no extension-cancellation. Indeed -- employing Steps I-III above at one point of a straight length of string, and the mirror-images (√R\L↗) of these Steps to a nearby point -- we produce a string-position whose schema is given by

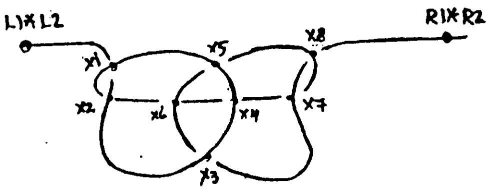


Fig. 108.H': Example; non-extension-cancellation.

Here, the associated linear sequence is given by

|⇒ L1*L2: x1(U): x2($\emptyset$): x3($\emptyset$): x4($\emptyset$): x5($\emptyset$): x1($\emptyset$): x2(U): x6(U): x4(U): x7(U): x8($\emptyset$): x5(U): x6($\emptyset$): x3(U): x7($\emptyset$): x8(U): R1*R2 ■

And, by inspection, absolutely no extension-cancellation is possible; of course, when the hands are widely separated, the figure -- being merely two "copies" of Fig. 108.H -- completely dissolves. Note that a move analogous to Step III, above, i.e. draw s;x6-x4 upward, under, and to the far side of the crossing x5 -- would produce the schema

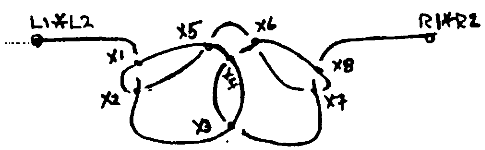


Fig. 108.H": Example; Continued.

The associated linear sequence,

|⇒ L1*L2: x1(U): x2($\emptyset$): x3($\emptyset$): x4($\emptyset$): x5($\emptyset$): x1($\emptyset$): x2(U): x5(U): x6(U): x7(U): x8($\emptyset$): x6($\emptyset$): x4(U): x3(U): x7($\emptyset$): x8(U): R1*R2 ■

admits complete extension-cancellation.

It is simply an unfortunate fact that the above analogue of Step III cannot be realized as any finite sequence of $\emptyset_1$, $\emptyset_2$-type moves or their inverses -- whence the string-position of Fig. 108.H" is not cancellation-equivalent to that of Fig. 108.H'. But, of course, the two string-positions involved are two presentations of the "same" figure, in that only local movement of a single frame-independent arc differentiates the two. That is, any "reasonable" definition of "equivalent string-positions" must include Fig's. 108.H' and H" in the same equivalence class. Thus it is time to broaden our definition of "cancellation-equivalence" to a more general type of "equivalence" between string-positions. Simply stated, at present there is no $\emptyset$-justification for drawing a string ℓ across a crossing x -- both of whose constitutent strings the string ℓ lies above (or below). Henceforth we shall explicitly allow such an operation, referring to any such move as an "$\emptyset_3$-type" move on the relevant string-position.*

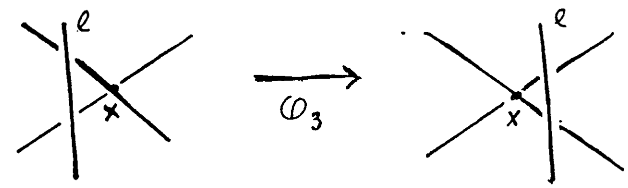


Fig. 108.H"': The $\emptyset_3$ -operation.

Definition: Two linear sequences (or schemata) are $\emptyset$-equivalent if either may be obtained from the other by any finite sequence of the operations $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ or their inverses.

In terms of moves of $\emptyset_3$-type, Step III, above, may be simply rendered

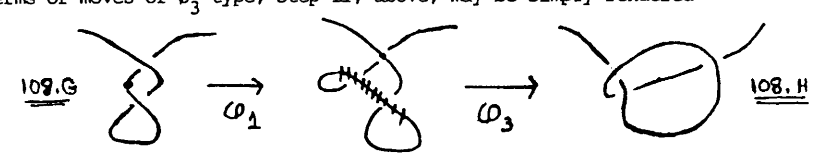
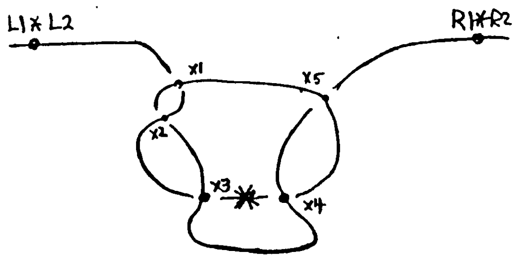


Fig. 108.H$^{(iv)}$: Step III

* Reversing the "arrow" in Fig. 108.H"' gives the $\emptyset_3^{-1}$-move; we shall not draw this distinction, in these notes.

This is simple, direct; <u>and</u> it accounts for the "extra" crossing in Fig. 108.H. The cancellation-equivalence of Fig's. 108.G and H is a curiosity, extraneous to the current construction.



<u>Fig. 108.I</u>: Elusive Knot; Alternate Construction, IV.

<u>IV</u>. Seizing the upper s;x2-x3 of Fig. 108.H (at ✱), draw this down and over (to the near side of) the bottom string of the design. [Slack for this move may be obtained from the R1*R2 string].

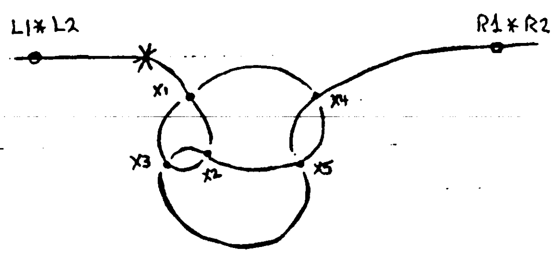
This is an $\emptyset_2$-type move giving rise to the crossing-analysis

<u>Fig. 108.H</u>		<u>Fig. 108.I</u>
x1	→	x1
x2	→	x2
x3	→	x5.

The crossings x3, x4 of Fig. 108.I are created by the $\emptyset_2$-manipulation on Fig. 108.H. The linear sequence associated to Fig. 108.I is

|⇒ L1*L2: x1(U): x2(∅): x3(U): x4(U): x5(∅): x1(∅): x2(U): x3(∅): x4(∅): x5(U): R1*R2 ■

It is cancellation-equivalent to that of Fig. 108.H by Lemma 2.B.



<u>Fig. 108.J</u>: Elusive Knot; Alternate Construction, V.

<u>V</u>. Seizing the upper s;x3-x4 (at ✱), draw this string up, under all intervening strings, to the top of the figure.

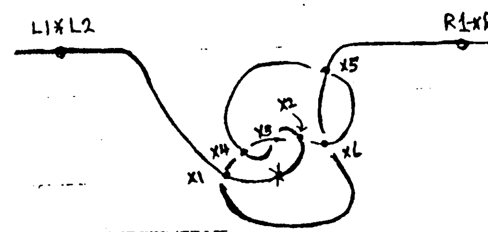
This is accomplished <u>via</u> three $\emptyset_3$-type moves (<u>cf</u>. Fig. 108.H$^{(iv)}$), one for each of the crossings x1, x2, x5 of Fig. 108.I past which the given string must travel. The crossing-analysis of this transition is given by

<u>Fig. 108.I</u>		<u>Fig. 108.J</u>
x1	→	x2
x2	→	x3
x3	→	x1
x4	→	x4
x5	→	x5

and the linear sequence associated to Fig. 108.J is, explicitly

|⇒ L1*L2: x1(∅): x2(U): x3(∅): x1(U): x4(U): x5(∅): x2(∅): x3(U): x5(U): x4(∅): R1*R2 ■

No extension-cancellation is possible here; in particular, this linear sequence is <u>not</u> cancellation-equivalent to the empty string.



<u>Fig. 108.K</u>: Elusive Knot; Alternate Construction, VI.

<u>VI</u>. Seizing the s;L1*L2-x1 near the crossing x1, draw this string down, and to the near side of crossings x3, x2 (<u>i.e</u>. over them) to the center of the design.

This requires two $\emptyset_3$-moves; for the crossing-analysis we have

<u>Fig. 108.J</u>		<u>Fig. 108.K</u>
x1	→	x1
x2	→	x3
x3	→	x4
x4	→	x5
x5	→	x6,

the crossing x2 of Fig. 108.K being created by the indicated manipulation.

The linear sequence associated to Fig. 108.K is given by

⊨> L1*L2: x1(Ø): x2(Ø): x3(U): x4(Ø): x5(U): x6(Ø): x2(U): x3(Ø): x4(U): x1(U): x6(U): x5(Ø): R1*R2 ■

in which no extension-cancellation is possible. The sequence is not cancellation-equivalent to that of Fig. 108.J.

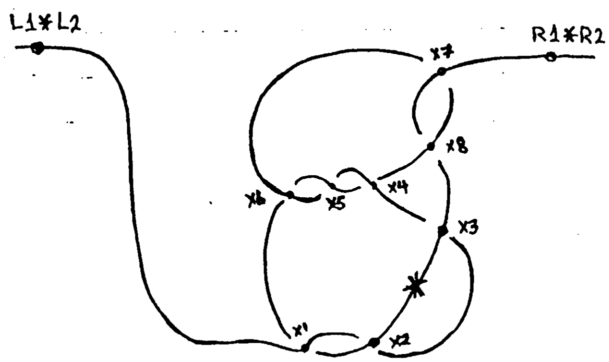


Fig. 108.L: Elusive Knot; Alternate Construction, VII.

VII. Now, seizing the s;x1-x2 near the center of the design (at ✳), draw this string to the right and down through the lowest loop of the figure, so that it extends slightly to the (lower-) right of this loop. [Slack for this move may be obtained from the L1*L2-string.]

This $\emptyset_2$-move gives rise to the crossing-analysis

Fig. 108.K		Fig. 108.L	Fig. 108.K		Fig. 108.L
x1	→	x1	x4	→	x6
x2	→	x4	x5	→	x7
x3	→	x5	x6	→	x8,

the crossing x2, x3 of Fig. 108.L being created by the indicated $\emptyset_2$-move. The linear sequence associated to Fig. 108.L is given by

⊨> L1*L2: x1(Ø): x2(U): x3(U): x4(Ø): x5(U): x6(Ø): x7(U): x8(Ø): x4(U): x5(Ø): x6(U): x1(U): x2(Ø): x3(Ø): x8(U): x7(Ø): R1*R2 ■

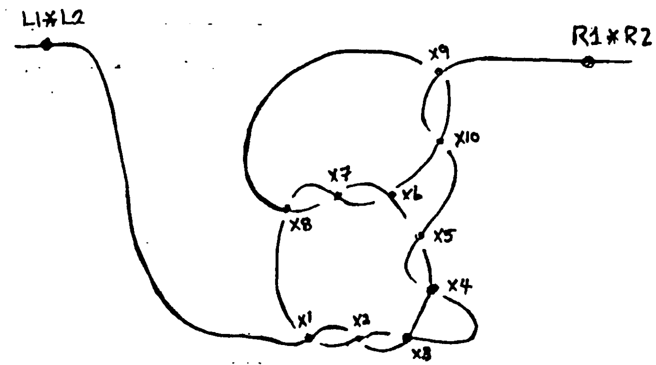


Fig. 108.M: Elusive Knot; Alternate Construction, VIII.

VIII. Seize up the upper s;x2-x3 near its center (at ✳), and draw this string down and under the lower rightmost string of the design, so that it extencds slightly to the right of this string. [Slack for this move may be obtained from the R1*R2-string.]

This $\emptyset_2$-move has attendant crossing-analysis given by

Fig. 108.L		Fig. 108.M	Fig. 108.L		Fig. 108.M
x1	→	x1	x5	→	x7
x2	→	x2	x6	→	x8
x3	→	x5	x7	→	x9
x4	→	x6	x8	→	x10,

the crossings x3, x4 of Fig. 108.M being created by the indicated $\emptyset_2$-manipulation. The linear sequence associated to Fig. 108.M is given by

⊨> L1*L2: x1(Ø): x2(U): x3(Ø): x4(Ø): x5(U): x6(Ø): x7(U): x8(Ø): x9(U): x10(Ø): x6(U): x7(Ø): x8(U): x1(U): x2(Ø): x3(U): x4(U): x5(Ø): x10(U): x9(Ø): R1*R2 ■

We are now ready to produce the Elusive Knot. The final manipulation has the effect of passing the upper and lower loops of Fig. 108.M -- together with their attendant strings -- through one another, so that they exchange places; the lower loop passes to the <u>outside</u> of the upper loop during this transition.

IX. Using the line determined by crossings x6, x7, x8 of Fig. 108.M as a pivot, fold the lower loop of the figure up and to the outside of the entire upper loop. Then fold this upper loop -- with its attendant strings -- up, through this loop and down towards you, to become the (new) lowest loop of the figure.

Note: Fig. 108.M is, essentially, a "figure eight" with a single right string in lateral involvement; Step IX, applied to this string-position, produces the Elusive Knot, Fig. 108.E -- after minimal string-rearrangement -- again, essentially, a "figure eight" with (distinct) right lateral involvement. For the crossing-analysis of this complicated transition, we find

Fig. 108.M		Fig. 108.N	Fig. 108.M		Fig. 108.N
x1	→	x1	x6	→	x6
x2	→	x2	x7	→	x7
x3	→	x3	x8	→	x8
x4	→	x4	x9	→	x9
x5	→	x5	x10	→	x10,

the crossings x11, x12 being created by the indicated manipulation. The linear sequence associated to the Elusive Knot was given earlier (see page 247). It is ∅-equivalent -- in the new sense -- to each of the linear sequences encountered in the construction -- since Step IX, above, is entirely decomposable into $\emptyset_1$, $\emptyset_2$, $\emptyset_3$-type moves; each string may be moved, individually, across each intermediate crossing. The linear sequence under discussion, however, is cancellation-equivalent to none of these earlier sequences. That is, depending upon the coefficient of friction of the string being worked, the Elusive Knot will dissolve "down the chain" of associated linear sequences as the hands are widely separated; in but six of the nine steps involved in this dissolution will Lemma 2 be the entire justification.

We remark that the Alternate Construction of the Elusive Knot may be performed in a loop of string -- mirabile visu -- whence its ultimate dissolution is assured by appeal to the Theorem that a plain loop of string is not homeomorphic to a loop containing an intrinsic knot (See Osage Diamonds, Appendix A.2). The reader is encouraged to construct the Elusive Knot by this method in his own, personal String-Figure loop -- not doubled to approximate a single string. The Alternate Construction -- 'tho lengthy to detail -- is really very easy to learn, and may be accomplished with great speed (surely the equal of the original construction). The effect is visually stunning, even after the foregoing analysis is completely mastered. [For further discussion of ∅-equivalence and the dissolution of knots, see Appendix B, pages 345-351 ; herein will be found, in particular, a continuance of the discussion of the material briefly introduced in Appendix A.2 (pages 102-105). This material is presented with the same caveat that accompanied Appendix A (c.f. page 72).]

We close our investigation of the Elusive Knot with a discussion of how an $\emptyset_3$-type move, applied to a given schema, affects the linear sequence associated to that schema.* We remark that this analysis is fundamental to the ultimate objective of achieving schema-independence in our emerging String-Figure theory. For definiteness, we shall choose Step III (cf. Fig. 108.$H^{(iv)}$, page 251) of the Alternate Construction of the Elusive Knot as an example of the general situation.

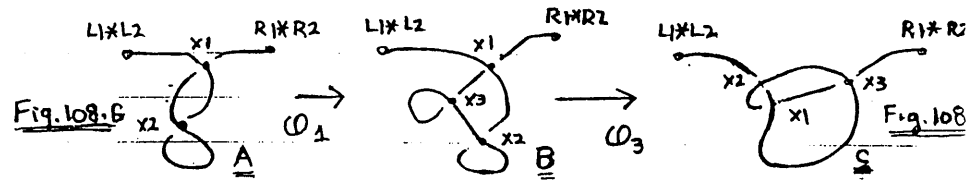


Fig. 108.N: Step III, $\emptyset_3$-analysis.

In Fig. 108.N, above, may be found the alibi schematization of the passage from Fig. 108.G to Fig. 108.H via Step III of the Alternate Construction of the Elusive Knot. The three associated linear sequences are, respectively

A. |⇒ L1*L2: x1(∅): x2(U): x2(∅): x1(U): R1*R2 ■

B. |⇒ L1*L2: x1(∅): x2(U): x2(∅): x3(∅): x3(U): x1(U): R1*R2 ■

C. |⇒ L1*L2: x2(U): x1(∅): x3(∅): x2(∅): x1(U): x3(U): R1*R2 ■

And, the passage from the linear sequence in A to that in B (i.e. by $\emptyset_1$) being well-understood, we concentrate on the similar transition from B to C. Note first that, in order to be a candidate for an $\emptyset_3$-type move, the crossings involved -- x1, x2, x3 -- must be mutually adjacent in pairs, in the schema under consideration and, therefore, in its associated linear sequence. Further, applying $\emptyset_3$ to B, above -- i.e. drawing the s;x2-x3 string up and across the crossing x1 -- results in a string-position whose schema still lists the crossings x1, x2, x3 as mutually adjacent in pairs, although their relative positions have, of course, changed. Specifically, crossing x2 has migrated up the s;x1-x2 to the far side of x1 (on this selfsame string) and, likewise, crossing x3 has moved up s;x1-x3 to the far side of x1 on its constitutent string. We may generalize this observation to the following formal assertion: If the $\emptyset_3$-type move "Draw the s;xi-xj string across the crossing xk" is admissible for a given string-position,

* See pages 44-45 of these notes for an analogous discussion of $\emptyset_1$, $\emptyset_2$-type moves

then its associated linear sequence must include the triple of distinct pairwise adjacencies

xi adj xj, xi adj xk, xj adj xk

in some order. Further, the $\emptyset_3$-image of this string-position has an associated linear sequence with the same triple of pairwise adjacencies, the order within each pair being reversed. Thus, for example, we have x1 adj x2 in both B and C = $\emptyset_3$(B) of Fig. 108.N, but in B we find

B. ⊨ ... x1(∅): x2(U) ... ■,

whereas in C we find the order reversed within this adjacent pair, *viz.*

C. ⊨ ... x2(U): x1(∅) ... ■

Second, we remark that the above "necessary" condition for the admissibility of the indicated $\emptyset_3$-type move is rendered both necessary and sufficent by the additional requirement that -- within the mandated adjacency "xi adj xj" in the associated linear sequence -- we have the "parity"-agreement $\pm$(xi) = $\pm$(xj). This insures, for example, that of the eight possible associated sub-schema

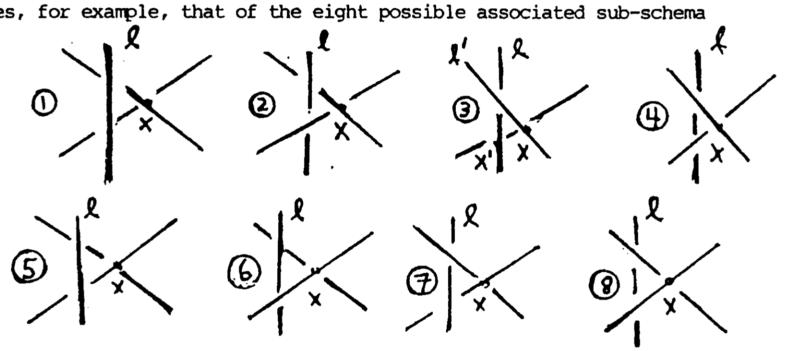


Fig. 108.∅: $\emptyset_3$-analysis, continued

we are dealing with only (1), (4), (5), (8) -- where $\pm$(xi) = $\pm$(xj) = ∅, U, ∅, U, respectively -- in each of which the indicated $\emptyset_3$-move is possible. [We remark that ℓ will not "pass" x in cases (2), (7), above. In case (3) (and (6)) for example, ℓ will -- technically -- pass between the constituent strings of x to its far side; we prefer to view this passage as one of ℓ' across x' -- which *is* a classic $\emptyset_3$-move, equivalent thereto. Thus we may preserve the "tight" definition for the class of $\emptyset_3$-type moves with no loss of generality].

From the above several remarks/observations, we are able to characterize those string-positions which admit $\emptyset_3$-type moves in terms of their associated

linear sequences:

The $\emptyset_3$-move "Draw s;xi-xj across xk" is admissible for a given string-position if and only if its associated linear sequence has the properties

i). xi adj xk, xj adj xk

ii). xi adj xj and, in that adjacency, $\pm$(xi) = $\pm$(xj).

In this case, the linear sequence associated to the $\emptyset_3$-image of the given string-position is precisely the original linear sequence with the order in each of the above pairwise adjacencies reversed.

It remains to check that each of the (six) individual crossing-parities for xi, xj, xk is preserved under the indicated $\emptyset_3$-move. And this is immediate from an inspection of (1), (4), (5), (8) of Fig. 108.∅.

We close the present subsection (1) with two examples of disappearing knots which arise from String-Figures -- as opposed to "String Tricks" -- by way of comparison with the foregoing tricks.

4. LITTLE BIRD [A.C. Haddon: "String Figures From South Africa", No. 3, "Inyoni"]

In this figure we are presented with a representation of a little bird making a getaway through the underbrush.

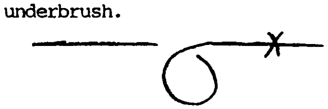


Fig. 109.A: Little Bird, I.

I. Throw a small, inverted loop in the string, passing right side towards you and to the left.

Fig. 109.B: Little Bird, II.

II. Seizing the rightmost string emanating from this loop (at ✳), pass it to the far side of -- and then back up through (towards you) -- this loop, until it protrudes as a second small loop (loop 2) to the near side of the figure. Then fold it down to the right -- *i.e.* flat -- over the rightmost string of the figure [Figure. 109.B].

Fig. 109.C: Little Bird, III.

III. Reaching down into the last-formed loop, seize up the rightmost string of the figure (at ✱), draw this through to become a new small loop to the near side of the figure. Then fold it directly down to the right -- i.e. flat -- over the rightmost string of the figure, [Fig. 109.C].

Fig. 109.D: Little Bird, IV.

IV. (~Step III)^n, where n is a function of the length of the string [Fig. 109.D, n=4].

To observe the motion of the Little Bird running away, seize up the rightmost string of the figure (to the right of the rightmost loop, at ✱), and pull out. All loops will dissolve in turn (by Lemma 2) in a "puddling" motion to the left.

Notes on the figure: (1). The left- and rightmost strings of the figure ultimately meet, as the construction is being effected in a loop of string ('though it may equally well be made in a single straight length of string). This closure is not indicated in the accompanying diagrams. (2). The figure is also initiated on the toe, or in the mouth, in different cultures. A contemporary, fully-looped figure (i.e. one which employed all the straight string possible) was observed being worn as an attractive headband, recently, by the author. (3). The representative aspect of this String-Figure is of a distinctive motion -- rather than the more typical person, place, or thing.

5. A LOCUST [W.A. Cunnington: "String Figures and Tricks From Central Africa", No. 19, "Nzige".]

Here an insect (knot) is constructed in the string between the hands. The hands are clapped sharply about him, but to no avail; he always escapes.

O.1:□L5:R15∞→R2 | ::HL2345(L1∞):<L2345(#):: HR1(R2∞):>R1(#):
R2345(R1f$^{(2)}$) # |: R1(Lp$^{(2)}$) :: L2↓(R1∞$^{(2)}$): L2(L1p$^{(2)}$)# |:□ IBd$^{(2)}$ |:
HL2345(L2n$^{(2)}$): HR2345(R1f$^{(2)}$)#(H2345) |(widely):□2345#□L2:
□R1] (very gently)

This gives the Locust; illustrated below.

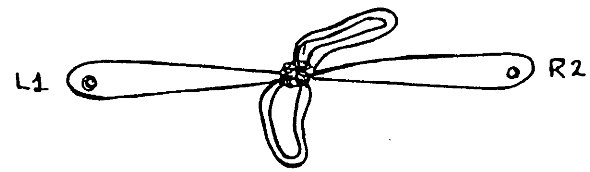


Fig. 110: The Locust.

To "catch" the locust, widen hands slightly apart to give the knot some upward momentum, then sharply clap hands about the central "bug". Quickly extend hands to their utmost extent in the string, and the knot will dissolve -- representing the escape of the locust from certain death.

Notes on the construction: (1). This figure -- with minor variations -- is known worldwide. The presentation given here is representative. (2). After both hands relinquish their double dorsal strings --□ IBd$^{(2)}$ | -- the 2345-fingers of either hand momentarily hook over the major constituent strings of the central knot, thereby "cinching" it up preparatory to the (very gentle!) final extension.

This concludes our look at I. KNOTS as String Tricks. The next major subcategory of String Tricks to be considered is that of II. PLAITS; these are string-designs laid out on a flat surface to effect an open "trellis-work" type of configuration which is often aesthetically pleasing in its own right. The two distinct -- but related -- examples we present below are also characterizable as "releases", since, in each, a finger is inserted in one of the finite, enclosed subdivisions of the plane induced by the plait -- and then "released" by pulling on the appropriate string (arc) thereof. They may equally be characterized as "swindles", as they combine to form the basis of a virtually unbeatable con-game effectively employed by the street-hustler (who often manipulates a short chain, garter, or flexible wire instead of a loop of string). The present view, in these notes, is that -- just as Knots lie just outside of one end of the String-Figure spectrum (because of the absence of a representational aspect) -- so, too, Plaits lie just outside the other end of this spectrum (because they are not made "on the hands"). Thus we shall henceforth distinguish String-Figures from Plaits by stipulating that the former are made on the hands -- i.e. that their associated linear sequences involve frame-nodes in some fundamental way -- while the latter, by definition, involve frame-nodes (if at all) only peripherally -- during the laying-out of the design.* The string-loops involved in most Plaits are invariably shorter -- on the order of 1 to 1.5 meters in total length -- than those employed in the construction of String-Figures. Many performers, therefore, double their String-Figure string (to produce a "double-loop" of 1/2 the length of the original loop) when demonstrating a particular Plait, and find that this works quite well. With these few words of introduction, then we shall move on to the category of Plaits.

*See, for instance, J. Elffers and M. Schuyt: Cat's Cradles and Other String Figures, pp. 140-147, and F.D. McCarthy: "The String Figures of Yirrkalla", No's. 72, 94, 110-116, 128, 130-146, 149, where many examples of plaits may be found.

II. PLAITS

The two examples of Plaits that we present in this section are representative of their genre in that they involve the laying-out of the loop of string on a flat surface, viewed as having been "discretized" in some definite way. The first involves the "Compass discretization" into the cardinal directions, viz

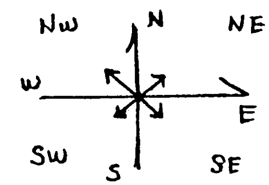


Fig. 111: The Compass Discretization.

1. ANSA [See P. Jattilainen: Antero Vipunen, page 375].

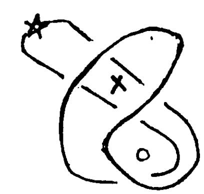


Fig. 112.A: Ansa.

Construction :

I. Holding one string of the loop between R1*R2 (R2 uppermost) draw the loop diagonally from NW to SE on the flat surface: continue drawing the loop horizontally from SE to SW: then turn it up and fold it back over itself from SW to NE: □ R1*R2. [Fig. 112.A]

II. Onlooker now places his finger into the design (i.e. onto the flat surface) at X (or, at O). Performer then seizes the string at ☆, and pulls it off to the NW; onlooker's finger comes free of all strings.

This release may be completely justified by appeal to the Lemma 2 and, while the whole appears to be entirely trivial -- hardly "a trick" at all -- many untutored onlookers will be completely mystified by it at first presentation. This confusion may be aggravated by occasionally giving the loop an extra 180°-twist when turning it up and folding it directly back over itself during its passage from SW to NE, at the end of Step 1. The resulting configuration has the schema

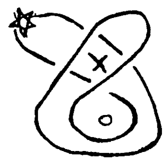


Fig. 112.B: Ansa (variation).

Now onlooker's finger, placed at X , is caught as the string is pulled away at ✡; the finger, placed at O , is released as before. We remark that the "extra" twist of the second construction -- done smoothly -- becomes apparent only after frequent repetition, the two motions involved being masked by the gross (translational) movements of the right hand.

2. SNARE(2) [See C.W. Ashley: The Ashley Book of Knots, No. 2594].

This plait involves the visualization of the following lattice-configuration being superimposed upon the constructional surface:

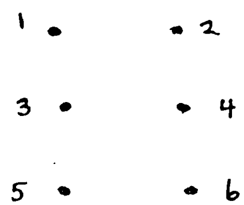


Fig. 113: Snare (2) scaffold.

Construction:

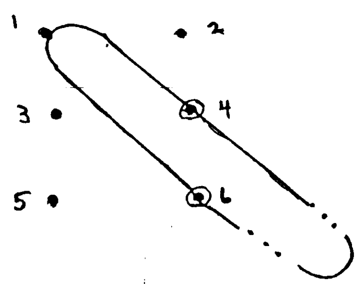


Fig. 114.A: Snare (2), I.

I. Lay loop from NW to SE on (imaginary) scaffold-configuration, as pictured in Fig. 114.A, above. The string should be incident with the lattice-points 1, 4, 6 of this scaffold, as depicted.

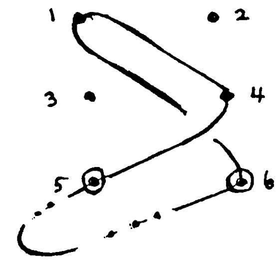


Fig. 114.B: Snare (2), II.

II. Using scaffold-points 4,6 as pivots, fold the lower (SE) portion of the loop over, and directly down, to SW; so that the upper string of this loop passes through scaffold-point 5, the lower string being parallel thereto.

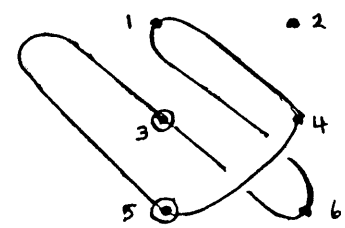


Fig. 114.C: Snare (2), III.

III. Now using scaffold-points 5, 6 as pivots, fold the lower (SW) portion of the loop over, and directly up to NW; so that the upper string passes through scaffold-point 3, the lower string being parallel thereto.

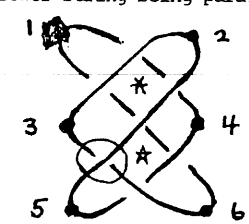


Fig. 114.D: Snare (2), IV.

IV. Finally, using scaffold-points 3,5 as pivots, fold the upper (NW) portion of the loop over, and directly across to NE, so that its terminus comes to rest on the scaffold-point 2 [Fig. 114.D].

Onlooker now places his finger into the design at ✱; performer seizes the string at scaffold-point 1 (i.e. at ● , above) and pulls it off to the NW. Onlooker's finger will be caught.

If, now, the above construction is carried out, with the word "over" in Step IV replaced by the word "under", we obtain the variant figure

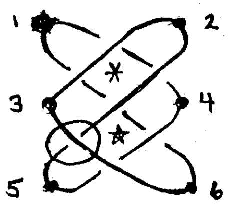


Fig. 114.E: Snare (2), variation

whose schema differs from that of Fig. 114.D only as to the parity of the circled crossing. When onlooker places his finger into this design at ✱, and performer pulls the string at scaffold-point 1 (i.e. at ● , above) off to NW, this time onlooker's finger will be released. As with the previous figure, Ansa, the two constructions -- when mastered through practice -- will appear to all but the most sophisticated observers* as identical; as, indeed, will the final designs.

We close this section on plaits by remarking that both Snare(2)-designs admit an "on hands" construction. For example, the string-position of Fig. 114.D has the contemporary alternate construction

O.A:□5|: L2∞→L5: L1∞ →L2#|: L1(L2f)#R5(L1f)# >>L2]:Lay flat, fingers pointing up.

This gives the design of Fig. 114.D. Et cetera.

This concludes our brief foray into the subject of II. PLAITS. The next major subcategory of String Tricks to be considered in these notes is III. DO-AS-I-DO; that is, challenges. Here, performer executes a quick, usually simple maneuver in the string, which he then challenges his onlooker to duplicate. This category often borders closely on the subsequent one (i.e. IV. SWINDLES), as the given designs frequently involve some subtlety which performer obscures during his performance thereof.

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* as well as to many "sophisticated" observers!

III. DO-AS-I-DO

Herein, performer executes a "simple" maneuver in his String-Figure loop, which he then challenges onlooker to duplicate.

1. THE IMPOSSIBLE KNOT: see Osage Diamonds, Appendix A.2, pages 104-105. This is a genuinely effective "challenge"-figure, owing to the virtual undetectability of the "release and regrasp" movement of R during the final manipulation (Step IV, page 105) of the construction. An extra dimension may be added, here, by the following amusing piece of business.

After onlooker's anxiety-level has risen sufficiently due to the frustration of repeated failures, performer slowly -- and very patiently -- performs Steps I-III of the construction in as fair and open a manner as is possible. He then offer the ends of the string -- secured between his R1*R2 and L1*L2, respectively to onlooker to take up.* Performer then relinquishes the string ends to onlooker and slips the remaining wrist loops. When onlooker extends his hands, the Impossible Knot -- which has so far eluded his best efforts -- visibly "ties itself" in the center of his strings. ②

2. THE LIZARD [P. Beaglehole: String Figures from Pukapuka, page 27, "Handslip trick (Nameless)".]

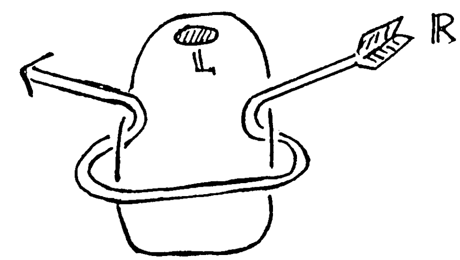

Fig. 115: The Lizard.

I. Hold the loop of string in the L fist, with fingers (knuckles) pointed away from you, so that you are directly facing a wide, dependent loop.

II. Pass R away from you into this Lhoo, then down, around the rightmost string, and back to the near side thereof. Continue passing R to the left --

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* Often words to the effect, "Here, you take it off my hands" are effective at this point.

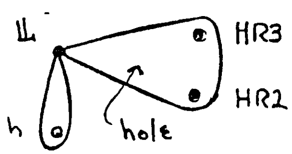
just past the leftmost string of the Lhꝏ (still to the near side thereof) -- then pass ℝ away around, and to the far side of the Lhꝏ. Finally, turning ℝ up, bring it directly back through the Lhꝏ.

The complete ℝ-motion is depicted in Fig. 115, above. As ℝ is drawn towards you and left -- away from 𝕃 -- all strings incident with ℝ will come free.

A minimal variation has ℝ turning palm away -- at the conclusion of Step Ⅱ, before ℝ is separated widely from 𝕃 -- and encircling both strings of Lhꝏ (near L) within the "ring" formed by bringing the tips of R1 and R3 together around these strings. The whole then comes free of ℝ as ℝ "milks" the two strings of Lhꝏ.

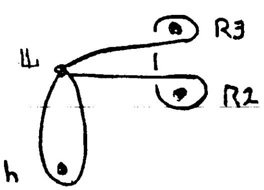
<u>3</u>. RABBIT IN THE HOLE [R.M. Abraham: <u>Easy To Do Entertainments and Diversions</u>, No. 129, "Reversing the loop".]

This time performer displays a small loop --representative of a rabbit hole -- in a larger loop of string. Reaching in with two fingers, he catches the "ear" of a rabbit who was hiding therein.



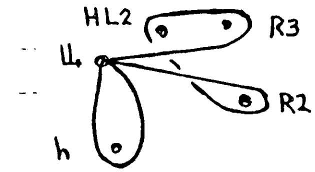
<u>Fig. 116.A</u>: Rabbit in the hole, I.

<u>I</u>. Hold both strings of the loop in 𝕃 about 6" from one end, allowing the remainder of the string to depend as a long hꝏ beneath 𝕃. The small upper loop represents the "rabbit's hole" [Fig. 116.A]. Now hook R2 and R3 down into this small upper loop.



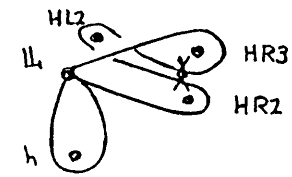
<u>Fig. 116.B</u>: Rabbit in the hole, Ⅱ.

<u>Ⅱ</u>. Now, separating HR2 and HR3 widely, >HR2(#): <HR3(#), thus bringing ℝ back to #. [Fig. 116.B]



<u>Fig. 116.C</u>: Rabbit in the hole, Ⅲ.

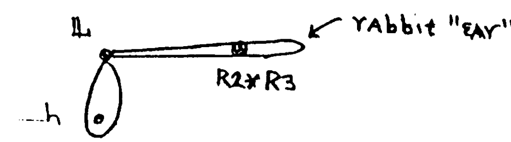
<u>Ⅲ</u>. Releasing L2 momentarily from its grip on the strings retained in 𝕃, pass it to the right -- on the far side of the central transverse strings -- and hook it, from above, down over R23p; draw this string out, slightly, to the left on HL2.



<u>Fig. 116.D</u>: Rabbit in the hole, IV.

<u>IV</u>. Now, separating R2 and R3 widely, <R2 [#(HR2)]: >R3 [#(HR3)], and pinch HR2f and HR3n between HR2*HR3 (at X, above).

<u>Note</u> that Step IV is, essentially, the inverse of the earlier Step Ⅱ.



<u>Fig. 116.E</u>: Rabbit in the hole, V.

<u>V</u>. □ HL2: #(R2,R3), maintaining the two strings pinched between R2*R3 during #.

The small loop protruding between R2*R3 on their dorsal aspect is presented as the "ear" of the rabbit who was hiding in the hole.

With a little practice, the five moves of the construction, above, will be found to blend into a single "flash", accomplished instantaneously. Experience has shown that the challenged onlooker will invariably fail in his attempts to duplicate the procedure -- for a time -- because of the wide separation of R2 and R3 required in Step Ⅱ. He will not be fooled for long. The effect, in and of itself, is especially suitable for small children, who -- at its completion--

may be encouraged to catch the proffered "rabbit ear". As the child grabs for the small loop, which represents this "ear", on the back of R2*R3, a quick "tug" by L will pull it free of R -- thus eluding his grasp; the rabbit has retreated safely back into his hole.

4. BEAR TRAP [G. Tessmann: Die Bubi auf Fernando Poo, Fig. 161, "We'sa".]
The final figure of the present section which we present appears to be widely known as a contemporary trick, but has been recorded only rarely. It is a good "challenge"-figure for the fledgling String-Figure enthusiast, who yet believes all String-Figures begin with "Opening A".

Fig. 117: Bear Trap

Construction:

O.1:>R15ꝏ: O.A::<2ꝏ—>1:□5]

Here the twist of R15ꝏ is disguised in the presentation, and the last two manipulations are effected more-or-less simultaneously -- as the 2ꝏ's are "tossed" towards you onto 1 -- the extension being already in progress. To onlooker it often appears that the opposite 2ꝏ's release to encircle the 1f$^{(2)}$-strings in an entirely natural and reasonable manner; and his usually-produced string-position,

O.A:<2ꝏ—>1:□5]

i.e. 1ꝏ$^{(2)}$, comes as a genuine surprise.(3)

IV. SWINDLES

In this section are listed several String Tricks of a more serious nature, whose purpose is to defraud, discomfort, or humiliate the onlooker. The first two of these are (the basis for) "razzles" -- a word common to old time carnival workers and street-hustlers, meaning "a game of chance (!) whose odds are worse than a million-to-one against the player".

1. FIND THE CENTER [R.M. Abraham: Easy To Do Entertainments and Diversions, No. 145, Trick o' the Loop".]
This trick traditionally employs a man's leather belt* in the working, although leather straps, ropes, webbing, and -- once -- a decorative ribbon have been so observed. A thick String-Figure loop, doubled to approximate a straight length of single string, will also suffice.

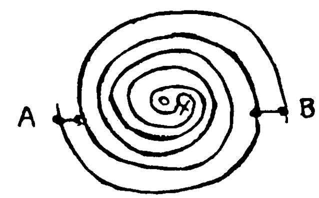


Fig. 118: Find the Center.

Construction:

I. Double the string -- just slightly to one side of its center -- and roll the ends up about the small central loop thus formed, to produce a flat spiral. Lay this on the surface in front of you. [Fig. 118]
At this point two small, central "teardrop"-shaped loops will appear (demarked X and ◯ in Fig. 118). Invite onlooker to stick a pencil (or other pointed object) into whichever of these he believes to be "the center".

II.A. Onlooker chooses X: performer seizes the two outermost strands at B (in Fig. 118), and pulls them away, right.

B. Onlooker chooses ◯: performer seizes the two outermost strands at A (in Fig. 118), and pulls them away, left.

In both cases, as performer pulls the two indicated strands, the spiral will unwind about, and ultimately come free of, onlooker's pencil. He has clearly not

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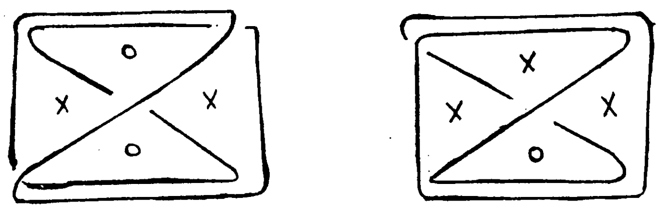
* Note: the two sides of the belt must be identical, or the trick will be soon found out.

found the spiral's center.

Notes: ①. The actual spiral produced in the construction -- unlike its depiction in Fig. 118 -- will have several more layers of coils; and these will lie much more closely together than is here pictured. ②. Of course, performer must know which small, central loop is X, and which is O. This is no problem, of course, since he constructs the spiral and lays it out. ③. Note that the string-position of Fig. 118 may be rotated 180^{o} in its plane, to produce a very similar-appearing figure. This fact may be put to good advantage in subsequent repetitions of the trick with the same onlooker.

2. THE FINGER TRAP [G. Budworth: The Knot Book, Fig. 103.]
This swindle has as many variations as it has practitioners; we present but four here, based on the plaits of Section II.

A.



B.

Fig. 119: Finger Trap, I.

Construction:

A. Construct II.1.A, Ansa (Fig. 112.A); place both indices down into the design at X, and both thumbs at O. Extend indices and thumbs widely, and separate hands. (Leave figure flat, as a plait).

B. Construct II.1.B, Ansa variation (Fig. 112.B); place indices at X, thumbs at O. Extend indices and thumbs widely, and separate hands. (Leave flat).
Onlooker in now invited to insert a finger into one of the four regions of the design's interior, and either lateral single string is pulled away. Onlooker loses if his finger is caught by the string, and wins if the string comes free. These outcomes are indicated for the various regions of the two designs in Fig. 119 by

X = caught, O = free,

respectively.

The same "game" may also be played with the two plaits of II.2, Snare(2), Fig's. 114.D and E.

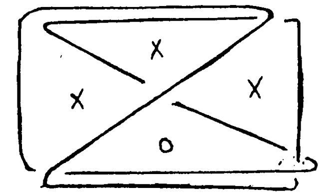


Fig. 120: Finger Trap, II.

Construction:

A. Construct II.2, Snare(2) (Fig. 114.D); place both indices down into the design at ✳, and both thumbs at ★. Extend indices and thumbs widely, and separate hands. (Leave flat). [Fig. 120]

B. Construct II.2, Snare (2), variation (Fig. 114.E); place indices at ✳, thumbs at ★. Extend indices and thumbs widely, and separate hands. (Leave flat). This gives Fig. 119.A.
As before, the outcomes of the previous game played on the design of Fig. 120 are indicated for the various regions by

X = caught, O = free.

Figures 119.B and 120 are seen to be "functionally" equivalent, in that they have identical outcomes (payoffs). We remark that if Snare (2) is made on the hands (see page 266), and laid flat by turning the fingers away and down to release the figure onto the flat surface, then construction A., above, produces a turned-over version of Fig. 120. To the onlooker, this will be indistinguishable from Fig. 120, itself -- but, note, that the upper X and lower O have exchanged places. Et cetera.

We conclude the present section with three examples of swindles which arise from String-Figures -- as opposed to String Tricks -- by way of comparison with the foregoing tricks. The first of these is innocuous enough, although the occasional, sensitive child will break into tears over it.

3. WILL YOU HAVE A YAM? [I. Vinton: The Folkways Omnibus of Children's Games, Pages 203-204, "I Have a Yam", or T. Wu: T'iao Hsien Yushi, Pages 50-53, "Yam". The plate, below, is from C.F. Jayne: String Figures, Fig. 800.]

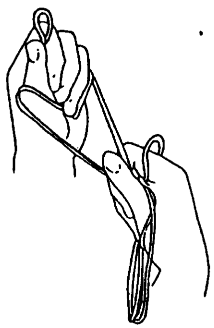


Fig. 121: Will You Have a Yam?

Construction :

O.1: 2(5f)#1(2n)#N1|:: 5(2f-s): 5(s;L12-R12)# 1(2f-s; center)#|:
L2∞->L1*L2]

Now proffer the L1*L2-loop to onlooker as a "Yam". When he reaches for it, pull down sharply with R -- and the "Yam" disappears; all strings come free of L.
The same procedure may then be repeated with R.

The last two figures of the present section have as their object the causing of some minor discomfort to onlooker as befits the various events they seek to portray.

4. THE EEL [K. Haddon: String Games for Beginners, No. 5.]

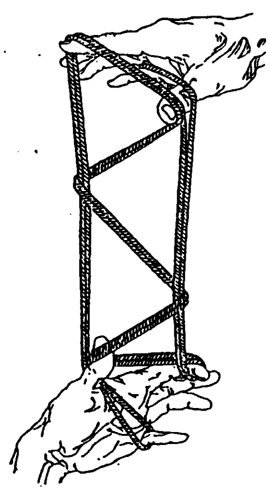


Fig. 122: The Eel

Construction :

O.1:□1|: R1(R5∞)#|:: L1(R1f(2)) :L1(Rp(2))# |: 1(5∞)# P: >L]
(Rotate figure 90°-counterclockwise).

This gives Fig. 122, above. Onlooker is now challenged to "Catch the eel" by encircling all strings of the central configuration with either of his hands. This being done, performer looses all strings except the 5∞, and sharply extends his hands. The "eel" escapes, leaving the distinctive "feel" of all such encounters in onlooker's palm, i.e. something akin to "rope burn". There is a related, alternate construction:

O.1:□1 |: L1(L5∞)# |:: R1(L1n(2)) : R1(Lp(2))# |: 1(5∞)# P:>L]
(Rotate figure 90°-counterclockwise).

5. FLINT AND STEEL [C.F. Jayne: String Figures, pages 320-324.]

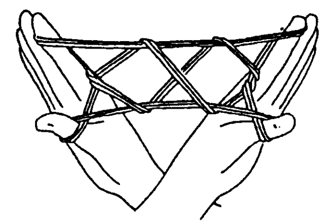


Fig. 123: Flint and Steel,

Construction:

O.1: R1(Lp)#L1(L5n)# |: 2(1f)# N1:□5 |::<R:>L: Simultaneously pass L towards you to the right, and R away from you to the left -- keeping strings taut -- until hands assume the back-to-back position of Fig. 123, above]
Onlooker is now encouraged to bring his face between L1*L2 to "blow on the fire". This being done, performer pinches onlooker's nose (or lips) to simulate the latter's being "burned by the fire".

Note: The unusual final extension encountered in this figure -- the movements subsequent to "□5 |"-- occurs frequently enough among the String-Figures of the world to warrant its own notation. We define

I_{BB}(L) -- <R: >L: Pass L to right, R to left -- until hands assume a back to-back position, fingers pointing up.

The symmetric extension will be denoted by I_{BB}(R).

V. RELEASES

Fully half of the source-identified String Tricks of the world belong to this category. And because of this multiplicity, we have broken-up this section into three sub-classifications: Releases from

A. Openings,
B. Single Functor,
C. Multiple Functors.

Wherever possible, we include examples of releases arising from String-Figures -- as opposed to String Tricks -- by way of comparison.

A. Openings. Here performer effects a "release" directly from an opening position for the String-Figures of his culture. The number of possible tricks in this sub-category, itself, is legion; and there is much to be learned about the string from such experimentation.

1. THUMB RELEASE (1) [L. Dickey: String Figures From Hawaii, page 154, "Thumb Slip Trick, A".]

Construction:

O.A: $\overleftarrow{2}$∞→1:□5 |: R1∞$^{(2)}$ ⇒uL1

This gives four L1∞'s; the ℓL1∞ being distinct from the uL1∞$^{(3)}$. Continue

$\overleftarrow{\overline{R1*R2}}$(ℓL1n) I.

All loops come free of L1.

2. THUMB RELEASE (2) [cf. L. Dickey: String Figures From Hawaii, page 154, "Thumb Slip Trick, B".]

Construction:

O.1:: $\overleftarrow{\overline{HR2}}$($\overline{Lp}$):<R2# $\overleftarrow{L5}$∞→L2:□ R5 |: <$\overleftarrow{2}$∞→1: R1∞$^{(2)}$⇒ ℓL1

This, again, gives four L1∞'s; this time the uL1∞ is distinct from the ℓL1∞$^{(3)}$. Continue

$\overleftarrow{\overline{R1*R2}}$(uL1n) I,

and all loops come free of L1. But note that the alternate (contemporary) construction

O.1:: $\overleftarrow{\overline{HR2}}$($\overline{Lp}$): <R2# $\overleftarrow{L5}$∞→L2:□R5|:<$\overleftarrow{2}$∞→1: R1∞$^{(2)}$ ⇒uL1,

which results in a single ℓL1∞, distinct from the uL1∞$^{(3)}$-- as in V.A.1,

above -- gives III.4 (left), i.e. Fig. 117, Bear Trap (left half of figure) -- upon the continuation

$\overleftarrow{\overline{R1*R2}}$(ℓL1n) I.

We remark that the Opening under discussion, here, is that of the Kwakiutl figure "Bear's Den" (Fig. 94.A, page 201 of these notes) -- and it is one which enjoys world-wide popularity.

3. INDEX RELEASE (Contemporary)

Construction:

Throw a small, inverted loop in the string -- passing right string towards you and to the left. This gives Fig. 124.A, below.

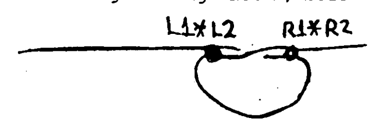


Fig. 124.A: Index Release, 1.

Now hook (both) 2's directly towards you into this small loop from the far side of the figure, and continue >2 up to #: |

This gives Fig. 124.B, below.

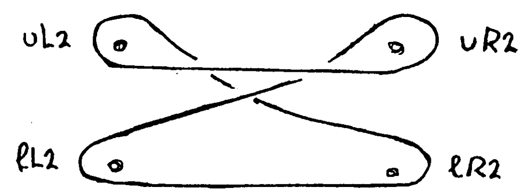


Fig. 124.B: Index Release, 2.

This is another widely-known opening for many of the String-Figures of the world [see, e.g. C.F. Jayne: String Figures, Fig. 528, "Little Fishes"]. Continue

R2∞$^{(2)}$ ⇒ uL2: $\overleftarrow{\overline{R1*R2}}$(ℓL2n) I .

All loops come free of L2.

4. THUMB-INDEX RELEASE [H. Noguchi: "String Figures in Japan, IV", No. 100, "Right Hand Slip Trick".]

Construction:

O.JA:□5 |: 3∞→1: R2↑(R1∞$^{(2)}$)# |: Pass L1∞$^{(2)}$ between the crotch of R1*R2 and --□L1 -- allow these loops to depend from the dorsal aspect of R. This gives Fig. 125, below.

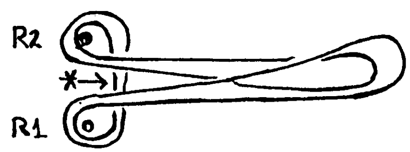


Fig. 125: Thumb-index release.

Continue,

Now pass L1*L2 and seize up the R12d$^{(2)}$-strings (at *)].

All strings will come free of R. A minimal variation continues wrapping the final double right dorsal loops across the back of R, and then around to the front and across the Rp, where they are encircled within the "ring" formed by bringing the tips of R1 and R2 together around them. The excess is allowed to suspend from this R1*R2-ring as a h∞$^{(2)}$. The final release is executed exactly as before.

The final three examples of the present subsection involve two people; Mr. ØC (performer) and Ms. 𝔖 (onlooker): That is, onlooker's role is (slightly) less passive than heretofore. This is in contradistinction to figures to be encountered in future sections which require two performers ("Two-person figures"). All constructions, below, will be given from the perspective of the performer, Mr. ØC.

5. WRIST RELEASE (1) [W.H. Ingrams: Zanzibar, page 232.]

Construction:

Mr. ØC: O.A
Ms. 𝔖: R↓2∞
Mr. ØC: □2:□5# |: 5↑(1∞)# O.A |
Ms. 𝔖: R↑2∞
Mr. ØC: □2:□5]

All strings come free of Ms. 𝔖's right hand. This very easy trick has exceptionally wide currency among the children of modern urbanites, and enjoys a favored status among them.

6. WRIST RELEASE (2) [D. Jenness: "Papuan Cat's Cradles", No. 24A.]

Construction:

I. Mr. ØC: O.1: >>R15∞: O.A:□5 |

This gives Fig. 126, below.

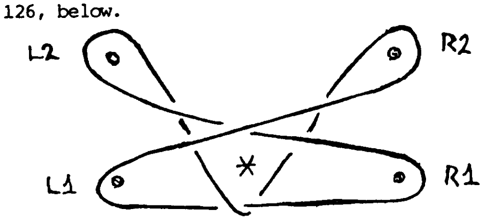


Fig. 126: Wrist Release (2).

Continue,

II. Ms. 𝔖: R↑(c-◊ ; at *)
Mr. ØC: □2 |: 5 (1f)# O.A |
Ms. 𝔖: R↑(2∞)
Mr. ØC: □5]

All strings come free of Ms. 𝔖's right hand. Step II of the construction may now be repeated -- as many times as desired -- for an "instant replay" of the release.

7. WRIST RELEASE (3) [D. Jenness: "Papuan Cat's Cradles", No. 24B.]

Construction:

I. Mr. ØC: O.1: >>R15∞: O.A |

This gives the (complex) schema below.

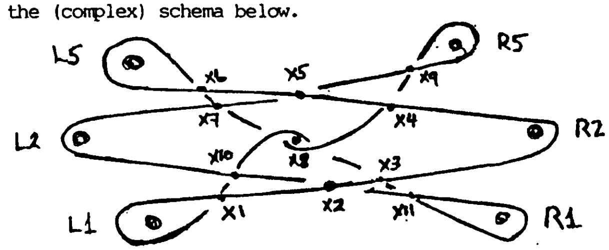


Fig. 127: Wrist Release (3).

Note: The central crossing, x8, is considered to be complex, even though "fine-structure" is shown.

II. Ms.𝔅: pass R toward Mr.𝔄, in the "space" below x2, x5 and above x8, to near side of figure

Mr.𝔄: □5 ⌶ [𝔅's R caught]
□2 ⌶ [𝔅's R free].

This concludes our discussion of A. Releases; Openings. In the next section, we continue with an examination of single-functor releases in the context of slightly more complicated constructions, i.e. ones that are more than slight modifications of well-known openings.

B. SINGLE FUNCTOR. Here performer loops and/or laces the String-Figure loop about a principal functor, and effects an unexpected release therefrom. Other functors may play a temporary, constructional role in the manipulations.

1. THUMB RELEASE (3) [cf. C.F. Jayne: String Figures; pages 344-345, "A Dravidian Trick".]

This contemporary trick is unusual in that the hand from which the release is effected is manipulated during the smooth pulling-off motion of the other hand.

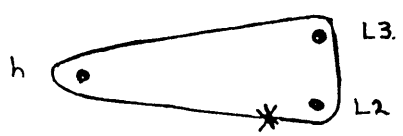


Fig. 128.A: Thumb Release, Opening.

Opening: Hang the loop on L2, L3 so that a long hanging loop depends from their dorsal aspect.

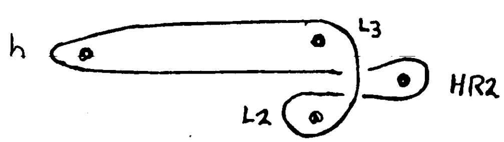


Fig. 128.B: Thumb Release, I.

I. HR2(L23p): Pass HR2 between L2, L3 to back of L and hook up L2n (at ✱) from above: pull this latter string (on HR2) back to palmar side of L, beneath

L23p [Fig. 128.B].

The remainder of the construction is done, in a continuous manner, as R slowly and smoothly draws the (current) HR2∞ away from L.

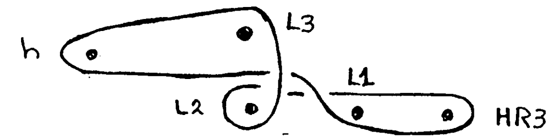


Fig. 128.C: Thumb Release, II.

II. R3↑(HR2∞):□HR2:: <R3: L1↑(HR3∞)

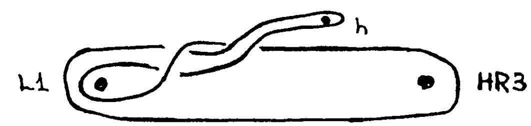


Fig. 128.D: Thumb Release, III.

III.□L3: <L2∞→L1 ⌶

All strings come free of L1. Note, performed smoothly, with a continuous "drawing-out" motion of R, this is a very effective little trick, bordering on the category of "Illusions". Move III of the construction is best accompanied by a "down-up" motion of L.

2. THUMB RELEASE (4) [J. Averkieva: Kwakiutl String Games, No. 110.]

Construction:

I. Hang string-loop on L1 so that a long h∞ depends therefrom on the palmar side of L.

II. Hook R45 over both strings of h∞ about 10" from the base of L1: insert R1, R2 down into L1∞ and spread them widely (still pointing down). This gives Fig. 129.A, below.

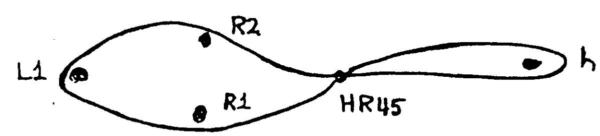


Fig. 129.A: Thumb Release, after II.

III. > R12ꝏ and place far string of loop (R1f) directly onto L2: << R12ꝏ and place near string of loop (R2n) directly onto L1:□ ℝ.
This gives Fig. 129.B, below.

Fig. 129.B: Thumb Release, after III.

IV. (~II & III)^2:□ L2: draw out hꝏ with ℝ].
All strings come free of L1. Note: The "exponent" 2 in Step IV may be replaced by any even number (including 0). The distinctive ℝ-position of Fig. 129.A is typical of many Inuit "lacing"-type manipulations.

3. THUMB RELEASE (5) [P.E. Victor: "Jeux d'Enfants et d'Adultes Chez les Eskimo d'Angmagssalik", No. 28, "Cutting the Thumb".]
Construction:
I. Hang the string-loop on L1, so that the long hꝏ depends from the palmar aspect: give L1n an additional turn about L1.
This gives Fig. 130.A, below.

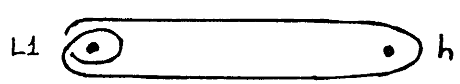


Fig. 130.A: Thumb Release, I.

II. Put a simple twist in hꝏ (about 10" above its nethermost point) by passing the far string towards you over the near string of this loop: turn the resulting small loop directly back on itself at the crossing so formed.
This gives Fig. 130.B, below.

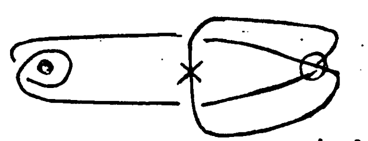


Fig. 130.B: Thumb Release, II.

III. Seize up the nethermost crossing of hꝏ (at O), turn this directly up, and place behind L1: pull out the central hꝏ depending from L1 (at X)].
All strings come free of L1.

4. WRIST RELEASE (4) [W.A. Cunnington: "String Figures and Tricks From Central Africa", No. 18, "A String Trick".]
Construction:
O.1: $\overrightarrow{15ꝏ}$→W# | :: M (Wn) : $\overrightarrow{M}$ (Wf) # | : Twist Mꝏ 180°-counterclockwise: 𝔹↓(Mꝏ) :□M# |
This gives a string-position with Wꝏ$^{(2)}$. Now □ℝ and hold 𝕃 back uppermost, fingers pointing to right. A tug with ℝ on LWn$^{(2)}$ will free LW.

5. WRIST RELEASE (5) [J. Averkieva: Kwakiutl String Games, No. 112.]
Construction:
I. Hang string-loop from LW: give LWn an additional turn about LW: seize nethermost point of hꝏ between R1*R2.
This gives Fig. 131, below.

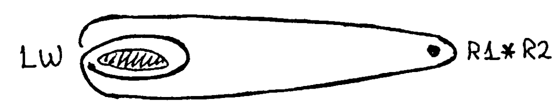


Fig. 131: Wrist Release, I.

II. Draw R1*2ꝏ to left and just over the tips of the 𝕃 fingers: < R1*2ꝏ: draw small R1*2ꝏ just created back to right over tips of the 𝕃 fingers].
All strings come free of 𝕃.

6. FLY ON THE NOSE (1) [H. Krauss: "Lufambo", No. 7.]
A classic, whose principal variation (also given below) -- although apparently less well-known -- enjoys a similar contemporary currency.
Construction:
Throw a small, inverted loop in the string by passing right string towards you and to the left: take the crossing thus formed in the mouth and □ 𝔹: now pass ℝ away from you and up into the long hanging Mꝏ, and HR2 towards you and down into the short hanging Mꝏ: twist R2 180°-counterclockwise --thus putting a twist in the small hanging Mꝏ (only) -- and place the tip of R2 against the tip of the nose.

This gives Fig. 132, below.

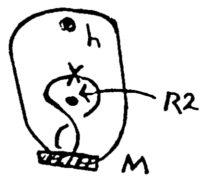


Fig. 132: Fly on the Nose (1).

Now seize up the R2d-string (at X) between L1*L2, ☐ M, and pull L1*L2 away from body.

All strings will come free of IR; the "fly" has escaped.

This trick has the following interesting variation [H. Noguchi: "String Figures in Japan, IV", No. 104, "Nose Slip".]

Construction:

Throw a small,inverted loop -- passing left string towards you and to the right -- and take the crossing so formed in M, ☐ IB: now pass R2 away from you and up into both hanging M∞'s: pressing R2 away, against the nethermost point of the short hanging M∞, draw this loop down and away to the right (under the rightmost string of the long hanging M∞): now bring this small loop back to the center of the figure and (pointing R2 down) directly down and through the long hanging M∞: continue drawing the small loop down and away to the left (under the leftmost string of the long hanging M∞): finally, turn R2 directly back up, and place its tip against the tip of the nose.

This gives Fig. 133, below.

Fig. 133: Fly on the Nose (2); variation.

Now seize up the R2d-string (at ✱) between L1*L2, ☐ M, and pull L1*L2 away from body.

As before, all strings will come free of R.

7. CUTTING OFF THE HEAD; another classic, with many variations. We shall indicate a few of the principal constructions.

Variation 1 [D.S. Davidson: "Aboriginal Australian String Figures", No. 69.]

Construction:

Place loop over head: take up right string of h∞ in IR and left string in L, and cross these in front of face (left over right): seize up crossing so formed in M: now uncross hands (left over right) and throw h∞ directly back over head, ☐ B: place each 1 up into lateral h∞ on its respective side]. This gives Fig. 134.A, below.

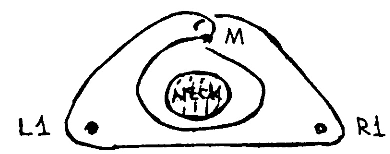


Fig. 134.A: Cutting off the Head, 1.

Maintaining 1's in their loops, clap hands sharply in front of face, then spread hands widely -- and back -- while ☐ M.

All strings will come free of the head.

Variation 2 [D. Levinson and D. Sherwood: Tribal Living Book, page 145, "Hanging Trick".]

Construction:

Place loop over head and, seizing up right string of h∞, give this another twist about the neck: make a small, inverted loop in the bottom of the long h∞, passing left over right: seize up the crossing so formed and pass it back over the head, so that the head passes up into the (lower) small h∞: ☐ B.

This gives Fig. 134.B, below.

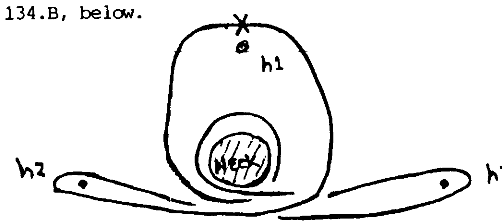


Fig. 134.B: Cutting off the Head, 2.

Seize up the nethermost point of the long h∞ lying in front of the torso (at ✕), and pull this string sharply away from the neck.

All strings will come free of the neck. (cf. V.B.3)

Variation 3 [H.C. and E. Helfman: String On Your Fingers, Pages 26-27, "A Noose".]

This release is a minimal variation of the one immediately preceding. We specifically include it because of its current wide popularity.

Construction:

Place loop over head and, seizing up right string of h∞, give this another twist about the neck: pass 5 away and up into h∞: $\underrightarrow{1}\uparrow$(5∞) # | (as in O.1): O.A: pass head away, under all intervening strings, and up into 2∞ [releasing 5 to enlarge 2∞, if string is otherwise too short]: □ B.

This gives Fig. 134.B, above. The release is that of the previous variation, i.e. pulling out the long h∞.

Variation 4 [P.G. Brewster: "Some String Figures and Tricks From the United States", Fig. 11.*]

Construction:

O.1: □5 |: place both transverse strings behind neck: $\overleftarrow{\text{HR5}}(\overline{\text{L1f}})$ # (HR5): $\overrightarrow{\text{HL5}}(\overline{\text{R1f}})$ # (H5) |.

This gives Fig. 134.C, below.

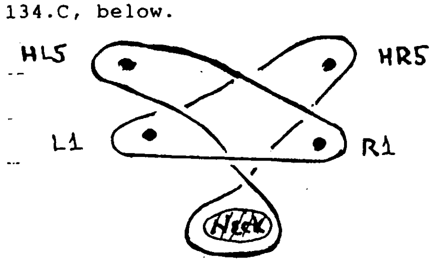


Fig. 134.C: Cutting off the Head, 4.

Continue,

Simultaneously □ L1, □ HR5 and extend hands widely and sharply.**

All strings will come free of neck.

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* Note: the text figure is incorrect.

** Beware of rope-burns with this variation!

Variation 5 [F.J. Rigney: Cub Scout Magic, pages 88-89, "Rope loop through your neck".]

As with No. 3 of this sequence, this is a minimal variation of the preceding release. Of the two, the present variation is perhaps the more effective -- as a trick; however, it lacks primary-source citation.

Construction:

O.1: □5 |: place both transverse strings behind neck: $\underleftarrow{\text{R1}}\uparrow$(L1∞): □L1 # $\underrightarrow{\text{L1}}\uparrow$(R1∞$^{(2)}$) # |: $\overleftarrow{\text{H45}}(\overline{\text{1f-s}})$ # (H45): $\overleftarrow{\text{H2}}(\overline{\text{u1f-s}})$ # (H2, 45): □1 |

This gives Fig. 134.D, below.

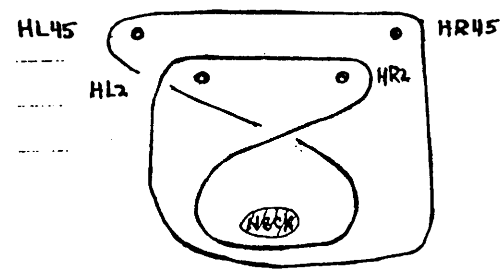


Fig. 134.D: Cutting off the Head, 5.

Continue,

□H2 ⌶ (sharply).

All strings will come free of neck.

Variation 6 [T.Saito: Ayatori Itotori, Vol. 3, Trick No. 11, "Cutting the Head".]

Construction:

Place loop over head: make a small, inverted loop in the bottom of the long h∞, passing right over left: pass 1 towards you and down into small, inverted loop and -- picking it up -- turn it over towards you and place it over the head (do not □1): slide 1's (in their loops) to front of body.

This gives Fig. 134.E(1), below.

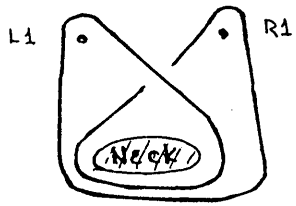


Fig. 134.E(1): Cutting off the Head, 6.1.

Continue,

R1↑(L1co) # NR1|

This gives Fig. 134.E(2), below.

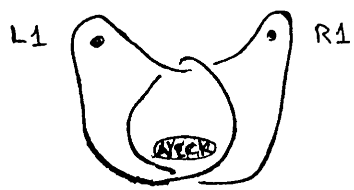


Fig. 134.E(2): Cutting off the Head, 6.2.

Continue,

R1↑(L1co):□L1|: L1↑(R1co$^{(2)}$) #|

This gives Fig. 134.E(3), below.

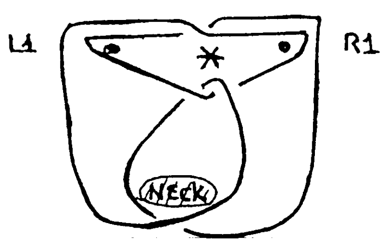


Fig. 134.E(3): Cutting off the Head, 6.3.

Continue,

Pass the head forward and down into the 1co$^{(2)}$'s (at ✱), so that this double loop passes over to the back of the neck.

This gives Fig. 134.B. To release, pull either (or both) string(s) in front

of the neck; all strings will come free.

Variation 7 [T.H. Centner: "L'Enfant Africain Et Ses Jeux", Jeu-Probleme No.1, "Le Sorcier et le Prisonnier".]

Construction:

Performer seated on ground, right leg straight: place loop around neck and right great toe.

This gives Fig. 135.A, below.

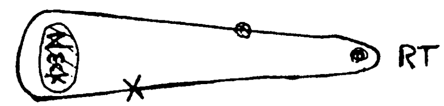


Fig. 135.A: Cutting off the Head, 7.1.

Continue,

Pass IL to right over left string of neck-loop and seize up right string (at ✕) between L1*L2: pass R to left over right string of neck loop and seize up left string (at ○) between R1*R2 #|

This gives Fig. 135.B, below.

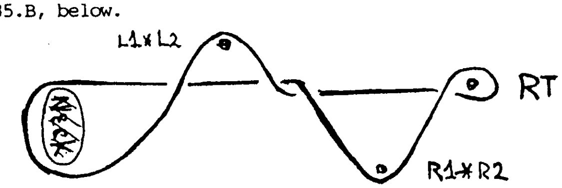
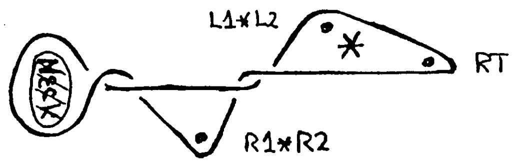


Fig. 135.B: Cutting off the Head, 7.2.

Continue,

Transfer R1*R2co directly to R5: using neck and RT to maintain necessary string-tension, circle L1*L2co counterclockwise beneath left string of neck loop to right of figure, and take up this string between R1*R2: □IL # |: Pass L to right over all strings and, seizing up R5d between L1*L2, remove the loop from R5 #|.

This gives Fig. 135.C, below.



Fig, 135.C: Cutting off the Head, 7.3.

To release the head, pass head forward, to the left, and down into the L1*L2-RT∞: circle head down, left, and back up again -- to normal position -- releasing L1*L2: pull neck sharply away from RT and □R1*R2 I.

All strings will come free of the neck.

We shall content ourselves with these few variations of this widely known String Trick, "Cutting off the Head". In the author's experience, Variation 3 is the most widely known, 5 is the most effective as a trick, and 6 provokes the most discussion among String-Figure enthusiasts. In Variation 7, the finger of a second person is sometimes substituted for the right great toe.

The final three examples of String Tricks in the present subsection involve two people; Mr. α (performer) and Ms. β (onlooker), the latter providing the (single) functor for the release. All constructions (below) will be given from the perspective of the performer, Mr. α.

8. NAVAHO RELEASE [A. van Oorschot: Touwfiguren, pages 84-87, "Ontsnapping 4".]

Construction:

Opening: Hang string as a simple (untwisted) loop on Ms. β's R2 and Mr. α's L2, |.

Mr. α : Pass R3 to left over rightmost string of this shared loop, about 10" from Ms. β's finger, and hook the leftmost string back -- and just to the right of -- said rightmost string: now turn R3 up to #, picking up rightmost string on its back (□former HR3∞).

This gives Fig. 136.A, below.

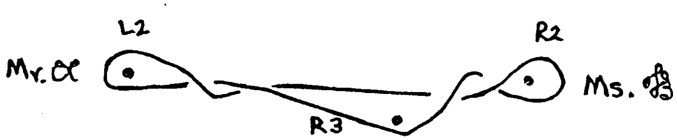


Fig. 136.A: Navaho Release, 1.

Continue,

Mr. α: Rotate R about R3 (in its loop) -- so that, viewed from above, R2 proceeds counterclockwise about R3 -- and put R2 directly up into Ms. β's R2∞ from below: now, simultaneously, hook R2, R3 to left over the respective strings on their leftmost aspect.

This gives Fig. 136.B, below.

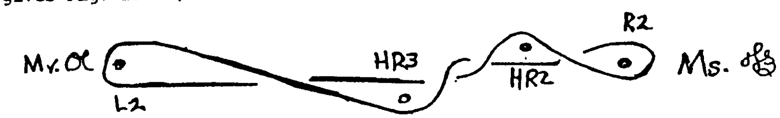


Fig. 136.B: Navaho Release, 2.

Continue,

Mr. α: Rotate R about HR2 (in its loop) -- so that, viewed from above, HR3 proceeds counterclockwise about HR2 -- and place the tip of HR3 directly on the tip of Ms. β's R2.

This gives Fig. 136.C, below.*

Fig. 136.C: Navaho Release, 3.

To release, Mr. α performs □HR2 I, and all strings come free of Ms. β's R2.

This is a most effective release, and is usually performed very quickly. A finger-ring is also sometimes employed in the working [M. Gardner: "Mathematical Games: The Ancient Lore of String Play", page 149].

9. INDEX SLIP TRICK [H. Noguchi: "String Figures in Japan, IV", No. 102.]

This wonderful String Trick has a more modest distribution than many here met with, and appears but rarely in the literature. Occasionally, even someone well-acquainted with the string will be completely fooled by it.

Construction:

---

* Fig. 136.C does not show Mr. α's HR3 atop Ms. β's R2; i.e. this "composite functor" has a single schema-node.

Hang string as a simple (untwisted) loop on Ms. [symbol]'s R2 and Mr. [symbol]'s R2, |.

Mr. [symbol]: Place L2 (pointing right) over both transverse strings, about 1/4 of the distance from Mr. [symbol]'s R2 to Ms. [symbol]'s R2: keeping strings taut, roll R2 up and over L2 and place R2 (pointing left) over both transverse strings: similarly, roll L2 up and over R2 and place L2 (pointing right) over both transverse strings: finally, roll R2 one last time about L2 and -- turning R2 directly down -- place all (three) of its loops on the tip of Ms. [symbol]'s R2.

This gives four R2∞'s for Ms. [symbol], her ℓR2∞ being distinct from the uR2∞(3); Mr. [symbol] retains four L2∞'s. Mr. [symbol] now seizes up Ms. [symbol]'s <u>ℓR2n</u> and -- releasing his L2 -- pulls this string towards him.

All strings will come free from Ms. [symbol]'s R2.

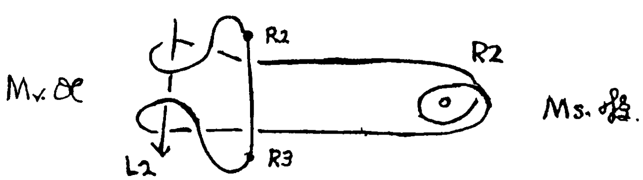
<u>10</u>. PAWNEE RELEASE [K. Haddon: <u>Cat's Cradles From Many Lands</u>, Trick No.11.]

Construction:

Hang string as a simple loop on Ms. [symbol]'s R2 and Mr. [symbol]'s R2,|: take the far string of Ms. [symbol]'s R2∞ and give it an additional turn about her R2.

Mr. [symbol]: Place L2 (pointing right) over both transverse strings, about 1/3 of the distance from Mr. [symbol]'s R2 to Ms. [symbol]'s R2: keeping strings taut, roll R2 up and over L2, and -- releasing R2 -- drape its loop over both transverse strings.

This gives Fig. 137.A, below.



<u>Fig. 137.A</u>: Pawnee Release, 1.

Continue,

Mr. [symbol]: Pass R2, R3 away from you below **L** and -- spreading these fingers widely (<u>i.e</u>. outside of the transverse strings) -- pass them up into the central "draped" loop, hooking them over the far string there-of (at <u>R2</u> and <u>R3</u>, respectively): <uL2∞: transfer L2∞(2) directly to Ms. [symbol]'s R2 |.

This gives four R2∞'s for Ms. [symbol], and single HR2, HR3 loops for Mr. [symbol]. Continue,

Mr. [symbol]: □HR2 ⌶

All strings come free of Ms. [symbol]'s R2.

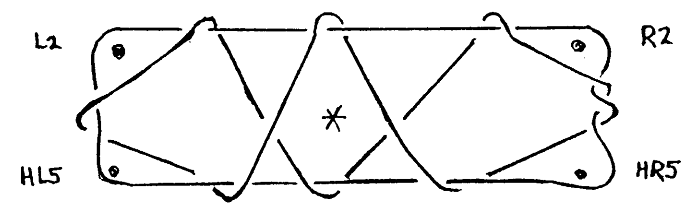
We conclude the present subsection with a single-functor release proceeding from a String-Figure -- as opposed to a String Trick -- by way of example, for comparison with the previous String Tricks.

<u>11</u>. DUGONG [C.F. Jayne: <u>String Figures</u>, pages 39-43, "A King Fish (Also a Catch)".]

Construction:

<u>O</u>.A:□R2|: $\overrightarrow{\overline{HL2}}$($\overline{Lp}$):□L1&L5|: $\overrightarrow{\overline{HL2\infty}}$→L15 [<u>O</u>.1(L)]: $\overrightarrow{\overline{HL2}}$($\overline{Rp}$): >L2(#)|:: $\overleftarrow{\overline{HR2}}$($\overline{R1f}$): $\overrightarrow{HR2}$($\overline{R5n}$): >R2(#) [thus releasing R1f from R2]|: $\overleftarrow{\overline{HR5}}$($\overline{o;R2\infty}$): $\overleftarrow{HL5}$($\overline{L2f}$)#(H5): <R2∞:□1 ⌶

This gives Fig. 138.A, below.



Under

<u>Fig. 138.A</u>: Dugong, 1.

A second person is now asked to place his **R** through the central diamond (at ✱); if performer □**L** ⌶, onlooker's hand is caught. If □**R** ⌶, onlooker's hand is freed.

The string-position of Fig. 138.A admits an effective continuation [see J.C. Andersen: <u>Maori String Figures</u>, No. 10, "Komore" (discussion).]

$\underrightarrow{H1}$($\overline{o;2\infty}$): $\underrightarrow{1}$($\overline{o;5\infty}$)#(1): $\underleftarrow{5}$($\overline{o;5\infty}$)#□2 ⌶ (fingers pointed away)

This gives Fig. 138.B, below.

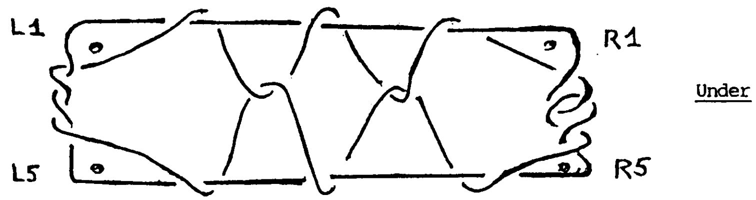


Fig. 138.B: Dugong, 2.

Again, a second person may be asked to insert his R into the central diamond -- and a "catch" or a "release" effected exactly as was done before. Or, this string-position may be continued [see G. Landtmann: "Cat's Cradles of the Kiwai Papuans", No. 15, "A Dugong".]

$\underleftarrow{2}$(o;1∞) : $\overrightarrow{\overline{H2}}$($\overline{1n}$) : <2(#) : □1 | : $\overleftarrow{\overline{2∞}}$→1 :: $\overrightarrow{\overline{2}}$↓(△;5∞) : $\underleftarrow{H2}$($\overline{1n}$) : <2(#) : □1 |
$\overleftarrow{\overline{2∞}}$→1 ⌶ (fingers pointed away)

This gives Fig. 138.C, below.

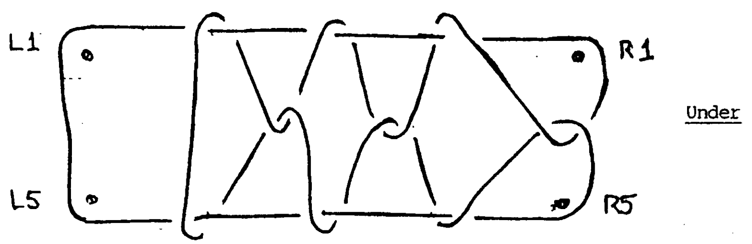


Fig. 138.C: Dugong, 3.

If, now, the hands are widely extended -- with an accompanying "sawing" motion -- the Dugong will "swim" away towards R with a characteristic jerky undulation. Again, an onlooker may be invited to "spear the Dugong" -- passing his IR through the central diamond of the three which comprise the "body" of the Dugong. And, as before, if performer executes □L ⌶ -- onlooker's IR is caught; he has speared the Dugong; if performer executes □R ⌶ --onlooker's IR comes free of the strings; the Dugong has escaped. (4)

We close the present subsection at this point, contenting ourselves with the above several examples of string-releases from a single functor. We next turn to the situation of string-releases involving multiple functors.

C. Multiple Functors. Here performer loops and/or laces the String-Figure loop about several functors -- none of which can be distinguished as "principal" -- and effects an unexpected simultaneous or serial release from all. The examples below, as a group, comprise perhaps the most universally recognized group of String Tricks.

1. ATTRAPE [M. Griaule: "Jeux et Divertissments Abyssins", Plate VII, No's. 3-6.]

The final position, just prior to the release, in the three variations we present here, are all illustrated in Fig. 139.A, below.

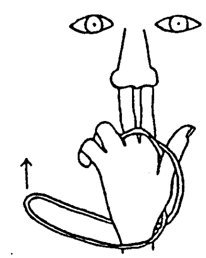


Fig. 139.A: Attrape; frame configuration.

The detailed string-positions associated to the three distinct variations of this String Trick are given in the subsequent diagram, Fig. 139.B.

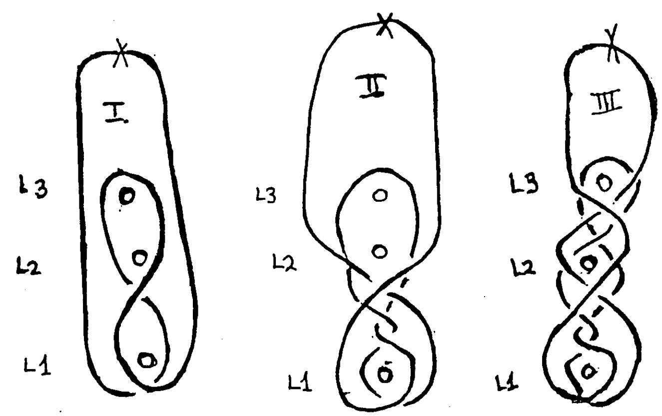


Fig. 139.B: Attrape; the three variations.

Variation 1: Construction:

Hang the string as a simple loop on L23, the long hꝏ depending from the palmar aspect of 𝕃: seize up the long hꝏ, about 10" from 𝕃, in the right fist and, ☐ R2, hook ⟸R2↓ into the L23ꝏ: now* < R2, and then L1⟹↑(R2ꝏ) :: > R2, and then draw the resulting HR2ꝏ directly up and over L1 -- thus putting two loops onto L1, crossed on its dorsal surface -- place the rest of the R2ꝏ over L2, L3 [Fig. 139.B.I] = place L2, L3 in their respective nostrils [Fig. 139.A].

To release, ☐ L1], and all loops come free of 𝕃.

Variation 2: Construction:

Hang the string as a simple loop on L23, the long hꝏ depending from the palmar aspect of 𝕃: seize up the long hꝏ, about 10" from 𝕃, in the right fist: now R1⟸↑(L23ꝏ) : >> R1: L1⟹↑(R1ꝏ) : < R1, and then draw the resulting HR1ꝏ directly up and over L1 (to become an uL1ꝏ) : < R1 and place resulting R1ꝏ directly onto L2, L3 [Fig. 139.B.II]: place L2, L3 in their respective nostrils [Fig. 139.A].

To release, ☐ L1], and all loops come free of 𝕃.

Variation 3: Construction:

Hang the string as a simple loop on L3, the long hꝏ depending from the palmar aspect of 𝕃: seize up the long hꝏ, about 10" from 𝕃, in the right fist and, ☐ R2, hook ⟸R2↓ into the L3ꝏ: now** < R2, and then L2⟹↑(R2ꝏ) : >>R2: L1⟹↑(R2ꝏ) : < R2 and then draw the resulting HR2ꝏ directly up and over L1 (to become uL1ꝏ) : < R2: L2⟹↑(R2ꝏ) : > R2: L3⟹↑(R2ꝏ)] [Fig. 139.B.III]: place L2, L3 in their respective nostrials [Fig. 139.A].

To release, ☐ L1], and all loops come free of 𝕃.

2. CUTTING OFF THE FINGERS. Each of the three variations of the previous String Trick, Attrape, admits an extension to all the fingers of the left hand (no fingers of which, in these cases, are customarily inserted into the nostrils). We shall indicate the construction for the extension of Variation 3, below, omitting the easily deduced remaining two in favor of the major variations of the current figure to follow.

* The move "< R2", here, results in a somewhat awkward position, at first encounter.

** Here's that awkward position again.

Variation 1: [R.M. Abraham: Easy To Do Entertainments and Diversions, No. 127, "The Five Boy Scouts".]

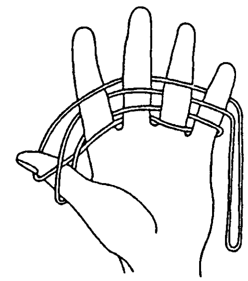


Fig. 140.A: Cutting off the Fingers, I.

Construction:

Hang the string as a simple loop on L5, and grasp the long hꝏ in the right fist, about 10" from 𝕃: ☐R2: ⟸HR2↓(L5ꝏ) :: <R2: L4⟹↑(R2ꝏ) :: >R2: L3⟹↑(HR2ꝏ) :: <R2: L2⟹↑(R2ꝏ) :: >>R2: L1⟹↑(R2ꝏ) :: <R2 and then draw the resulting HR2ꝏ directl up and over L1 (to become uL1ꝏ) :: <R2: L2⟹↑(HR2ꝏ) :: >R2: L3⟹↑(R2ꝏ) :: <R2: R4⟹↑(HR2ꝏ) :: >R2: L5⟹↑(R2ꝏ) :: ☐ ℝ# (𝕃)

This gives Fig. 140.A. To effect the release, ☐ L1 and pull the L5hoo out, briskly. The left hand will be freed of all strings.

Variation 2: [D. Jenness: Eskimo String Figures (C.A.E.), Trick No. 2.]

This is a two-handed version of this class of String Tricks.

Construction:

O.1: ☐ 1 | : →4(f-s) # | : →3(f-s) # | : →2(f-s) # | : →1(s(2)) [keep f-s uppermost] ←2(u1n-s) : →2(ℓ1n-s) # | : ←3(n-s) # | : ←4(n-s) # | : ←5(n-s) #]

This gives a two-handed version of Fig. 140.A. For the release, simply ☐ 1] (widely), and all strings will come free of 𝔹.

Variation 3: [R. Martinez-Crovetto: "Juegos de Hilo de los Aborigenes del Norte de Patagonia", Trick No. 26, "Desenrollar la Tripa".]

In this interesting variation, the String-Figure loop is doubled, to approximate a single straight string, and then interlaced four times through the fingers of the left hand -- preparatory to the final release.

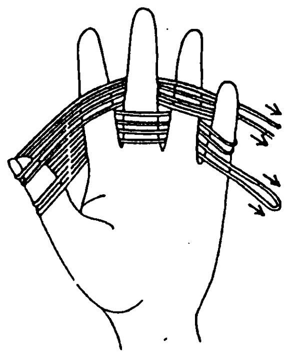


Fig. 140.B: Cutting off the Fingers, III.

Construction:

Leaving a small "tail" at the base of L5, lace the doubled string in front of L5, behind L4, in front of L3, behind L2, in front of L1. Wrap it around behind L1, behind L2, in front of L3, behind L4, in front* of L5. Then wrap it around behind L5, in front of L4, behind L3, in front of L2, behind L1. Finally, wrap it around to the front of L1, in front of L2, behind L3, in front of L4, and behind L5 -- to become a second small "tail" at the base of L5.

This gives Fig. 140.B. To effect the release, simply □ L1 and sharply pull both "tails" away from IL. All strings will come free. ⑤

Variation 4: THE MOUSE [I. Kattentidt: Fadenspiele aus Aller Welt, pages 29-30, "Die Maus Entspringt"; or W.W. Rouse Ball: String Figures, No. 21 "The Yam Thief".]

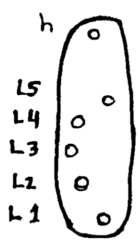


Fig. 140.C.1: The Mouse, I.

Over

* i.e. back the way it came.

Construction:

Hold IL with thumb and fingers pointing away: hang String-Figure loop well up on L1, so that a long, h∞ depends therefrom -- its rightmost (palmar) string passing to the palmar side of the L fingers, its leftmost (dorsal) string passing to the dorsal side thereof [Fig. 140.C.1].

Continue,

R2(L12p): pass R2 up between L1*L2 to back of L: HR2(L12d): and pull this L12d back to form a small HR2∞, just to the palmar side of L: > R2: R2∞ ⇒ L2: grasp both strings of Lh∞ in IR, and pull tight: □ R:: R2(L23p): pass R2 up between L2*L3: HR2(L23d): pull back: > R2: R2∞ ⇒ L3: pull tight:: R2(L34p): pass R2 up between L3*L4: HR2(L34d): pull back: > R2: R2∞ ⇒ L4: pull tight:: R2(L45p): pass R2 up between L4*L5: HR2(L45d): pull back: > R2: R2∞ ⇒ L5: pull tight I

This gives the left-hand string-position illustrated in Fig. 140.C.2, below.

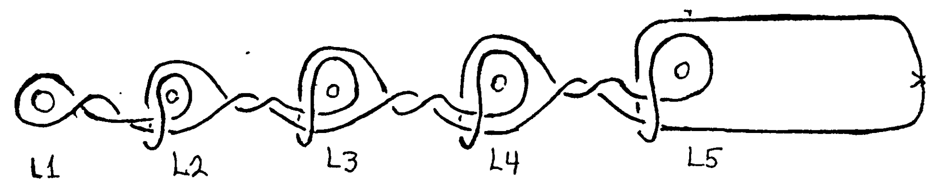


Fig. 140.C.2: The Mouse, II.

For the release, □ L1 and show the resulting small loop as the "ear" of the mouse. Then grasp the long h∞ in IR and pull it out with a loud "Squeak!" The L will come free of all strings; the Mouse has escaped!

Variation 5: THE RAVEN'S PLUMAGE [T.F. McIlwraith: The Bella Coola Indians, Vol. II, String Trick No. II, "The Fastening of Raven's Plumage".]

We present this variation as an example of a non-sequential lacing of the IL fingers which leads to a release. The plate, Fig. 141 below, is from C. Grysk Super String Games, page 49, "The Fastening of the Raven's Plumage".

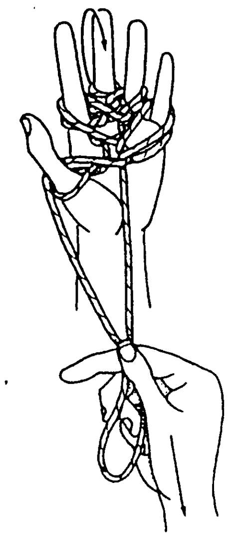


Fig. 141: The Raven's Plumage.

Construction:

I. Hang string-loop on L1 so that a long hꝏ depends therefrom on the palmar side of 𝕃.

II. Hook R45 over both strings of hꝏ about 10" from the base of L1: insert R1, R2 down into L1ꝏ and spread them widely (still pointing down).

This gives the position of Fig. 129.A, preparatory to the lacing of the 𝕃 fingers. Continue,

III. >R12: $\overrightarrow{\text{HL5}}$↓(R12ꝏ): <L5(#) :: <<R12: $\overrightarrow{\text{HL2}}$↓(R12ꝏ): >L2(#) :: >>R12: $\overrightarrow{\text{HL4}}$↓(R12ꝏ): <L4(#) :: <<R12: $\overrightarrow{\text{HL3}}$↓(R12ꝏ): >L3(#) :: >>R12: $\underrightarrow{\text{L4}}$↑(R12ꝏ) :: <<R12: $\underrightarrow{\text{L2}}$↑(R12ꝏ) :: >>R12: $\underrightarrow{\text{L5}}$↑(R12ꝏ) :: <<R12: $\underrightarrow{\text{L1}}$↑(R12ꝏ)#(L): straighten loop being pulled by R ⌶. [Fig. 141]

To effect the release, □L3 ⌶, pulling Rꝏ smoothly away from 𝕃; all strings will come free of L.

We shall content ourselves with the above several examples of the String Trick(s) "Cutting off the Fingers" as illustrative of their genre, and continue with another classic complex of "cutting" tricks -- "Cutting the Hand".

3. CUTTING THE HAND. Here performer loops and laces the string-loop about the fingers and the "trunk" of one hand -- here, 𝕃 is chosen -- using the other hand solely as a manipulator of the strings; a release is then effected in the usual way, wherein the string appears to "pass through" and come free of the laced hand. And while this complex of figures is also cited as an example of "Cutting off the Fingers" in several sources -- e.g. W.E. Roth: "String Figures, Tricks, and Puzzles of the Guiana Indians", No. 710 -- we feel that it is distinct enough from the previous complex of figures to warrant its own, individual classification. In particular, the complex relationship between the hands in the various intermediate string-positions of the construction have a direct bearing on the description of highly-asymmetrical String-Figures not enjoyed by the previous complex of tricks. We shall discuss this in some detail, below, at the propitious time.

Variation 1 [F. Mindt-Paturi: "Three String Tricks from the Wolof Tribe in Senegal", page 10, "Index and Middle Finger Trick".]

Construction:

I. Hold the loop with HR2345 so that the back of R is uppermost, fingers (knuckles) pointing left, the long hꝏ depending from the fingers: # (L):: R draws the hꝏ left until its lowermost string engages L as a L2345p-string; continue passing ℝꝏ back over the 𝕃-fingers -- the near string passing to the near side of L2, the far string to the far side of L5: now draw Rꝏ back over the L-fingers, the near string passing between L2*L3, the far string between L4*L5 |: draw Rꝏ towards you between L1*L2 and around L1, then pass it once more over the L-fingers (to the back of 𝕃), the near string passing between L2*L3, the far string between L4*L5: finally draw this loop directly back over the L-fingers (to the front of 𝕃), the near string passing to the near side of L2, the far string passing to the far side of L5: draw Rꝏ away to far side of figure (to get it out of the way) and □ R [Fig. 142.A].

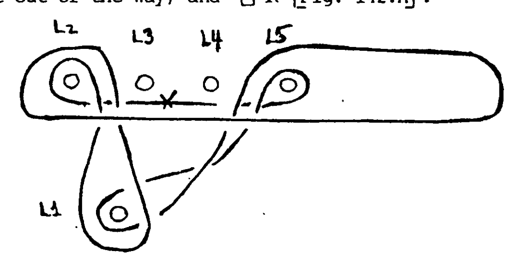


Fig. 142.A: Cutting the Hand, I.

Continue,

II. Remove L1ꝏ$^{(2)}$, and fold them directly up and back between L3*L4, where they are securely held.

To "Cut the Hand', seize up the Lp (at X) between R1*R2, and pull this string firmly away from IL; all strings will come free of L.

An effective alternate to Step II, above, is afforded by Step II', [see J. Elffers and M. Schuyt: Cat's Cradles and Other String Figures, page 43, "The Smith's Secret".]

II'. HR3↓(L1ꝏ$^{(2)}$):□L1|

This gives Fig. 142.B, below.

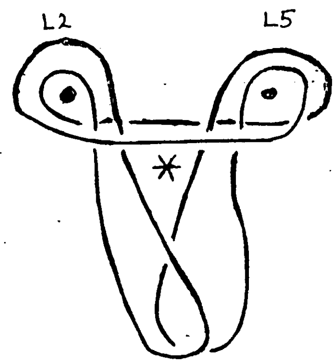


Fig. 142.B: Cutting the Hand, II.

Continue,

(II'). Pass Rꝏ back over L-fingers, the (two) near strings passing between / L2*L3, the (two) far strings between L4*L5, and L3, L4 -- themselves -- passing into the triangle (at ✱) between these two pairs of strings.

The release is executed exactly as in the parent figure. Before proceeding to a general discussion of the hand-positions involved in the construction, we give a variant of Step I which -- while functionally identical thereto -- proceeds from a very different perspective. [See T. Saito: Ayatori Itotori, III, Trick No. 8].

I'. O.1(L): L1ꝏ→L2: HR23(L34p)|: L1(L5n & L2f)#(L1): L5↑(HR23ꝏ)
#(L5): >R23(#):R23ꝏ⇒L2: HR3↓(L1ꝏ$^{(2)}$):□L1|

This gives the string-position of Fig. 142.B.

Let us now perform the following gedanken-experiment on Step I of the original construction: visualize an imaginary deck of cards, on each of which appears only a representation of both hands and the loop of string -- that is, no arms, body, direction of gravity, etc. The first card illustrates the opening position of Step I; L(#), the loop held in HR2345 some distance from L. The second card shows IR having moved one inch towards L, preparatory to drawing HR2345ꝏ over the back of L. The third card shows IR having moved another inch -- and so on, through the manipulations of Step I. The last card shows the hands supporting a string-position whose schema is given in Fig. 142.A. Thus, the deck of cards shows the stop-action motion of Step I, with space discretized to a fundamental unit of one-inch width; in particular, when this deck is riffled -- front to back -- we shall perceive a fairly detailed (if somewhat "jerky") execution of Step I. We remark that, during this riffling-process,. the left hand remains essentially stationary -- suffering only momentary flexions ("deformations") to allow the various passages of the string being worked about it by the right hand. We shall call this ordered collection of cards, "Deck A".

We now construct a second deck -- Deck B -- from Deck A, as follows. Deck B has the same first card as Deck A. To get the second card of Deck B, flip one card of Deck A -- by translation and rotation, make R's hand-position on card 2 of Deck A coincide with IR's hand-position on card 1 of Deck A (or Deck B). This (or a "snapshot" thereof) is card 2 of Deck B.. In general, for the generic natural number, n, to get card n of Deck B we take card n of Deck A (if any) and-by translation and rotation -- make IR's hand-position on card n of Deck A coinci with R's hand-position on card 1 of Deck A; this (or a "snapshot" thereof) is ca n of Deck B: We stipulate that Decks A and B have the same cardinality. Note that, when the Deck B is riffled -- front to back -- the right hand stays, essentially, stationary, and the string is, apparently, worked by the left hand durinc the stop-action progress of Step I. But these two decks represent the same movement in three-dimensional space, viewed from two different perspectives!

We shall refer to the perspective of this motion afforded by (the riffling of) Deck A as the IL-Dual of that afforded by Deck B; by symmetry, we refer to the Deck B-perspective of the motion afforded by Deck A as the IR-Dual thereof.

Let us now take a somewhat closer look at the motion afforded by riffling tl the Deck B; i.e. at the $\mathbb{R}$-Dual of Step I. We find

I. (Deck B): Hold loop in HR2345 with back of R up, fingers left, and the long h∞ depending from the R-fingers: $\overrightarrow{HL2345}\downarrow$(RH∞) :: >L2: <L5:#(L23) :: $\underrightarrow{L1}$(L5n & L2f) #(L1) :: $\overrightarrow{HL34}\downarrow$(R∞) : $\underrightarrow{L2}\uparrow$(HL34∞) : $\underleftarrow{L5}\uparrow$(HL34∞) :□HL34#(L) :: draw R∞ away to far side of figure: □R.

The resulting description, above, is far more concise than the original (for its $\mathbb{L}$-Dual) in that L performs movements entirely ordinary for the String-Figure Calculus, and so admits immediate, easy encoding therein. We shall, henceforth, take advantage of this "Dual"-perspectivity by introducing the following notational device:

$\mathfrak{D}_{\mathbb{L}}$[...] — perform the $\mathbb{L}$-Dual of the bracketed (instruction-) sequence.

In particular, L will remain stationary -- in whatever initial position it finds itself at the start of the indicated movement -- throughout the subsequent manipulations. The manipulations, themselves -- the expressions in "[...]"-- will, of course, indicate movements for $\mathbb{L}$; these are to be "mentally translated" into the L-Dual perspective, wherein $\mathbb{L}$ remains stationary. For completeness, we define the symmetric construct

$\mathfrak{D}_{\mathbb{R}}$[...] — perform the R-Dual of the bracketed (instruction-) sequence.

Here, then, is the original construction for "Cutting the Hand", employing these concepts:

I. Hold loop in HR2345 with back of $\mathbb{R}$ up, fingers left, and h∞ depending from $\mathbb{R}$-fingers:
$\mathfrak{D}_{\mathbb{L}}$[$\overrightarrow{HL2345}\downarrow$(RH∞) :: >L2: <L5:#(L23) :: $\underrightarrow{L1}$(L5n & L2f)#(L1) :: $\overrightarrow{HL34}\downarrow$(R∞) : $\underrightarrow{L2}\uparrow$(HL34∞) : $\underleftarrow{L5}\uparrow$(HL34∞) :□HL34#(L)]: draw R∞ away to far side of figure: □ $\mathbb{R}$

II. $\mathfrak{D}_{\mathbb{L}}$[$\overleftarrow{HL3*L4}$($\overline{L1\infty}^{(2)}$) :□L1#(L)].

To "Cut the Hand"

$\overleftarrow{R1*R2}$(Lp) I

All strings will come free of L.

We shall employ the "$\mathbb{L}$/R-Dual" concept in the figure below.

Variation 2: [A.C. Haddon: "String Figures and Trick", No. 28, "Au Mokeis (the Rat)".]

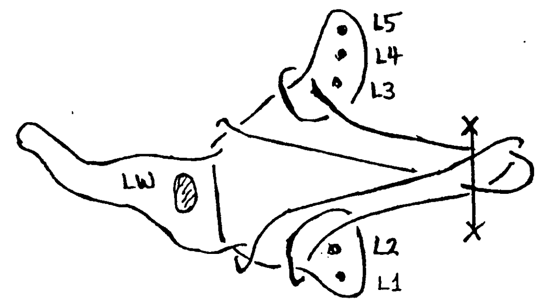


Fig. 143: The Rat.

Construction:

Hold 12" section of string in $\mathbb{R}$ as a h∞, facing $\mathbb{L}$: $\mathfrak{D}_{\mathbf{L}}$[$\overrightarrow{H\mathbb{L}}\downarrow$(RH∞) : >L2: <L3#(L)] : $\overleftarrow{R2}\downarrow$($\mathbb{R}$∞) : $\underleftarrow{HR2}$($\overline{Lp}$)#(R2) | : $\mathfrak{D}_{\mathbb{L}}$[$\overrightarrow{HL}\downarrow$(R2∞) : >L2: <L3#(L) : $\underrightarrow{L1}$(L2f & L3n)# (L1) : $\overrightarrow{HL2*L3}$(R2∞) : $\underrightarrow{L}\uparrow$(R2∞) # (L)] : thread R2∞, from above, down behind lower Rp-string, and back to R2: $\mathfrak{D}_{\mathbf{L}}$[$\overrightarrow{\mathbb{L}}\downarrow$(R2∞)] :□ R2: $\mathfrak{D}'_{\mathbf{L}}$[$\overleftarrow{HL2*L3}$(L1∞$^{(2)}$) :□ L1 # (L)].

This gives Fig. 143, above -- except that the two L2*L3 loops have not been folded back between L2*L3 (at ✱) in the interests of diagrammatic clarity. This having been done, the two small loops protruding dorsally between L2*L3 are interpreted as the Rat's "ears" -- the whole construct of strings on the left hand, with fingers slightly curved, being a really evocative representation of such a rodent.

The release is effected by seizing the left-hand dorsal h∞ in $\mathbb{R}$ -- left palm facing body -- and pulling this string directly away from $\mathbb{L}$. All strings will come free of L with an amazing "scurrying" motion, round and about the hand.

We remark, again, that because of the fine representational aspect of the final string-position on $\mathbb{L}$, we consider this release to be an example of a String-Figure, rather than a String Trick, per se, [See discussion, page 232].

Comparison Figure 3: THE SEA SNAKE [K. Haddon: Cat's Cradles from Many Lands, No. 9, "Pagi".]

We close the present subsection with a String-Figure whose final extension displays a "motion" every bit as reminiscent of the animal it seeks to represent as that met with in the previous figure. Of course, the two various motions -- like the two types of creatures involved -- are very distinct, each being brilliant in its own right. Their juxtaposition provides eloquent testamonial to the representational versatility of the String-Figure medium.

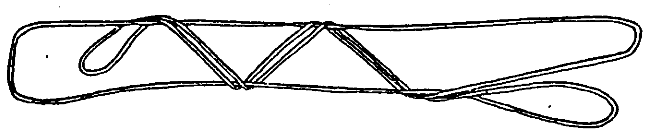


Fig. 144: The Sea Snake.

Construction:

O.A:: pass R away from you, around behind L, then to the near side of L, and back to # (R) :: ⇉L↓(R2∞):□R2: unwrap R from L, retracing its circuit about L, back to #(R)::□L2|:⇉L2(Rp)#□LW∞Ɪ□L1: ⇇L2∞→L1|: ⇉2(5f)#:□L1 (gently): →H1(5n) [Fig. 144]:Ɪ (widely).

... and the Sea Snake slithers off in characteristic fashion.

4. THE PILOT FISH [A.C. Haddon: "String Figures and Tricks", No. 29, "Zermoi".]

Construction:

Place the string on L3 as a simple loop, the long hoo depending from its palmar aspect: seize L3d-string and pull this out, until equal loops hang down from both its palmar and dorsal aspects: seizing up the L3p-h∞ at its nethermost point, turn this directly up and hang it on (the back of) L5: now pull this L5d-string across the back of L, and allow it to depend -- as a h∞ -- from between L1*L2: draw the L3d-h∞ directly up over the tip of L3, and release it to become a second h∞ on the palmar aspect of L: now, seizing up L4n near the base of L4, push about 2" of this string up and through the L5∞, and bring the small loop so formed towards you, finally placing it over L3n: reach down through this small loop and seize up L3n -- near the base of L3 -- and pull up about 2" of this string through the small loop, and bring the (new) small loop so formed towards you, finally placing it over uL2n: similarly pull up 2" of uL2n, draw it towards you as a (new)

loop, and place it over ℓL2n: finally, pull up 2" of this latter string and place it, as a loop, directly onto L1.

This gives Fig. 145, below; both front and back views of L are illustrated.

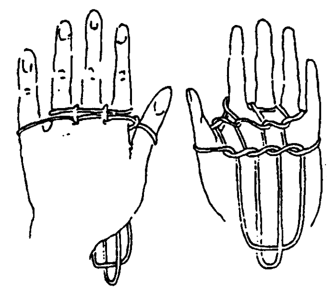


Fig. 145: The Pilot Fish.

The release proceeds in three stages.

With R, pull on the Lp-h∞'s.

1. □L1 Ɪ.

2. □R::□L5: with R pull L2n(2) until the slack from □L5 loop is absorbed.

3. □L3: pull L2n(2) firmly out Ɪ.

All strings come free of L.

This concludes our look at V. RELEASES as String Tricks. The final major subcategory of String Tricks which we shall consider is that of VI. ILLUSIONS.

VI. ILLUSIONS

The String Tricks of this section are characterizable as direct attempts to engender sensory deception through the visual realization of the "plausible impossible". Thus, for example, closed loops are magically threaded and inelastic strings are stretched. And, unlike IV. SWINDLES -- whose aim is to create in onlooker a "profitable" confusion -- the motive here is pleasant amazement for all concerned.

1. THREADING A CLOSED LOOP [B. Sutton-Smith: "The Folkgames of Children", page, 184, "Threading the Needle".]

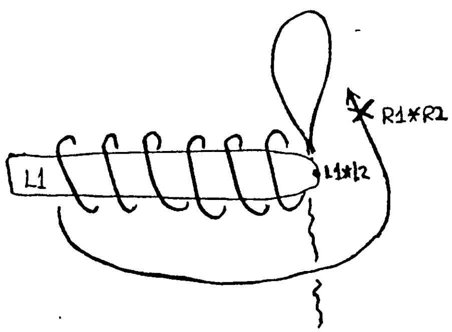


Fig.146: Threading a Closed Loop.

In this construction, as in several previous figures, the String-Figure loop is doubled to approximate a straight, single length of string [this is not shown in Fig. 146, above].

Construction:

Leaving about a 10-inch "tail", begin wrapping the string about L1 -- starting at the base of the digit. Begin by passing string under L1, then over-- and to the near side of -- L1; then under again ... and so on, making at least six turns about the thumb, proceeding from base to tip. The last turn of the string about L1 should be under, and to the far side of that digit, where the remaining string is turned directly down upon itself -- to form a small, upright loop; this is then held pinched between the tips of L1*L2.

This gives Fig. 146; the small, upright loop there pictured is the "closed loop" of the figure's title. To thread this "closed loop", seize up the 10-inch "tail" at the base of L1 between R1*R2 at its terminus, and make several feints at the "closed loop" -- as if trying unsuccessfully to pass the tail through this standing loop. Suddenly slip the tail-string round the tip of L1 -- while making a vain stabbing-motion in the general direction of the "closed loop" -- and it will appear as if the loop has been magically threaded by the "tail" held in the right hand. The trick may be immediately repeated, but notice (as onlooker eventually will) that each threading of the "closed loop" is accompanied by a lost turn in the winding about L1.

2. THE ELASTIC BAND [T. Saito: Ayatori itotori, vol 1, No. 8, "Iron Bridge, Turtle, Rubber Band, Yo-Yo".]

This serial String-Figure lists a version of one of the world's best-known, most widely-distributed string-illusions as its penultimate design.

Construction:

$$\underline{O}.JA: \underrightarrow{1}(\underline{3n})\#\underleftarrow{5}(\underline{3f})\#|:\overrightarrow{1}(\underline{5n})\#\square 5|:\overleftarrow{5}(\underline{1f}^{(2)})\#\square 1|:\overrightarrow{1}(\underline{5n}^{(2)})\#|:$$
$$\underline{O}.JA(\text{with } p^{(2)}):(\ell 3\infty)N(u3\infty^{(2)})\mathrm{I}.$$

This gives Fig. 147.A, below.

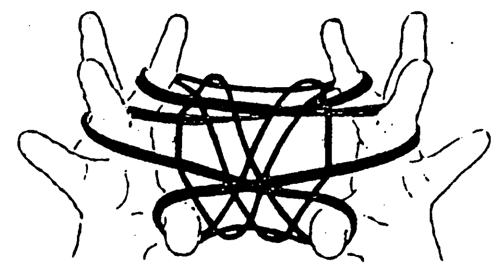


Fig. 147.A: Iron Bridge.

Continue,

$$\overrightarrow{H3}(\overline{o};3\infty^{(2)})[\sqcup\text{former } 3\infty^{(2)}]\mathrm{I}.$$

This gives Fig. 147.B, below.

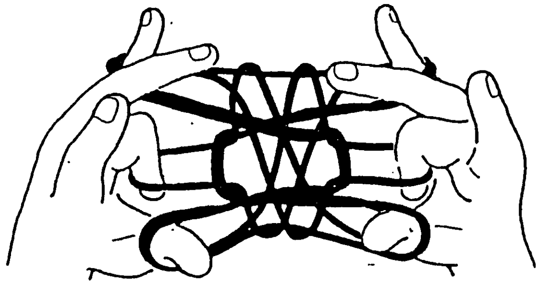


Fig. 147.B: Turtle.

Continue,

□H3 # | (so that central strings separate).

This gives the string-position of Fig. 147.C, below.

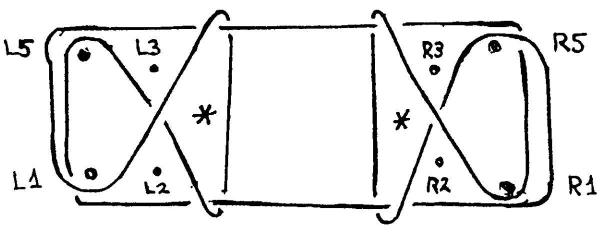


Fig. 147.C: Intermediate position.

Continue,

Insert 2,3 into central configuration, as indicated, and bring them directly up -- on their respective sides -- at *.

This gives Fig. 147.D, below.

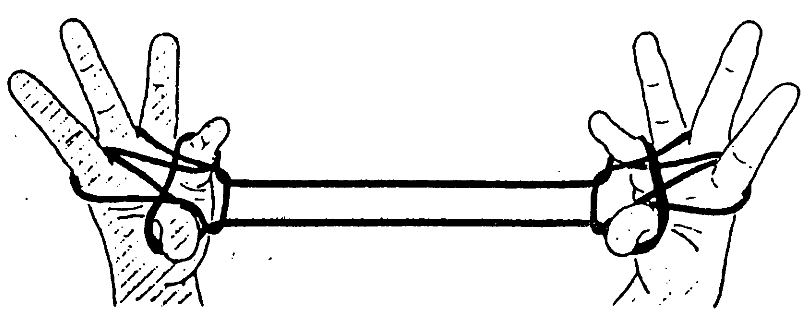


Fig. 147.D: The Elastic Band.

By alternately closing together and widely separating 1, 5 on each hand, individually, the string between the hands will appear to lengthen and shorten -- "stretching" like a rubber-band.

Continue,

□1:□2:□3|: 1→(5n-c) # O.JA: throw the hoo towards you over all strings, then away from you under all strings, and once more towards you over all strings again -- finally picking it up with M I.

This gives Fig. 147.E, below.

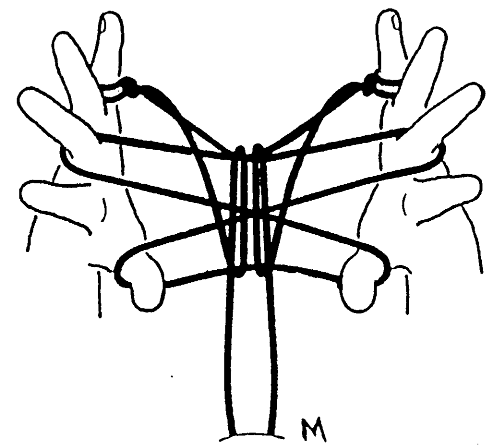


Fig. 147.E: The Yo-Yo.

Now, by alternately stretching hands away from the mouth and returning them close thereto again, some flavor of the up-and-down motion of the yo-yo is captured.(6) We remark that the string-position of Fig. 147.C may be produced by the sequence

O.JA: 1→(3n) # 5←(3f) # |: 1→(5n) # □5 |: 5←(1f$^{(2)}$) # □1 |: 1→(5n$^{(2)}$) # □3 |,

from which the Elastic Band is constructed in a single further move [cf. the manipulation following Fig. 147.C]. Other versions of this illusion are only slightly simpler (as to construction), but are materially less satisfying in their final effect [see e.g. E.S. Rogers: "Material Culture of the Mistassini page 20.]

3. MOON SHADOW; It is the final configuration in this contemporary String Trick which constitutes the "illusion", and there are many ways to produce this string-position. We have chosen one such for presentation here; the construction is divided into three parts.

Construction, I [T. Saito: Ayatori Itotori, Vol. 2, No. 6, "Moonrise on Fujisan".]

O.1: 5∞⟶2 :: H1(2f) : >1(#) | : 1(2n) # 2(1f) # | : R2(L12p) # L2↓(uR2∞) : L2(R12p) # | : (ℓ2∞^(2))N2: 2(1f-s) # | : □L :: from near side of R, pass L1↑(R1∞^(2)) : from far side of R, pass L2↑(R2∞^(2)) # | :: □12d-string from 2 (over) | : 5(f-x) [formed by both (single) 2f-strings] # □1: 2∞⟶1].

This gives Fig. 148.A, below.

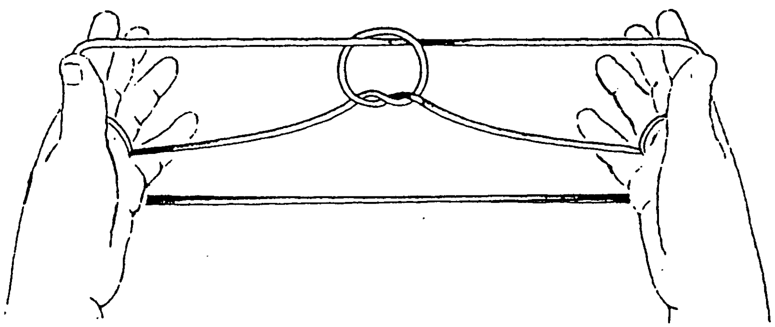


Fig. 148.A: Moonrise on Fujisan.

Continue,

□5 | (gently): lay flat (fingers pointing up).

This gives Fig. 148.B, below.

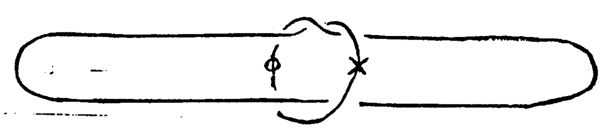


Fig. 148.B: Moon.

Construction, II:

L1*L2 pick up left central string (at ◐): R1*R2 pick up right central string (at ✗) | (gently): lay flat.

This gives Fig. 148.B, above, again. The above move may be repeated several times, for effect.

Then, continue,

Construction, III [cf. P.D. Noble: String Figures of Papua New Guinea, No. 51, "Uka Ma'u'ua".]

Pass 1's away under near transverse string of figure, and up into their respective sides of the long loop, thus picking the design up with a single loop on each 1: slide the small, central loop, to the right, near the base of R1: HRT(1f-c) | (gently): 5↑(1∞) # | (gently): O.A :: □1:□5: 2∞⟶O.1 # | (gently): O.A' :: □1:□5: 2∞⟶ O.1 # | (gently) ... et cetera.

This gives the alternating pair of figures of Fig. 148.C, below.

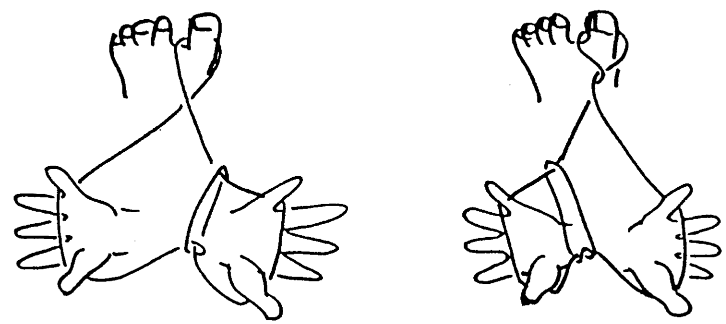


Fig. 148.C: Moon Shadow.

If, at any stage of the construction (III), the small loop representing the Moon [right -- or its "shadow", left] closes-up, it may be easily enlarged by inserting two fingers into it, and separating them until the ideal dimensions are realized. We remark that, although this is an extremely simple illusion, it is surely a most effective one, which invariably elicits comments/questions from the onlooker. When performed smoothly -- with just the right string-tensions -- the "Moon" appears to glide smoothly from one extreme lateral position to the other, while "rotating" about its vertical axis: Truly a beautiful effect, well worth the practice required for its achievement!

4. INTERLOCKING LOOPS [J.C. Andersen: Maori String Figures, Trick no. 9.]

This final example of "Illusions" is one of the most conceptually sophisticated effects recorded in the String-Figure/Trick literature, as it single-handedly virtually demolishes the tempting premise that "loops" are the fundamental concept for string-designs.

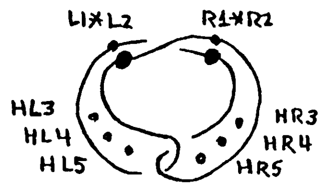


Fig. 149.A: Interlocking Loops, I.

Construction:

0.1: □5 :: R345↑(R1∞) : □R1 :: HL345↓(L1∞) : □L1 | : HR345 (R345n) :: bring B towards each other, passing R to the near side of L: grasp upper strings (the 3n-strings) between 1*2 as in Fig. 149.A, above.

This gives a pair of interlocked loops at the bottom of the figure.

Note: The principal strings held between 1*2 are indicated by black nodes; the strings temporarily held (to display figure) are indicated by red nodes. These latter strings are to be released whenever the upper loops are separated.

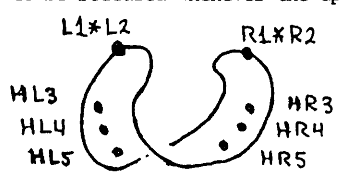


Fig. 149.B: Interlocking Loops, II.

Continue,

Separate upper loops, drawing hands slightly apart [Fig. 149.B]:: now bring B towards each other, this time passing R to far side of L: grasp upper strings between 1*2 as in Fig. 149.C, below.

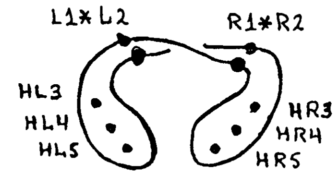


Fig. 149.C: Interlocking Loops, III.

Note that, at this point, two lower "loops" appear to have separated -- the rightmost one having been to the near side of the leftmost one -- the two upper "loops" having vanished.

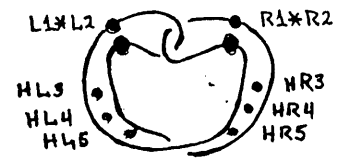


Fig. 149.D: Interlocking Loops, IV.

Continue,

Bring B towards each other, and pass the R lower loop to the far side of the L lower loop: grasp upper strings between 1*2 as in Fig. 149.D, above.

At this point it will be noticed that the two upper "loops" have reappeared, interlocked one with the other. The lower "loops" are no longer in evidence. The whole sequence of steps, above, may be retraced, in inverse order, until the starting-position of Fig. 149.A is reached, et cetera. (7)

This concludes our discussion of String Tricks from the category of VI. ILLUSIONS. Another example may be found in the "Trick Sequence" which concludes the section. We remark the diversity and mutual dissimilarity of one trick to another that marks the four examples of the present section -- surely greater than that encountered in any of the other five subcategories of String Tricks. This is not entirely accidental as, indeed, one of our intentions is to indicate the breadth of this denomination. Their sole common link -- except, perhaps, for the large amount of practice required to master the smooth production of each illusion -- is that each, in its own highly individualized way, presents a sensory realization of the "plausible impossible"; and, for the author that is what it is to be an "illusion".

The six subcategories of String Tricks we have imposed are largely matters of developmental expedience and -- looking back over the 70 or so individual examples, given above -- we see that the "structure" of String Tricks emerges more as a "tapestry" of interconnections, rather than as six mutually exclusive boxes of unrelated individuals. An equally interesting grouping into "series" of tricks -- analogous to "Serial String-Figures" -- is also readily employed by the skillful, knowledgeable performer. Here, individual tricks are not the fundamental items of memory but, rather, ordered sequences of tricks whose egress from one flows naturally to the ingress of the next -- thus binding them into a "Serial String Trick". Occasionally, such a serial String Trick has an accompanying story [see e.g. A. Pellowski: The Story Vine, pages 22-30, "Lizard and Snake".]; more often they are well-loved series used to "warm up the string" when first it is removed from performer's pocket, or to "keep it warm" during the interval between String-Figures. As an example of the above alternate grouping, one of the author's favorite serial String Tricks is presented below.

Serial String Trick: This series is composed of three individual tricks; I. an Illusion, II. a derivative of a Swindle, and III. a Single-Functor Release.

Construction, I [T. Saito: Ayatori Itotori, Vol. III, No. 14, "String Transfer".]

Hang string as a simple loop on L1, the long h∞ depending from the palmar side: seize up h∞ at its nethermost point between R1*R2 (R1 above R2): < R1*R2: spreading L1, L2, bring h∞ directly up between them, so that R1*R2∞ protrudes some 4 inches above their dorsal aspect: now bend R1*R2∞ directly down over L1, L2 -- so that the near string thereof passes to the near side of L1, the far string to the far side of L2].

The loop on IL has transferred from L1 to L2. In like manner, the loop may be transferred from L2 to L3, and so on, fluidly "walking" down the hand; and back again (reversing the direction of twist on R1*R2 in the above construction). For definiteness in what follows, let us suppose that -- having "walked" the L-loop up and down the hand several times -- we have attained the position described in the opening of Construction, II, below.

Construction, II [H. Noguchi: "String Figures in Japan, IV", No. 105, "Changing Loop from Index to Middle Finger".]

Hang string as a simple loop on L2, the long h∞ being held at its nethermost point between R1*R2 (R2 above R1): identify the L2n-string, and keep this string

uppermost during the following manipulation; wrap the L2∞ about L2, L3 by circling L2, L3 clockwise with the loop held between R1*R2: after two rounds of L2, L3 by R1*R2, perform a single round with the upper (L2n) string only: continue wrapping L2, L3 with both strings of the R1*R2∞ until only a small "tail" remains: immediately change directions and begin unwrapping the R1*R2∞ from about L2, L3: continue until a single loop remains about L3.

The loop on L has transferred from L2 to L3. We remark that the above construction is an "on-the-hands" version of the Swindle IV.1, Find the Center (see page 271). It will be found that it is natural to terminate these manipulations in the position detailed in the opening of Construction, III, below.

Construction, III [H. Noguchi: "String Figures in Japan, III", No. 72, "Middle Finger Slip Trick".]

Turn Lp away and hang string as a simple loop on L3, the long h∞ depending from the dorsal aspect to the near side of LW: pass R3 towards you and up into the L3d-h∞ (same orientation in this loop as L3) and turn R3p towards you |: now, simultaneously turn Lp towards you, wrap the R3∞ once around IL (including L1) in the counterclockwise direction, and <<R3: finally pull the R3∞ directly over the back of L3 and over onto it (as a second loop), and continue drawing R3, in its loop, to the bottom of the figure, over the $Lp^{(2)}$-strings □R3 momentarily, thread its loop -- from above -- down behind the $Lp^{(2)}$-strings, and replace R3 in this loop: □ $Lp^{(2)}$-strings: pull R3 away from L].

All strings will come free of IL. With practice, the three constructions given above will blend into one attractive series, whose performance will become almost "unconscious".

------- End String Tricks Discussion -------

STRING TRICKS NOTES

① (page 258). For the more sophisticated reader, who wishes to apply the concepts developed in conjunction with the presentation of I.B.3, The Elusive Knot, in the context of a concrete example — at a roughly commensurate level of difficulty — we offer the Kwakiutl Dissolving Knot [see J. Averkieva: Kwakiutl String Games, No. 111, "String Trick"]. The three diagrams, below, are self-explanatory and uniquely specify the construction, without the usual attendant verbiage.

I.B.6. THE KWAKIUTL DISSOLVING KNOT

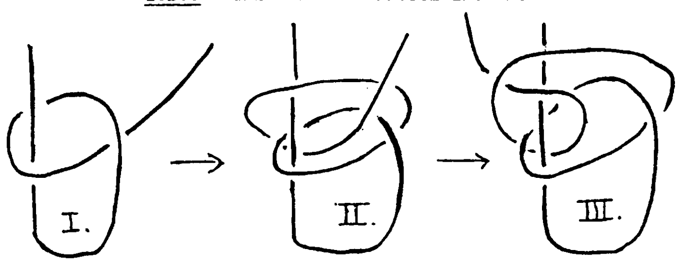


Fig. 108.P: Kwakiutl Dissolving Knot.

In particular, after becoming thoroughly familiar with the above construction, one should look for an alternate construction — performable in a String-Figure loop — write down the associated linear sequences, and develop the ∅-equivalence between these and the "empty" string.

② (page 267). A wonderful, but only marginally germane to String-Figures proper, piece of business comes to us from the province of "Close-up Magic", where it is a classic, intimately related to the Impossible Knot.

III.1 (variation): THE KNOT OF ENCHANTMENT [P. Abbott: Abbott's Encyclopedia of Rope Tricks for Magicians, pages 20-23.]

Construction:

I. Performer proffers a 1-meter length of rope, or string, for examination, and then has onlooker tie the ends, securely but not too tightly, about perfor-

mer's wrists.

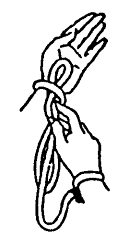
This gives Fig. 114.5.A, below.

Fig. 114.5.A: Knot of Enchantment, I.

Performer now causes a genuine knot to appear in the center of the rope between his hands, without disturbing the knots at either wrist, or removing the loop from either wrist. Here's how this is done.

II. Throw a small, upright loop in the center of the rope, passing R towards you over L — and then give this small loop an additional (180°-) twist. Now, with Lp down, fingers pointing away, thread this loop away under the rope ring about LW.

This gives Fig. 114.5.B, below.



Under

Fig. 114.5.B: Knot of Enchantment, II.

Continue,

III. As the small loop emerges on Lp, pass L completely through it and, reaching to the back of L, pass the strings of this loop under the rope on the back of L. Then, immediately, draw these strings over the rope on the back of L — and completely off the left hand.

This gives Fig. 114.5.C, below.

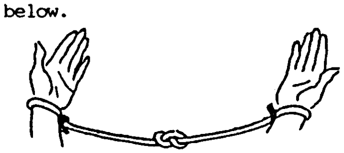


Fig. 114.5.C: Knot of Enchantment, III.

We remark that performer's attitude towards this trick should be one of, "Look, now i can't let go of the ends; they are securely tied to my wrists." Of course, topologically speaking, the present string configuration (Fig. 114.5.A) is radically different from a single, straight length of string; there is no "trick", here, as in the case of the Impossible Knot (which is impossible).

③ (page 270). String Trick III.4, Bear Trap, is closely related to another type of String Trick, not represented in our categorization. This is the Inuit "Hearth"-figure, "The Little Finger" [D. Jenness: Eskimo String Figures (C.A.E.), No. 15]. And, even though there are perhaps as many ways to construct this figure as there are for "Crows Feet" (pages 108-149, these notes), nevertheless, the introduction to The Little Finger we present here must be adjudged "non-standard".

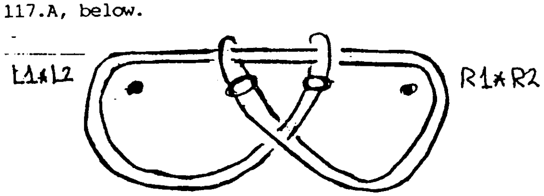
First note that the final string-design of the trick "Bear Trap" may be alternately constructed via the manipulation sequence

O.1: $\overrightarrow{15\infty}$ --> W:: bend B towards each other and down over both transverse strings, encircling these with the "ring" formed by putting the tips of 1*2 together: □ W: > 2(#) [thus picking-up a double-loop on 2, and □1]: $\overleftarrow{2\infty}^{(2)}$ -> 1 ⌶.

This gives the string-position of Fig. 117, the Bear Trap. If, now, we vary the above construction, slightly, viz.

O.1: $\overrightarrow{15\infty}$ -> W:: bend B towards each other and down over both transverse strings, encircling these with the 1*2-ring: □ W in such a manner that these loops cross -- right over left -- on the transverse strings.

This gives Fig. 117.A, below.



Under

Fig. 117.A: Bear Trap, alternate.

Continue,

Now, □1, pass 2 away and up, and pick up both strings of the central loop

depending from the double transverse strings (at ○) on their back: $\overleftarrow{H345}$↓$(2\infty^{(2)})$: □2 ⌶.

This gives the final design of The Little Finger, pictured in Fig. 117.B, below.

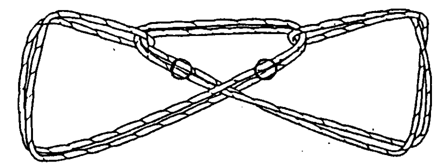


Fig. 117.B: The Little Finger.

To recover the Bear Trap from this figure, of course, one need only pass 1's away, under, and take up on their backs the double lateral strings of the central, inverted double triangle (at ○), releasing all other strings from the hands, and ⌶. This gives Fig. 117, again.

In the more normal course of events, one usually constructs The Little Finger -- as, for example --

O.A: $\overrightarrow{H4}$↓(5∞) :□5 : $\underleftarrow{H4}$ $(\overline{1n-s})$:□1 | : $\overleftarrow{H5}$↓(4∞) :□4 #(H5) | : $\overrightarrow{1}$(2n) : $\underrightarrow{1}$$(\underline{H5n}^{(2)})$ #(H5) :□2 | :: $\underrightarrow{2}$$(H5n^{(2)})$: $\overleftarrow{H2}$$(\bar{o}^{(2)}$:1∞) #(H2,5)□1:□H5 | : $\overleftarrow{H345}$↓(H2∞) ⌶

and then produces the Bear Trap -- by the above method -- as an amusing and somewhat surprising continuation of this figure.

As was mentioned earlier, The Little Finger is the "Hearth"-figure of a rich String-Figure cycle -- and so continuations and variations of it abound. Here is another, of deep, thought-provoking beauty.

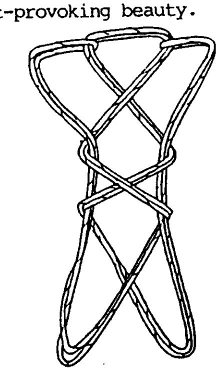


Fig. 117.C: The Little Finger Made Double.

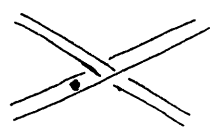
Construction [see D. Jenness: <u>Eskimo String Figures</u> (C.A.E.), No. 16]:

Make "Little Finger": $\underset{\rightarrow}{2}$(<u>o</u>$^{(2)}$; lateral strings of c-∇) (#) : $\overrightarrow{1}$(H3n$^{(2)}$) : $\underset{\Rightarrow}{1}$(<u>2f</u>$^{(2)}$) # (1) : K : pass 1 away, under, and pick up both strings of the central loop depending from 2n-s$^{(2)}$: □ 2: < 1ꝏ$^{(2)}$] (arrange figure as in Fig. 117.C).

Some practice is required to get the feel of the tensions needed in the intermediate string-positions of this construction in order to produce a "clean" final extension; once mastered, however, little final arrangement of the strings is necessary. To recover The Little Finger, either drop all but 1ꝏ$^{(2)}$, <u>or</u> drop all but H5ꝏ$^{(2)}$, and extend.

To explicate the manner in which The Little Finger may be used in a new type of String Trick, construct the figure -- as above -- so that it is extended between the hands, supported by H2345ꝏ$^{(2)}$ on either side. Now -- maintaining a firm tension in the strings -- perform the string-manipulation "<< H2345"; the central inverted double triangle will shrink (<u>i.e</u>. close-up) to a point. Similarly, perform ">> H2345", and the triangle returns to its original dimensions. Having remarked these facts, persuade onlooker to place his little finger into the central triangle, and then quickly close it up on him so that his little finger is tightly (!) trapped. The "trick" is -- for him to get away.

The secret of his escape is to found in the c-X$^{(2)}$ [formed by both H5f$^{(2)}$-strings] of the original figure.



<u>Fig. 117.D</u>: The Little Finger; c-X$^{(2)}$.

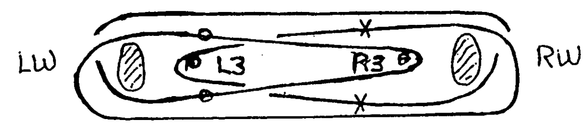
Both strings of the arm of this cross that are a continuation of HR5f$^{(2)}$ pass <u>inside</u> (<u>i.e.</u> between) the strings of the arm that are a continuation of HL5$^{(2)}$. Thus, no matter how tightly held is onlooker's little finger, he may always twist it around until he gains access to the space between the HL5f$^{(2)}$-strings (below the X, whose upper portion has closed, at •); then, thrusting his little finger down through this space and twisting it back round the central crossing, it will easily come clear of both strings.*

* <u>Note</u>: At some level, this may seem to be an example of a Single Functor Release; but the spirit is very distinct from the tricks met with in that section.

We close this note by observing that on several occasions (see, for example, D. Jenness: <u>Eskimo String Figures</u> (C.A.E.), page 26B) it has been remarked that an Inuit native -- upon being presented with the opening figure of the two-person game of Cat's Cradle -- took this figure off performer's hands in a manner that produced the String-Figure "Little Finger", much to performer's surprise.
<u>One</u> manner of accomplishing this is as follows:

<u>Player OC</u>: <u>O</u>.1: $\overrightarrow{15}$ꝏ→W: $\overleftarrow{B}$($\overline{Wn}$) # | : $\overleftarrow{R3}$ (<u>LWp</u>) # | : $\overrightarrow{L3}$↓(R3ꝏ) : $\underset{\Rightarrow}{L3}$ (<u>RWp</u>) #].

This gives Fig. 117.E, below.



<u>Fig. 117.E</u>: The Cat's Cradle (OC).

Player OC now presents the Cat's Cradle to Player [symbol], who takes the string off Player OC's hands as follows:

<u>Player [symbol]:</u> Approaching the proffered figure from above, pass L2 down into OC's R3ꝏ and hook up both continuations of OC's L3ꝏ (at ✗): similarly pass R2 down into OC's L3ꝏ and hook up both continuations of OC's R3ꝏ (at ✗): pull these H2ꝏ's out 2 inches above OC's respective 3ꝏ's and, rotating both 2's 180° in their respective loops, hook up both strings of OC's 3ꝏ (just passed through) on either side [thus □ [symbol]'s former H2ꝏ$^{(2)}$'s]: # (H2): $\overleftarrow{H345}$↓(H2ꝏ)].

When Player OC now relinquishes all strings, Player [symbol] is left with The Little Finger, Fig. 117.B.

④ (page 294). By now it will have been noticed that String Tricks in which a slapped, squashed, speared, or otherwise trapped creature gets away enjoy a wide popularity. Here is another example, of a type distinctive from others met with in these notes.

<u>V.B.12</u>. NAATI (Killing a Mosquito) [G. Landtmann: "Cat's Cradles of the Kiwai Papuans, British New Guinea", No. 1

$\underline{O}$.A: $\overrightarrow{M}$ and seize up all (six) transverse strings:: $\underline{L5}$↑(R5∞):□R5#(L5):: $\underline{L5}$↑(R2∞):□R2#(L5):: $\underline{L5}$↑(R1∞):□R1#(L5):: $\overrightarrow{L2\infty}$→L5: $\overrightarrow{L1\infty}$→L5|: $\overleftarrow{R1*R2}$(M∞$^{(6)}$):□M#|: <L5∞$^{(6)}$→M#|: $\underline{L2}$↑(R1*R2∞$^{(6)}$): M ⌶.

And L2 will come free of all strings. As L2 cuts through the released bundle of six loops, slap yourself in the forehead with the flat left hand (palm), as if killing a mosquito -- who has, of course, gotten cleanly away.

We remark that-- discounting figures in which the string is purposefully wound about a given functor -- this is the sole example known to the author of a String-Figure or Trick in which a single functor holds so many as six loops. The first time one sees such a thing, the natural reaction is to want to slap yourself in the forehead!

⑤ (page 298). An example somewhat related to the above, V.C.2, Var. 3, is included here, for completeness.

<u>V.C.2, Var. 6</u>: CUTTING THE FINGERS [S. Yuasa and T. Ariki: <u>Tanoshii Ayatori Asobi</u>, Trick No. 11.]

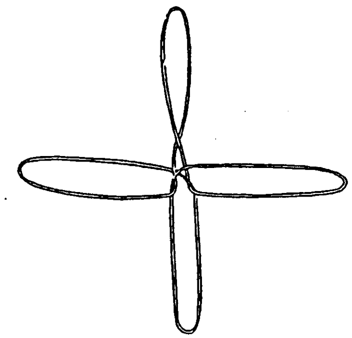
Construction:

$\underline{O}$.1:: $\overleftarrow{HR2}$($\overline{Lp}$): <R2(#)|: $\overleftarrow{L5\infty}$→L2:□R5|: <$\overleftarrow{2\infty}$→1ǂ|: $\overleftarrow{L2}$($\underline{L1n}^{(2)}$)#$\overleftarrow{L3}$($\underline{L1f}^{(2)}$)# $\overleftarrow{L4}$($\underline{L2f}^{(2)}$)#$\overleftarrow{L5}$($\underline{L3f}^{(2)}$)#|:□R:: $\overleftarrow{R1*R2}$($\underline{L1d}^{(2)}$) ⌶ (sharply).

All strings laced about the fingers of 𝕃 will come free.

We remark that the above variation of Cutting the Fingers also has a close affinity to the "one-handed" version of V.C.2, Var. 2, and may be thought of as a natural marriage of that variation and the present one (V.C.2, Var. 3).

⑥ (page 311). The distinctive <u>feeling</u> of "sawing" encountered in the action figures the Elastic Band (Fig. 147.D) and the Yo-Yo (Fig. 147.E) of this sequence is the compelling feature of another of the world's most widely distributed figures: Sawing Wood [<u>e.g</u>. K. Haddon: <u>Cat's Cradles from Many Lands</u>, No. 50.]



<u>Fig. 147.F</u>: Sawing Wood.

Construction: Two performers, 𝔄 and 𝔅, are required.

<u>Player 𝔄:</u> $\underline{O}$.A.

<u>Player 𝔅:</u> Seizing 𝔄's 5f between L1*L2, draw this string towards 𝔅 under all strings: now passing R away under all 𝔄's strings, seize up 𝔄's 1n between R1*R2, and draw this string towards 𝔅 under all strings ⌶

<u>Player 𝔄:</u> □1:□5 ⌶.

The two players now alternately separate their hands in imitation, and sensation, of sawing motions.

⑦ (page 315). Here we mention an illusion common to the discipline of "rubber-band magic" which is easily (and profitably) adapted to the string. It also has claims to membership in the categories of III. DO-AS-I-DO and V. RELEASES, as these are undeniable and cardinal aspects of its modal performance, but the overall effect of this trick roots it firmly in the category of VI. ILLUSIONS, to the author's best judgement.

<u>VI.5</u>: THE MAGIC GARTER [M. Gardner: "Mathematical Games: The Ancient Lore of String Play", page 149, "An Amusing Trick".]

Construction:

<u>I</u>. Fold String-Figure loop in half <u>three</u> times, to produce a small loop (on 8 strings) exactly one-eighth the size of the original loop (<u>i.e</u>. about 3 inches in diameter): insert indices, pointing towards one another, into this (simple) loop.

This gives Fig. 149.E, below.

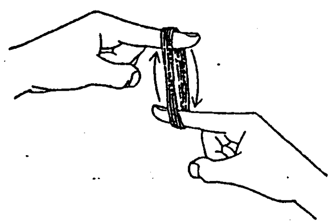


Fig. 149.E: The Magic Garter, I.

Continue,

II. Rotate hands around one another (either direction) several times -- keeping strings taut -- finally coming to rest with R (palm up) above L (palm down).

This gives Fig. 149.F, below.

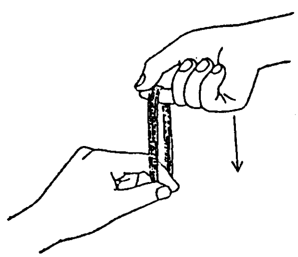


Fig. 149.F: The Magic Garter, II.

Continue,

III. Place tip of each 1 to tip of each 2: lower R directly to L and place tips of R1*R2 and L1*L2 together.

This gives Fig. 149.G, below.

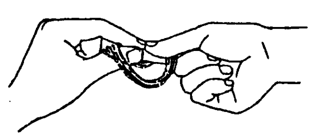


Fig. 149.G: The Magic Garter, III.

Continue,

IV. Now raise L2*R1 up and away from L1*R2, [Fig. 149.H, below], seize string with M, and pull it free of the hands.

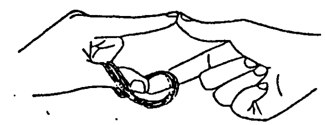


Fig. 149.H: The Magic Garter, IV.

Alternatively, maintaining intact the circle formed by fingers and thumbs, a slight forward toss of the hands will propel the loop free thereof, and out on the surface before you.

We remark that, as a "challenge"-figure, this is one of those String Tricks that will tolerate frequent repetition -- as most onlookers find it astonishingly difficult. In fact, even after many "demonstrations", most onlookers, for their turn, will still touch thumb to thumb and index to index at the final stage -- on which assumption release from whatever string-position they have achieved is impossible.

BIBLIOGRAPHY: STRING TRICKS

Abraham, A.J.: STRING FIGURES
- No. 2, "The Mouse, or, The Yam Thieves" (V.C.2, Var. 4)
- No. 3, "Navaho String Trick" (V.B.8)
- No. 4, "An African Hanging Trick" (V.B.7, Var. 2)
- No. 5, "The Disappearing Loops" (V.C.3, Var. 1)
- No. 8, "The Lizard" (III.2)
- No. 19, "The Sea Snake" (V.C.3, Var. 3)

Abraham, R.M.: DIVERSIONS AND PASTIMES
- No. 224, "Blowing a Knot on a Handkerchief" (I.A.3)
- No. 247, "String Release" (V.B.8)

Abraham, R.M.: EASY TO DO ENTERTAINMENTS AND DIVERSIONS
- No. 118, "Slipping the Lasso" (III.2)
- No. 119, "Crushing the Coconut" (cf. I.B.5)
- No. 120, "Tying the Knot" (cf. I.A.1 and Appendix A.2, Note (1), p.106)
- No. 121, "The Disappearing Loops" (V.C.3, Var. 1)
- No. 129, "Reversing the Loop" (III.3)
- No. 133, "The Fly on the Nose" (V.B.6)
- No. 136, "Cheating the Hangman" (V.B.7, Var. 2)
- No. 144, "Removing the Loop" (V.B.8)
- No. 145, "Trick o' the Loop" (IV.1)
- No. 146, "Threading the Needle" (VI.1)
- No. 127, "The Five Boy Scouts" (V.C.2, Var. 1)

Amir-Moéz, A.R.: MATHEMATICS AND STRING FIGURES
- Pp. 22-24, "Carrot Thieves and Induction" (V.C.2, Var. 4)

Andersen, J.C. MAORI STRING FIGURES
- No. 1, "Toemi" (cf. VI.2)
- No. 8, "Ikiiki, Comparison" (I.A.6)
- No. 10, "Komore" (V.B.11)
- Trick No. 1, "The German Trick" (V.C.3, Var. 1)
- Trick No. 2, "The African Hanging Trick" (V.B.7, Var. 2)
- Trick No. 3, "Fly on the Nose" (V.B.6)
- Trick No. 5, "The Button-Hole Trick" (V.B.7, Var. 4)
- Trick No. 6, "Tying a Knot" (I.A.5)
- Trick No. 7, "The Maori Mouse Trick" (cf. V.C.2, Var. 2)
- Trick No. 9, "The Interlocking Loops" (VI.4)
- Trick No. 10, "The Thumb Loop" (cf. V.B.8)
- Trick No. 11, "The Forefinger Loop" (V.B.10)
- Trick No. 12, "The Fore- and Middle-Finger Loop" (V.B.1)

Ariki, T.: AYATORI NYUMON
- Pp. 30-31, "Elastic Band, Turtle" (VI.2)
- p. 119, "The Saw" (cf. Note (6))
- p. 120, "Pounding Rice" (cf. Note (6))
- p. 133, "The Eel" (IV.4)

Ashley, C.W.: THE ASHLEY BOOK OF KNOTS
- No. 2534, "The Tom Fool's Knot" (I.A.2)
- No. 2536, "The Overhand Knot" (I.A.3)
- No. 2570, "Threading a Loop" (VI.1)
- No. 2571, "Rabbit in the Hole" (III.3)
- No. 2573, "An Elusive Knot" (I.B.3)
- No. 2576, "Puzzle" (cf. I.A.1 and Appendix A.2, Note (1), p. 106)
- No. 2581, "The Magical Knot" (I.A.4)
- No. 2593, "Snare(1)" (II.1)
- No. 2594, "Snare(2)" (II.2)
- No. 2596, "La Garrotte" (V.B.7, Var. 2)
- No. 2597, "Fingering the Nose" (cf. V.B.6)
- No. 2598-9, "Cutting the Fingers (1-2)" (V.C.3, Var. 1)
- No. 2600-1, "Cutting the Fingers (3-4)" (V.C.2, Var. 1)
- No. 2602, "Cutting the Fingers (5)" (V.C.2, Var. 4)
- No. 2603, Un-named (V.B.8)
- No. 2604, "Cutting off the Chair Knob" (V.B.10)
- No. 2605, "String from Buttonhole" (V.B.7, Var. 4)

Averkieva, J.: KWAKIUTL STRING GAMES
- No. 1, "A Fire Drill" (Note (3), Fig. 117.B)
- No. ~~102~~, 103 "A Catch" (cf. V.B.7, Var. 3)
- No. ~~103~~, 105 "Cutting off the Fingers" (V.C.2, Var. 5)
- No. 105, "Cutting off the Fingers" (V.C.2, Var. 1) ?
- No. ~~106~~, 107 "Cutting off the Fingers" (V.C.2, Var. 2)
- No. ~~107~~, 10a, Un-named (V.C.3, Var. 1)
- No. 109, "Threading a Closed Loop" (VI.1)
- No. 110, Un-named (V.B.2)
- No. 112, "Cutting off the Wrist" (V.B.5)

Ball, W.W. Rouse: FUN WITH STRING FIGURES
- No. 17, "The Mosquito or Fly" (I.B.5)
- No. 19, "The Elusive Loop" (V.C.3, Var. 1)
- No. 21, "The Yam Thief" (V.C.2, Var. 4)

Barrett, S.A. and E.W. Gifford: MIWOK MATERIAL CULTURE
- Plate Lxix, "Gopher" (V.C.2, Var. 4)

Bauer, C.F.: HANDBOOK FOR STORYTELLERS
- P. 176, "Ring in the String" (cf. V.B.8)

Beaglehole, P.: STRING FIGURES FROM PUKAPUKA
- No. 24, "The Gentle Wind (2)" (cf. V.B.11)
- No. 54, "The Yellow Anus" (cf. VI.2)
- P. 27, "Yele Moa, Tie the Chicken" (V.B.7, Var. 2)
- P. 27, "Hand Slip Trick, Nameless" (III.2)

Beart, C.: JEUX ET JOUETS DE L'OUEST AFRICAIN, I.
- Fig. 294, "The Mysterious Loop" (cf. V.C.2)

Brewster, P.G.: STRING FIGURES AND TRICKS FROM THE UNITED STATES
- Fig. 10(A), "Cutting off Fingers (A)" (cf. V.C.2, Var. 1)
- Fig. 10(B), "Cutting off Fingers (B)" (V.C.3, Var. 1)
- Fig. 11, "Cutting off Head" (V.B.7, Var. 4)

Buck, P.H.: SAMOAM MATERIAL CULTURE
P. 360, "Mongamonga" (V.A.7)

Budworth, G.: THE KNOT BOOK
Fig. 96, "Overhand Knots Galore" (V.A.4)
Fig. 98, "Threading the Needle" (VI.1)
Fig. 99, "Unfolding a Knot" (cf. I.A.1 and Appendix A.2, Note (1), p. 106)
Fig. 100, "A Disappearing Knot" (I.B.3)
Fig. 102, "Find the Middle" (IV.1)
Fig. 103, "The Finger Trap" (II.1)
Fig. 104, "Free the Ring" (cf. V.B.7, Var.4)

Centner, T.H.: L'ENFANT AFRICAIN ET SES JEUX
Fig. 199, "String-Problem No. 1: The Sorcerer and the Prisoner", (V.B.7, Var. 7)

Compton, R.H.: STRING FIGURES FROM NEW CALEDONIA AND THE LOYALTY ISLANDS
No. 5, "Nameless" (cf.V.B.11)
No. 14, "The Sugar Cane" (I.A.6)
No. 21, "A Fly" (cf. I.B.5)
No. 22, "Uprooting Maniania" (V.C.2, Var.4)
No. 23, "Putting on the Basket" (V.B.7, Var.2)
No. 24, "Nameless" (cf.I.B.4)
No. 25, "A Lifu Trick" (cf.V.B.8)

Culin, S.: GAMES OF THE NORTH AMERICAN INDIANS
P. 762, Un-named (V.B.7)
P. 762, "String Wrapped in and out Among the Fingers" (V.C.2)
Fig. 1059, "Sawing Wood" (cf. Note (6))

Culin, S.: GAMES OF THE ORIENT
P. 30, "Sawing Wood" (cf. Note (6))

Cunnington, W.A.: STRING FIGURES AND TRICKS FROM CENTRAL AFRICA
No. 2, "A String Trick" (V.B.7, Var.2)
No. 14, "A String Trick" (V.C.2, Var.4)
No. 17, "Tambukira" (cf. Serial String Trick, I, p. 316)
No. 18, "A String Trick" (V.B.4)
No. 19, "Nzige, a locust" (I.B.5)

Cunnington, W.A.: STRING TRICKS FROM EGYPT
No. 1, Un-named (V.B.7, Var.7)
No. 3, Un-named (cf. V.B.5)

Davidson, D.S.: SOME STRING TRICKS OF THE VIRGINIA INDIANS
Fig. 144, "Threading the Needle" (VI.1)

Davidson, D.S.: ABORIGINAL AUSTRALIAN STRING FIGURES
No. 3, "Two Water Holes" (IV.5)
No. 8, "Big River Bat" (cf. I.B.5)
No. 69, "Cutting off the Head" (V.B.7, Var.1)
No. 70, "Hanging" (V.B.7, Var.2)
No. 71, "Bullocky" (V.C.2, Var.4)
No. 72, "Cutting off the Fingers" (V.C.3, Var.1)
No. 73, "Cutting off the Nose" (V.B.6)
No. 74, "Threading the Needle" (VI.1)

Devonshire, C.W.: RAROTONGAN STRING FIGURES
No. 4, "Tunaeka" (cf.V.B.11)

Dickey, L.A.: STRING FIGURES FROM HAWAII
Pp. 118-120, "Okole-amo, A-C" (cf. VI.2)
P. 150, "Hand Slip Trick A" (V.A.5)
P. 150, "Hand Slip Trick B" (V.C.3, Var. 1)
P. 151, "Hand Slip Trick C" (V.C.2, Var. 2)
Pp 151-2, "Kele-opa" (V.C.2, Var.4)
Pp. 152-3, "E Loli E" (III.2)
P. 153, "Knot Slip Trick" (cf. I.B.5)
P. 154, "Wrist Slip Trick" (cf. V.B.7, Var. 2)
P. 154, "Thumb Slip Trick A" (V.A.1)
P. 154, "Thumb Slip Trick B" (V.A.2)
P. 155, "Index Slip Trick" (II.1)
Pp. 155-6, "Middle Finger Slip Trick" (Serial String Trick, III, p. 317
P. 156, "Changing Loop from Index to Middle Finger" (Serial String Trick, II, p. 316-317)
P. 157, "Toe Slip Trick B" (V.B.8)
P. 157, "Toe Slip Trick C" (cf. V.B.7, Var. 6)

Earthy, E.D.: VALENGA WOMEN
P. 101, "Tinyanyani" (I.B.4)
P. 101, "Kamba" (V.B.7, Var. 2)

Edge-Partington, J. and C. Heape: AN ALBUM OF THE WEAPONS, TOOLS, ORNAMENTS, ETC., OF THE NATIVES OF THE PACIFIC ISLANDS
Series I, Plate 341, No. 3, "A Water Snake" (V.C.3, Var. 3)

Elffers, J. and M. Schuyt: CAT'S CRADLES AND OTHER STRING FIGURES
P. 33, "String Around Your Neck and Pull" (V.B.7, Var. 2)
P. 38, "The Thumb-Catcher" (III.4)
Pp. 38-9, "The Fly" (cf. I.B.5)
P. 43, "The Smith's Secret 1" (V.C.3, Var. 1)
Pp. 44-5, "The Mouse" (V.C.2, Var. 4)
P. 46, "The Smith's Secret 2" (cf. V.C.2, Var. 2)
P. 112, "Sawing Together" (cf. Note (6))
Pp. 134-5, "Would You Like a Sweet?" (IV.3)
Pp. 138-9, "The Tinder Box" (IV.5)
Pp. 140-6", "Look, A Piece of String on the Ground" (II)
P. 165, "The Sunfish" (V.B.11)

Elliot, A.: WITHIN-DOORS
P. 94, "The Magic Bond" (V.C.3, Var. 1)

Emory, K and H.C. Maude: STRING FIGURES OF THE TUAMOTUS
No. 19, "Korairai" (IV.5)
No. 22, "Patanuku" (cf.V.B.11)
No. 28, "Matuarere" (cf. I.A.6)
No. 46, "Ko Peka" (V.C.2, Var. 4)
No. 60, "Potaka Puraga" (cf.V.B.11)
No. 77, "Fai Taamu" (cf. I.B.5)
No. 83, "Mangareva Slip Trick" (cf. V.B.8)

Farabee, W.C.: THE CENTRAL ARAWAKS
P. 125, "Pirara, A Worm" (V.C.2, Var. 4)
Pp. 128-9, "Rapur, A Vicious Fly" (V.B.7, Var, 2,3)
P. 130, "Tarabara, A Fly" (cf. I.B.5)

Farabee, W.C.: THE CENTRAL CARIBS
No. 2, "Watisa, A Fly" (V.B.7, Var. 3)
No. 19, "Kefulikaco, Mouse in a Trap" (V.C.2, Var. 4)

Firth, R. and H.C. Maude: TIKOPIA STRING FIGURES
No. 54, "Kai Poloki" (cf. V.B.11)

Fischer, H.: FADENSPIELE VOM UNTEREN WATUT, UND BANIR RIVER (OST-NEUGUINEA)
No. 25, "Kaŋg" (cf. VI.3)
No. 26, "Tam" (cf. V.B.11)
No. 31, "Petsek" (I.A.6)
No. 64, "Jatsetsaŋ Puk" (I.B.4)
No. 69, "Ku Katek" (V.B.7, Var. 3)

Fornander, A.: FORNANDER COLLECTION OF HAWAIIAN ANTIQUITIES AND FOLKLORE
P. 214, "The Puzzle" (cf. IV)

Gardner, M.: MATHEMATICAL GAMES: THE ANCIENT LORE OF STRING PLAY
P. 149, "Freeing a Ring from a Cord" (cf.V.B.8)

Griaule, M.: JEUX ET DIVERTISSMENTS ABYSSINS
Plate 7, No. 1, "Pour Tenir une Cruche" (I.B.4)
Plate 7, No's. 3-6, "Attrape" (V.C.1)

Griaule, M.: JEUX DOGONS
Fig. 39, "Nez Bouche" (V.C.1)
Fig. 52, "Tresse" (I.B.4)

Griffith, C.L.T.: GOLD COAST "STRING GAMES"
No. 27, "Weaving Cloth" (cf. Note (6))
No. 29, "Swat that Fly" (cf.I.B.5)

Grunfeld, F.: GAMES OF THE WORLD
Pp. 256-8, "The Mouse" (V.C.2, Var. 4)

Gryski, C.: CAT'S CRADLES, OWL'S EYES
Pp. 28-9, "Cutting the Hand" (V.A.5)
Pp. 32-3, "Cheating the Hangman" (V.B.7, Var. 3)
Pp. 36-7, "The Saw" (cf. Note (6))
Pp. 38-9, "The Twitcher" (cf. VI.2)
Pp. 50-1, "The Yam Thief" (V.C.2, Var. 4)
Pp. 54-5, "The Fly" (cf. I.B.5)

Gryski, C.: MANY STARS AND MORE STRING GAMES
Pp. 14-5, "A String Trick" (V.B.7, Var. 1)
Pp. 20-1, "The Eel or Catching Crabs" (IV.4)
Pp. 26-7, "Fire" (IV.5)
Pp. 48-50, "The King Fish" (V.B.11)

Gryski, C.: SUPER STRING GAMES
Pp. 16-17, "The Lizard" (III.2)
Pp. 18-23, "The Elastic Band" (VI.2)
Pp. 24-5, "The Sea Snake" (V.C.3, Var. 3)
Pp. 47-9, "The Fastening of the Raven's Plumage" (V.C.2, Var. 5)

Haddon, A.C.: A FEW AMERICAN STRING FIGURES AND TRICKS
No. 5, "Threading a Closed Loop" (VI.1)
No. 6, "An Omaha Trick" (V.C.2, Var. 4)

Haddon, A.C.: STRING FIGURES FROM SOUTH AFRICA
P. 143, "Another String Trick" (V.B.7, Var. 1)
No. 3, "Inyoni, A Little Bird" (I.B.4)

Haddon, A.C.: STRING FIGURES AND TRICKS
No. 7, "Geigi, King Fish" (V.B.11)
No. 10, "Pagi, A Sea Snake" (V.C.3, Var. 3)
No. 24, "Sinaur" (cf. Note (6))
No. 26, "Monan, A Lizard" (III.2)
No. 27, "Kebi Mokeis, the Mouse" (V.C.2, Var. 4)
No. 28, "Au Mokeis, the Rat" (V.C.3, Var. 2)
No. 29, "Zermoi, the Pilot Fish" (V.C.4)
No. 30, "Lewer, Food" (IV.3)
No. 31, "BUli, A Fly" (V.B.6)

Haddon, K.: CAT'S CRADLES FROM MANY LANDS
No. 7, "The King Fish" (V.B.11)
No. 9, "The Sea Snake" (V.C.3, Var. 3)
No. 17, "A Locust" (I.B.5)
No. 50, "Sawing Wood" (Note (6))
Trick No. 1, "The Lizard" (III.2)
Trick No. 2, "The Mouse" (V.C.2, Var. 4)
Trick No. 3, "Will You Have a Yam?" (IV.3)
Trick No. 4, "A Fly on the Nose" (V.B.6)
Trick No. 5, "A Dravidian Trick" (cf. V.B.1)
Trick No. 6, "A German Trick" (V.C.3, Var. 1)
Trick No. 7, "An English Hanging Trick" (V.B.7, Var. 4)
Trick No. 8, "An African Hanging Trick" (V.B.7, Var. 2)
Trick No. 9, "An Egyptian Trick" (V.B.7, Var. 7)
Trick No. 10, "A Navaho Indian Trick" (V.B.8)
Trick No. 11, "A Pawnee Trick" (V.B.10)
Trick No. 12, "Threading a Closed Loop" (VI.1)

Haddon, K.: SOME AUSTRALIAN STRING FIGURES
No. 24a, "The Scrub-Hen's Eggs" (cf. VI.3)

Haddon, K.: ARTISTS IN STRING
Fig. 32B, "The Scrub-Hen's Eggs" (cf. VI.3)

Haddon, K.: STRING GAMES FOR BEGINNERS
No. 5, "An Eel" (IV.4)
No. 14, "The Scrub-Hen" (cf. VI.3)

Haddon, K. and H.A. Treleaven: SOME NIGERIAN STRING FIGURES
No. 6, "Ayun, A Saw" (cf. Note (6))
No. 8, "Bush Fowl Running Away" (cf. I.B.5)

Hadfield, E.: AMONG THE NATIVES OF THE LOYALTY GROUP
Pp. 129-30, "Fikihalo" (V.C.2, Var. 4)

Hallet, J.-P. and A. Pelle: PYGMY KITABU
P. 289, "Musese" (V.C.2, Var. 4)

Handy, W.C.: STRING FIGURES FROM THE MARQUESAS AND SOCIETY ISLANDS
P. 44 and Plate 111.A, "Popo Tikaue" (cf. VI.2)
Pp. 51-2, "Kamo" (cf. V.B.11)
P. 52, "Koee" (III.2)
Pp. 52-3, "Another Trick Figure" (V.C.3, Var. 1)
Pp. 88-90, "Tuna Biro" (cf.V.B.11)
P. 90, "Amatara" (cf.I.B.5)

Harbison, R. and R. Reichelt: SOME STRING FIGURES FROM PAPUA-NEW GUINEA
Fig. 9, "Meto" (cf.V.B.11)

Hassrick, R., and E. Carpenter: RAPPAHANNOCK GAMES AND AMUSEMENTS
P. 23, "See-saw" (cf. Note (6))

Hathalmi, G.F.: GYERMEKMUVESZET ES GYERMEKJATEKOK
No. 2, "Gocsoles Szethuzassal" (cf. I.A.1 and Appendix A.2, Note (1), p.106)
No. 3, "Fonalrantas Gomblyukbol: (V.B.7, Var. 4)
No. 6, "Csavarintas A Kez Tenyere Felol" (V.C.2)
No. 7, "Csavarintas A Kez Tenyere Felol" (V.C.3, Var. 1)
No. 9, "Fonalatvetelek" (cf. Note (6))

Held, G.J.: THE PAPUANS OF WAROPEN
P. 368, "Ghaido, the River" (cf. V.B.11)

Helfman, H.C. and E.: STRING ON YOUR FINGERS
Pp. 18-9, "A Thumb Catch" (III.4)
Pp. 24-5, "A Fly" (cf. I.B.5)
Pp. 26-7, "A Noose" (V.B.7, Var. 3)
Pp. 32-5, "A Yam Thief" (V.C.2, Var. 4)
Pp. 37-9, "Threading a Closed Loop" (VI.1)

Holtved, E.: CONTRIBUTIONS TO ESKIMO ETHNOGRAPHY
No. 45, "Qaneq" (Note (3), Fig. 117.B)

Hornell, J.: STRING FIGURES FROM FIJI AND WESTERN POLYNESIA
Fig. 6, "Kalokalo" (cf.I.B.5)
Fig. 26-9, "Undu and Koro" (VI.2)
Fig. 62, "Ili" (cf. Note (6))
Fig. 66, "Mata Upenga" (cf. V.B.11)
Fig. 70, "Fetia" (cf. Note (6))
Fig. 72, "Manua" (cf. V.B.11)
Fig. 75, "Unnamed Figure" (cf. V.B.11)
Fig. 76, "Sawing Wood" (Note (6))

Hornell, J.: CAT'S CRADLES, THE WORLD'S MOST WIDESPREAD GAME
No's. 5-6, "Vale-ni-kula and Vonu" (cf. VI.2)

Hornell, J.: THE STRING GAMES AND TRICKS OF SIERRA LEONE
P. 9, "Cutting Your Throat" (V.B.7, Var. 2)

Hornell, J.: STRING FIGURES FROM SIERRA LEONE, LIBERIA, AND ZANZIBAR
Mende 1, "Ngenye, the Saw" (cf. Note (6))
Mende 3, "An-Geye and Bulonge" (cf. I.B.5)
Temne 1, "Sisa, See-saw" (cf. Note (6))
Sherbro 1, "Sik Bank, To Tie the Strings" (cf.I.B.5)
Sherbro 2, "Sa, the Saw" (cf. Note (6))
Mandingo 1, "Kadjulu Labo Ibolado, Pull String from Hand" (cf I.B.5)
Kru 1, "Sisa, See-saw" (cf. Note (6))
Kru 4, "Di Bagbi Sagbi, The String Tied and Loosed" (cf.I.B.5)
Zanzibar 1, "Sumeno, the Saw" (cf. Note (6))

Hornell, J.: STRING FIGURES FROM GUJARAT AND KATHIAWAR
No's. 2-6, "Krauat, the Saw" (cf. Note (6))
No. 9, "Ganth, the Knot" (cf. I.B.5)
No. 10, "Ghat, the Knot" (cf. I.B.5)

Hornell, J.: INDONESIAN INFLUENCE ON EAST AFRICAN CULTURE
P. 332, "Betsileo, String Figure" (cf. I.B.5)

Hornell, J.: STRING FIGURES FROM THE ANGLO- EGYPTIAN SUDAN
No's. 1-4, "Munshar, the Saw" (cf. Note (6))

Howard, D.: STRING GAMES OF AUSTRALIAN CHILDREN
Plate 6, No. 1, "Crow's Foot, View A" (cf. Note (6))

Ingrams, W.H.: ZANZIBAR
P. 232(1), Un-named (V.A.5)
P. 232(2), Un-named (V.B.7, Var. 1)

Jackson, M. and M. and K. Watkins: STRING GAMES
Pp. 32-3, "Snake and Lizard" (III.2)
Pp. 36-7, "The Escaping Thumb" (V.B.8)
Pp. 38-41, "The Knot Tying Competition" (I.A.4)

Jattilainen, P.: ANTERO VIPUNEN
P. 371, "Nuora Neulansilmaan" (VI.1)
Pp. 372-3, "Liikkuva Suu" (cf. Note (3), Fig. 117.B)
P. 374, "Hiiri Karkaa" (V.C.2, Var. 4)
P. 375, "Ansa" (II.1)

Jayne, C.F.: STRING FIGURES
Pp. 34-38, "A Sea Snake" (V.C.3, Var. 3)
Pp. 39-43, "A King Fish" (V.B.11)
Pp. 142-6, "A Caroline Islands Catch" (cf.V.B.11)
Pp. 320-4, "Flint and Steel" (IV.5)
Pp. 337-9, "A Torres Straits Lizard" (III.2)
Pp. 339-40, "Hanging" (V.B.7. Var. 3)

Maude, H.C.: STRING FIGURES FROM NEW CALEDONIA AND THE LOYALTY ISLANDS
No. 3, "Uo Lemadren" (cf. V.B.11)
No. 20, "Taro Patch" (I.A.6)
No. 35, "Shells" (VI.2)
No. 48, "O Ban" (I.B.4)
No. 51, "A Fly" (cf. I.B.5)
No. 52, "Uprooting Maniania" (V.C.2, Var. 4)
No. 53, "Putting on the Basket" (V.B.7, Var. 2)
No. 54, "No Name" (cf. V.B.8)
No. 55, "Trick" (III.2)

Maude, H.C.: STRING FIGURES FROM TONGA
No. 11, "Kato 'a Hina, Hina's Basket" (VI.2)
No. 16, "Trick" (V.C.2, Var. 4)
No. 17, "Trick" (V.C.3, Var. 1)

Maude, H.C. and G. Koch: POLYNESIER
Fig. 13, "Te Noti o Kulu" (cf. I.B.5)
Fig. 14, "Sele Paala" (cf. III,4)

Maude, H.C. and H.E.: STRING-FIGURES FROM THE GILBERT ISLANDS
No. 17, "Tenua ni Maniba, III" (cf. V.B.11)
No. 66, "Kabae Baara" (cf. V.B.11)
No. 115, "Kan Tebe Nako, I" (cf. V.C.3, Var. 3)
No. 118, "Bo Kabenga" (cf. I.B.5)
No. 119, "No Name Given" (V.C.2, Var. 4)
No. 120, "Kabaebae Baara" (cf. V.B.11)
No. 121, "Te Baara" (cf. III.4)
No. 122, "Te Ntabena" (IV.4)
No. 124, "Nei ni Kauki" (cf. VI.2)
No. 126, "Trick" (V.B.8)
No. 127, "Ring Trick" (cf. V.B.8)

Maude, H.C. and C. Wedgwood: STRING FIGURES FROM NORTHERN NEW GUINEA
No. 14, "Fire" (IV.5)

McCarthy, F.D.: STRING FIGURES OF AUSTRALIA
P. 280, "A Seagull" (II)

McCarthy, F.D.: THE STRING FIGURES OF YIRRKALLA
No. 6, "Ironwood Gum Cement" (cf. V.B.11)
No. 35, "Three Huts" (cf.V.B.11)
No. 72, "Tree" (II)
No. 94, "Stilt" (II)
No's. 110-116, various plaits (II)
No. 128, "Crow Walking" (II)
No's. 130-146, various plaits (II)
No. 149, "Girl Pounding Rakia Bulbs" (II)
No. 168, "Sawing Wood" (cf. Note (6))
No. 177, "Two Boys" (cf. III.4)

McIlwraith, T.F.: THE BELLA COOLA INDIANS, Vol. II.
No. 8, "The Fireseeker" (cf. Note (3), Fig. 117.B)
String Trick No. 1, "A Cutting" (cf. V.C.2, Var. 2)
String Trick No. 2, "The Fastening of Raven's Plumage" (V.C.2, Var. 5)

McToots, R.: THE KID'S BOOK OF GAMES FOR CARS, TRAINS AND PLANES
No. 5, "Breakfast Escapes!" (V.C.2, Var. 1)

Mindt-Paturi, F.R.: THREE STRING TRICKS FROM THE WOLOF TRIBE IN SENEGAL
No. 1, "Nose Trick" (V.C.1, Var. 1)
No. 2, "Index and Middle Finger Trick" (cf. V.C.3, Var. 1)
No. 3, "Opening A Trick" (V.A.5)

Mita, M.: A REMARK ON THE JAPANESE DESIGN "TURTLE"
P. 7, "Turtle" (cf.VI.2)

Muller, W.: YAP
No. 10, "Darui" (cf.V.B.11)
No. 17, Un-named (III.2)
No. 18, Un-named (V.B.8)
No. 19, Un-named (V.B.7, Var. 3)
No. 20, "Threading a Closed Loop" (VI.1)

Nevermann, H.: DIE JE-NAN
P. 199, "Waiol" and "Ohom" (cf. V.B.11)

Noble, P.D.: WHY DON'T YOU?: STRING GAMES
P. 7, "Mosquito Bite" (III.2)
P. 8, "Bamboo Cuts" (V.A.5)
P. 9, "Chopping Bamboo" (V.A.6)
P. 10, "Thumbscrew" (cf.V.B.8)
P. 18-19, "Eel" (IV.4)
P. 20-21, "Saw" (cf. Note (6))
P. 25, "Two Stars Twinkling" (cf. VI.2)

Noble, P.D.: STRING FIGURES OF PAPUA NEW GUINEA
No. 1, "Tarera" (I.A.6)
No. 14, "Anamura" (IV.4)
No. 22, "Tampara" (cf. V.C.3, Var. 3)
No. 48, "Malolo, Bamboo Cutting Hand" (V.A.5)
No. 49, "Variation on Malolo" (V.A.6)
No. 51, "Uka Ma'u'ua" (cf. VI.3)
No. 55, "? Rat" (V.B.11)
No. 58, "Neck Release" (V.B.7, Var. 2)
No. 128, "Snake" (cf.V.C.3, Var. 3)

Noguchi, H.: A PAPUA NEW GUINEA STRING FIGURE "TURTLE"
Pp. 3-4, "Turtle" (cf. VI.2)

Noguchi, H.: ZUKEI-ASOBI NO SEKAI
Plate 20, "Inter Locking Loopo" (VI.4)

Noguchi, H.: STRING FIGURES IN JAPAN, I.
No. 12, "Tekkyo Cycle" (VI.2)

Noguchi, H.: STRING FIGURES IN JAPAN, III
No. 67, "Kubikiri, Hanging" (V.B.7, Var. 3)
No. 68, "Yubinuki, Fingers Slip" (V.C.2, Var. 4)

Jayne, C.F. (Cont^d.)
Pp. 340-3, "The Mouse" (V.C.2, Var. 4)
Pp. 344-5, "A Dravidian Trick" (cf.V.B.1)
Pp. 346-7, "A Finger Catch" (V.B.8)
Pp. 348-9, "A Fly on the Nose" (V.B.6)
Pp. 350-1, "A Thumb Catch" (III.4)
Pp. 352-4, "Will You Have a Yam?" (IV.3)
Pp. 354-5, "Threading a Closed Loop" (VI.1)
Pp. 356-7, "A Saw Mill" (cf. Note(6))

Jenness, D.: PAPUAN CAT'S CRADLES
No. 22, "Ko'mako'ma, the Ear-Pendant" (I.B.4)
No. 23, "Sisiafwa'tu" (cf.I.B.5)
No. 24, "Ni'ma Dabada'bana, Cutting the Hand" (cf. V.B.11)
No. 35, "Kiloma'dumadu" (cf.V.C.3, Var. 4)
No. 43, "Nimo'ga, a Mosquito" (III.2)
No. 45, "La'bia, the Sago Palm" (V.C.3, Var. 1)

Jenness, D.: ESKIMO STRING FIGURES (C.A.E.)
No. 15, "The Little Finger" (Note(3), Fig. 117.B)
No. 16, "The Little Finger Made Double" (Note(3), Fig. 117.C)
Trick No. 1, "Cutting the Hand " (V.A.5)
Trick No. 2, Un-named (V.C.2, Var. 2)
Trick No. 3, Un-named (cf. V.C.2, Var. 4)
Trick No. 4, Un-named (V.C.2, Var. 5)
Trick No. 6, "A Point Hope Trick" (V.C.3, Var. 1)
Trick No. 9, "Another Copper Eskimo Trick" (cf. V.B.2)

Jochelson, W.: HISTORY, ETHNOLOGY AND ANTHROPOLOGY OF THE ALEUT
No. 3, "Ciniqitax' the Little Finger: (Note(3), Fig. 117.B)

Kattentidt, I.: FADENSPIELE AUS ALLER WELT
Pp. 19-22, "The Moon" (Note(3), Fig. 117.B)
Pp. 29-30, "Jumping Mouse" (V.C.2, Var. 4)
P. 31, "Another Trick" (VI.1)

Kidd, D.: SAVAGE CHILDHOOD
P. 176, "Another String Trick" (V.B.7, Var. 1)

Kramer, A.: TRUK
Fig. 218, "Derelap" (cf.V.B.11)

Kraus, H.: LUFAMBO
No. 4, "Lusazi" (V.B.7, Var. 1)
No. 7, Un-named (cf.V.B.6)
No. 9, Un-named (V.B.7, Var. 2)

Landtmann, G.: CAT'S CRADLES OF THE KIWAI PAPUANS, BRITISH NEW-GUINEA
No. 2, "A Snake" (cf. V.C.3, Var. 3)
No. 3, "Sirima, A Fish" (IV.4)
No. 4, "Gibubu, a Bumble Bee" (I.B.5)
No. 15, "A Dugong" (V.B.11)
No. 19, Un-named (cf. V.B.8)
No. 20, Un-named (cf.V.B.7, Var. 2)

Lantis, M.: SOCIAL CUSTOMS OF THE NUNIVAK ESKIMO
Fig. 70, "Tkulu'ŋax Nuŋi'kulu'ŋax" (Note(3), Fig. 117.B)

Leeming, J.: FUN WITH STRING
Pp. 157-8, "Cheating the Hangman" (V.B.7, Var. 3)
Pp. 158-9, "The Sawmill" (cf. Note(6))

Levinson, D. and D. Sherwood: THE TRIBAL LIVING BOOK
P. 145, "Hanging Trick" (V.B.7, Var. 2)
Pp. 145-6, "Cutting the Fingers" (V.C.3, Var. 1)

Lutz, F.E.: STRING FIGURES FROM THE PATOMANA INDIANS OF BRITISH GUIANA
P. 13, "Fly" (cf.I.B.5)
P. 14, "Hanging Trick" (V.B.7)
P. 14, "Warum, Snake" (V.C.2, Var. 4)

Maclagan, R.C.: THE GAMES AND DIVERSIONS OF ARGYLSHIRE
P. 189, "The Thumb Loop" (VI.1)
Pp. 189-90, "Buttonhole Trick" (cf. V.B.7, Var. 4)
P. 190, "See Saw, Johnnie Maw" (cf. Note(6))

Martinez-Crovetto, R.: JUEGOS Y DEPORTES DE LOS INDIOS GUARANIES DE MISIONES
Fig. 2, "Sierra" (cf. Note(6))

Martinez-Crovetto, R.: JUEGOS DE HILO DE LOS ABORIGENES DEL NORTE DE PATAGONIA
Fig. 48, "Sierra O Serrucho" (cf. Note(6))
Fig. 50a, "Desenrollar la Tripa" (V.C.3, Var. 1)
Fig. 50b, "Desenrollar la Tripa" (V.C.2, Var. 1)
Fig. 50c, "Desenrollar la Tripa" (V.C.2, Var. 4)
Fig. 51, "Desenrollar la Tripa" (V.C.2, Var. 3)
Fig. 52, "Koila Kuzen" (V.B.6)
Plate 10b, "Tornillo O Rosca" (IV.1)

Mary-Rousselière, G.: LES JEUX DE FICELLE DES ARVILIGJUARMIUT
No. 54, "Qanerjuk, the Mouth" (Note(3), Fig. 117.B)

Maude, H.C.: THE STRING FIGURES OF NAURU ISLAND
No. 12, "Eadum, Crayfish", (IV.4)
No. 16, "Etegerer me Etogarita", (cf. I.A.6)
No. 18, "Ngan, Two Breakers" (cf. VI.2)

Maude, H.C.: SOLOMON ISLANDS STRING FIGURES
No. 37, "Hato, Arm Ring" (cf.V.B.11)
No. 106, "Kare" (cf.I.B.5)
No. 107, "Ou'a, Crab" (IV.4)
No. 113, "Narihe, Centipede" (V.C.2, Var. 4)
No. 114, "Neck Loop Trick" (V.B.7, Var. 2)
No. 115, "Wrist Loop Trick" (III.2)

Maude, H.C.: STRING FIGURES FROM THE TORRES STRAIT
No. 4, "Wot" (I.A.6)
No. 11, "Dangal, Dugong" (V.B.11)
No. 20, "Ger, Sea Snake" (cf. V.C.3, Var. 3)

Noguchi, H.: STRING FIGURES IN JAPAN, III (Cont.[d])
No. 71, "Himoutsushi" (Serial String Trick, I, P. 316)
No. 73, "Yubinuki, III" (cf. V.C.3, Var. 1)
No. 74, "Tejina, Trick" (cf I.B.5)

Noguchi, H.: STRING FIGURES IN JAPAN, IV
No. 93, "Yama no Ue no Otukisan" (cf. VI.3)
No. 100, "Yubikiri" (V.A.4)
No. 101, "Yubikiri" (V.C.3, Var. 1)
No. 102, "Index Slip" (V.B.9)
No. 103, "Kubikiri, Hanging" (V.B.7, Var. 6)
No. 104, "Hananuki, Nose Slip" (cf. V.B.6)
No. 105, "Himoutsushi, II" (Serial String Trick, II. P. 316-317)

Oorschot, A. von : TOUWFIGUREN
Pp. 76-9, "Ontsnapping 1" (V.B.1)
Pp. 80-1, "Ontsnapping 2" (III.2)
Pp. 84-7, "Ontsnapping 4" (V.B.8)
Pp. 88-91, "Ontsnapping 5" (V.C.3, Var. 1)
Pp. 92-5, "Ontsnapping 6" (V.C.2, Var. 4)

Ould, J.; HINDU ROPE BOOK
P. 9, "The Puzzle Knot" (I.A.1)
P. 9, "One-Handed Knot" (I.A.5)
P. 10, "The Magic Shoe-Laces" (I.B.1)
Pp. 10-1, "The Bachelor's Needle" (VI.1)

Parkinson, J.: YORUBA STRING FIGURES
P. 141, "A String Trick" (V.C.2, Var. 4)

Paterson, T.T.: ESKIMO STRING FIGURES AND THEIR ORIGIN
No. 17, "The Mouth" (Note (3), Fig. 117.B)
No. 187, "The Little Finger Made Double" (Note (3), Fig. 117.C)
String Trick No. 1, "Cutting the Hand" (V.A.5)
String Trick No. 2, "The Mouse" (V.C.2, Var. 4)
String Trick No. 3, Un-named (V.C.2, Var. 5)

Pellowski, A.: THE STORY VINE
Pp. 5-8, "The Mosquito" (cf. I.B.5)
Pp. 9-15, "The Farmer and his Yams" (V.C.2, Var. 4)
Pp. 22-30, "Lizard and Snake" (III.2 + V.B.1 + V.C.3, Var. 1)

Pocock, W.I.: CAT'S CRADLE
Pp. 87-91, "Four Sawing Figures" (cf. Note (6))

Pocock, W.I. SOME ENGLISH STRING TRICKS
P. 353, "The Buttonhole Trick" (cf.V.B.7, Var. 4)
Pp. 354-6, "The Key-Ring Trick" (cf. V.B.8)
Pp. 356-8, "Cutting off the Head" (V.B.7, Var. 1, 2)
Pp. 358-9, "The Nose Trick" (V.B.6)
Pp. 362-3, "Gypsy Finger String Trick" (V.B.10)
Pp. 365-6, "A World-Wide Finger String-Trick" (V.B.8)
P. 367, "The Hand String-Trick" (V.C.3, Var. 1)
Pp. 369-70, "The Mouse Alternative" (V.C.2, Var. 1)

Pocock, W.I. SUPPLEMENTARY NOTES ON CAT'S CRADLE AND STRING TRICKS
No. 1, Un-named, (V.B.7, Var. 1)
No. 3, Un-named, (V.C.2, Var. 1)
No. 4, Un-named, (cf. V.B.7, Var. 4)

Popular Mechanics: EASY TRICKS WITH STRING
P. 819, (Inset), "The Dissolving Knot" (I.B.3)
P. 822, (Inset), "Knot that is not a Knot" (cf. I.A.4)
Pp. 822-3, "One-Hand Knot" (I.A.3)

Ransom, W.R.: PASTIMES WITH STRING AND PAPER
P. 11, "Mosquito" (cf. I.B.5)
P. 14, "Finger Weaving" (V.C.2, Var. 1)
P. 16, "Have a Yam" (IV.3)
P. 17, "Sawing Wood" (cf. Note (6))
P. 27, "The Fool Knot" (I.A.2)
P. 32, "A Handful Out" (V.C.3, Var. 1)
P. 33, "Through the Eye" (VI.1)
P. 61, "Cast off Three Loops" (V.B.10)
P. 64, "Sea Snake" (V.C.3, Var. 3)
P. 86, "Ring Off" (cf. V.B.8)
P. 90, "Trapped or Free" (cf. IV.2)
P. 92, "Neck or Buttonhole Trick" (V.B.7, Var. 4)
P. 93, "Strangling" (V.B.7, Var. 2)
P. 109, "Chain Stitch" (cf. I.B.4)
P. 111, "Surprise Snarl" (I.B.3)
Pp. 114-5, "The Yam Thief" (V.C.2, Var. 4)
P. 117, "The Lizard Twist" (III.2)
P. 118, "Caught or Got Away?" (cf. V.B.11)
P. 145, "String of Knots" (I.A.4)

Rasmussen, K.: INTELLECTUAL CULTURE OF THE COPPER ESKIMOS
No. 26, "Iter^{s}huk, Anus" (Note (3), Fig. 117.B)

Raymund, P.: DIE FADEN- UND ABNEHMSPIELE AUF PALAU
No. 44, "Cholanges" (cf.V.B.11)

Report of the Cambridge Anthropological Expedition to the Torres Straits =
Haddon, A.C.: STRING FIGURES AND TRICKS

Rigney, F.J.: CUB SCOUT MAGIC
Pp. 74-5, "The 'Is Not' Knot" (I.B.3)
Pp. 75-6, "Another Disappearing Knot" (I.B.2)
P. 76, "Can-You-Make-It Knot" (cf I.A.1 and Appendix A.2 Note (1) p. 10
P. 77, "Threading the Loops" (VI.1)
Pp. 80-1, "Lots of Knots" (I.A.4)
Pp. 88-9, "Rope Loop Through Your Neck" (V.B.7, Var. 5)
Pp. 89-90, "Double Looped Cord Through Finger" (V.B.6)
Pp. 92-3, "Looped String Through Buttonhole" (cf. V.B.7, Var. 4)
Pp. 93-4, "Loop from Finger of Helper" (V.B.8)

Rivers, W.H.R and A.C. Haddon: A METHOD OF RECORDING STRING FIGURES AND TRICKS
No. 7, "Geigi, King Fish" (V.B.11)
No. 9, "Pagi, A Sea Snake" (V.C.3, Var. 3)
No. 11, "Monan, Lizard" (III.2)
No. 12, "Kebe Mokeis, the Mouse" (V.C.2, Var. 4)

Robinson, A.: SOME FIGURES IN STRING FROM THE MAKUSHIS ON THE IRENG AND TAKUTU RIVERS
P. 144, "Mo, Worm" (V.C.2, Var. 4)
Pp. 144-5, "Toorer, Cowfly" (cf. I.B.5)
P. 145, "Awotiner, Cutting the Throat" (V.B.7, Var. 3)

Rogers, E.S.: THE MATERIAL CULTURE OF THE MISTASSINI
P. 128, "Second Type" (V.C.2, Var. 4)
P. 128, "Third Type" (cf. VI.2)

Rosser, W.E. and J. Hornell: STRING FIGURES FROM BRITISH NEW GUINEA
No. 1, "Lohi, Fire" (IV.4)
No. 17, "Ou, Water" (V.B.11)

Roth, W.E.: STRING FIGURES, TRICKS, AND PUZZLES OF THE GUIANA INDIANS
No. 692, "Cow Fly" (cf. I.B.5)
No. 708, "To Cut the Fingers" (V.C.2, Var. 1)
No. 709, "To Cut the Fingers" (V.C.2, Var. 4)
No. 710, "To Cut the Fingers" (V.C.3, Var. 1)
No. 711, "Hanging Trick" (V.B.7, Var. 2)
No. 712, "Hanging Trick" (V.B.7, Var. 3)

Saito, T.: AYATORI ITOTORI, I.
No. 3, "Easy Rubber Band" (cf. VI.2)
No. 8, "Iron Bridge, Turtle, Rubber Band, Yo-Yo" (VI.2)

Saito, T.: AYATORI ITOTORI, II
No. 6, "Moonrise on Fujisan" (VI.3)

Saito, T.: AYATORI ITOTORI, III
No. 4, "Sawing Wood" (cf. Note (6))
No. 5, "Pounding Rice" (cf. Note (6))
No. 7, "Cutting the Fingers, 1" (V.A.4)
No. 8, "Cutting the Fingers, 2" (V.C.3, Var. 1)
No. 9, "Cutting the Fingers, 3" (V.B.9)
No. 10, "Cutting the Wrist" (cf. V.A.5)
No. 11, "Cutting the Head" (V.B.7, Var. 6)
No. 12, "Cutting off the Nose" (cf. V.B.6)
No. 13, "Traveling Loops, 1" (Serial String Trick, II, p. 316-137)
No. 14, "Traveling Loops, 2" (Serial String Trick, I, p. 316)

Sbrzesny, H.: DIE SPIELE DER !KO-BUSCHLEUTE
No. 8, "The Saw" (cf. Note (6))

Seligman, C.G. (Ed.): NOTES AND QUERIES ON ANTHROPOLOGY
Pp. 338-9, "Kebe Mokeis, the Mouse" (V.C.2, Var. 4)

Senft, B. and G.: NINIKULA, FADENSPIELE AUF DEN TROBRIAND-INSELN, PAPUA-NEUGUINEA
No. 11, "Lakum" (IV.4)
No. 19, "Ka'ukwa Navivila" (cf. V.B.11)
No. 26, "Boku" (V.B.7, Var. 3)
No. 38, "Vayola" (V.C.3, Var. 1)
No. 50, "Ebwesi" (V.C.2, Var. 4)
No. 62, "Kakanukwa" (IV.4)
No. 74, "Vadila" (cf. I.A.6)

Shishido, Y. and H. Noguchi: SOME STRING FIGURES OF HIGHLAND PEOPLE IN PAPUA NEW GUINEA
No. 4, "Lightning" (cf. VI.2)
No. 12, "No Name Known" (cf. V.B.11)
No. 22, "Cuckoo Cuts a Treetop" (cf. I.A.6)
No. 26, "Cassowary Trap" (cf. I.B.5)

Smith, P.S.: STRING GAMES IN LITERATE SOCIETIES: A RESEARCH NOTE
Pp. 77-8, "The Magic Bond" (V.C.3, Var. 1)

Speck, F.G.: ETHNOLOGY OF THE YUCHI INDIANS
P. 90, "Sawing Wood" (Note (6))

Starr, F.: ETHNOGRAPHIC NOTES FROM THE CONGO FREE STATE
P. 135, No. 30, "String Tricks" (cf. V)
P. 136, No. 31, "Dingwengue, Seesaw" (Note (6))
P. 150, "The Mouse" (V.C.2, Var. 4)

Sutton-Smith, B.: THE GAMES OF NEW ZEALAND CHILDREN
P. 134, "Threading the Needle" (VI.1)

Tessmann, G.: DIE BUBI AUF FERNANDO POO
P. 163, No. 1, "Cutting the Neck" (V.B.7, Var. 2)
P. 163, No. 2, "Cutting the Wrist" (V.B.4)
P. 163, No. 4, "Cutting the Fingers" (V.C.2, Var. 4)
Fig. 161, "We'sa" (III.4)
Tessmann, G.: DIE BAFIA
P. 167, "Ndik, String" (cf. I.B.5)

Tessmann, G., E.M. von Hornbostel, and K. Haddon: CHAMA STRING GAMES (PERU)
No. 1, "Monoset" (V.C.2, Var. 4)
No. 9, "Rono Kaso, A Viper's Vertebra" (cf. Note (3), Fig. 117.B)

Tozzer, A.M.: A COMPARATIVE STUDY OF THE MAYAS AND THE LACANDONES
P. 76, "Tinbuhtse, Sawing Wood" (Note (6))

Tracey, H.: STRING FIGURES (MADANDI) FOUND IN SOUTHERN RHODESIA
No. 12, "Nyoka, the Snake" (V.B.7, Var. 2)

Victor, P.E.: JEUX D'ENFANTS ET D'ADULTES CHEZ LES ESKIMO D'ANGMAGSSALIK
No. 26, "Cutting the Fingers, I" (V.C.2, Var. 4)
No. 27, "Cutting the Fingers, II" (V.C.3, Var. 1)
No. 28, "Cutting the Thumb," (V.B.3)
No. 29, "Cutting the Head" (V.B.7, Var. 2)

Vinton, I.: THE FOLKWAYS OMNIBUS OF CHILDREN'S GAMES
Pp. 203-4, "I Have a Yam" (IV.3)

Wu, T-y.: T'IAO HSIEN YU SHI
Pp. 12-3, "Elastic Band" (cf. VI.2)
Pp. 43-6, "Thumb Catch" (III.4)
Pp. 50-3, "Yam" (IV.3)
Pp. 101-6, "Sea Snake" (V.C.3, Var. 3)
Pp. 107-14, "Little Finger" (cf. Note (3), Fig. 117.B)

Yuasa, S. and T. Ariki: TANOSHII AYATORI ASOBI
I. No. 6, "Treasure Hunt" (VI.2)
III. No. 5, "Pounding Rice" (cf. Note (6))
III. No. 6, "Sawing Wood" (cf. Note (6))
IV. No. 1, "Easy Gum and Tricks" (cf. VI.2)
IV. No. 2, "Cutting the Wrist, I" (III.2)
IV. No. 3, "Cutting the Wrist, II" (V.A.5)
IV. No. 4, "Cutting the Neck" (V.B.7, Var. 6)
IV. No. 9, "Traveling Loops" (Serial String Trick, I. p. 316)
IV. No. 10, "Cutting the Finger" (cf. V.B.8)
IV. No. 12, "Cutting the Fingers, II" (V.C.2, Var. 4)
IV. No. 13, "Cutting the Fingers, III" (cf. V.C.3, Var. 1)

Appendix B: Problems of Existence and Uniqueness of Schematic Representations of Three-Dimensional String-Positions.

Herein we discuss the geometry of "real" string-positions in three-dimensional Euclidean space ($\mathbb{R}^{(3)}$). We shall restrict ourselves to the case of smooth, totally finite, closed configurations -- i.e. the string-position is composed of smooth arcs, the string itself being of finite total length, and closed to form a loop ('though not necessarily a simple one) -- although, strictly speaking, much of the subsequent discussion will obtain in more generality. A good model for the discussion is that of a fly, sitting on a sugar-cube; being displaced, he buzzes around the room for several moments, finally to alight once more on the sugar cube. His (closed) path through space describes a potential string-position of the type we wish to discuss. Specific examples are afforded by the string-positions illustrated in Fig. A.14 (page 103) or Fig. 108.E -- with the nodes L1*L2 and R1*R2 identified -- or any of the String-Figures previously discussed.

The first step in our discussion will be to represent our three-dimensional string-position in a (two-dimensional) plane -- the method will be by "parallel projection". That is, we choose a plane in $\mathbb{R}^{(3)}$, and drop perpendiculars from each point of the string-position to this plane. The string will then appear on this plane as a closed curve of a certain width. We shall reject those projections in directions which result in "doubled strings" e.g.

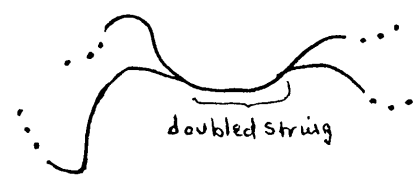


Fig. B.1: Example, disallowed projection, I,

or in crossings by more than two arcs, e.g.

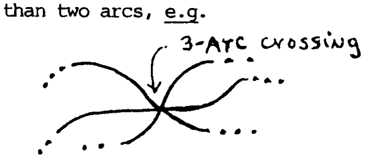
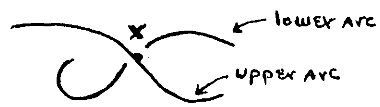


Fig. B.2: Example, disallowed projection, II.

although these may be treated by "variational" arguments, if desired. Because of the "total finiteness" restriction, above, parallel projections of a given string-position in $\mathbb{R}^{(3)}$ which avoid the above two "pathologies" always exist, in profusion. The resulting two-dimensional image of any such projection will be termed <u>regular</u>. In our former terminology, these are precisely the simple* schemata (minus frame-nodes) with fine-structure suppressed; <u>i.e</u>. we cannot identify the "over"-string at any crossing in the regular representation. Thus these <u>are</u> the elementary gross-schemata [<u>cf</u>. page 70] associated to the given, original string-position.

We now make a single, additional modification to a regular representation of a given string-position by stipulating that -- at any (simple) crossing therein -- the uppermost arc of the crossing is continuous, while the lowermost arc appears as a "broken" line segment.

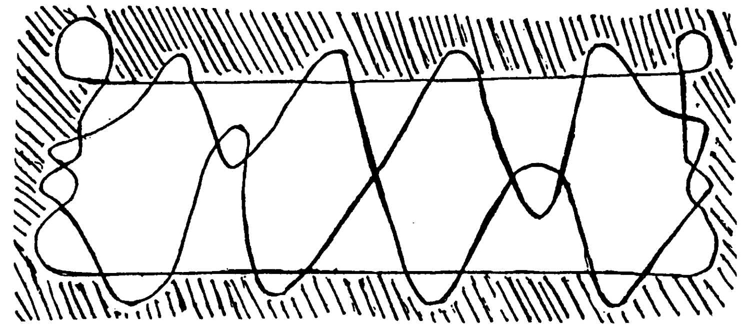


<u>Fig. B.3</u>: Example, fine-structure of a crossing, x.

This being done, the regular representation of the given string-position is said to be <u>normed</u>. Since the above modification effectively restores the fine-structure at each crossing, a normed, regular (NR-) representation of a given string-position in $\mathbb{R}^{(3)}$ is precisely the elementary schema (minus frame-nodes) of our previous discussion. The question we wish to discuss, in this Appendix B, is: How much information about a string-position in $\mathbb{R}^{(3)}$ is lost in the passage to any NR-representation thereof? The answer -- in terms of the central concern of the present discussion, the connectivity relations involved -- is, essentially: None at all!

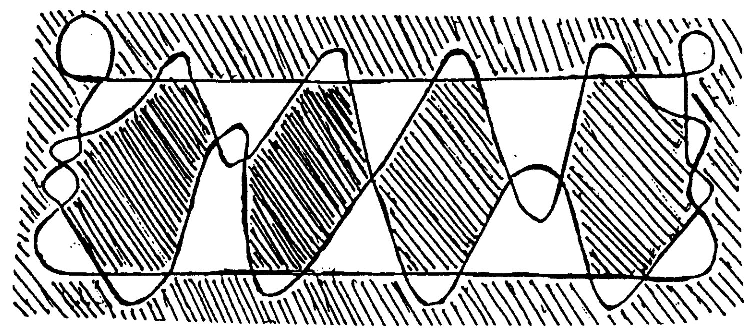
We begin our discussion with an alternate characterization of the fine-structure of any regular representation of a given "real" string-position in $\mathbb{R}^{(3)}$ <u>via</u> its uniquely associated "Seifert-Surface" [see H. Seifert: "Uber das Geschlecht von Knoten", Math. Annalen, Vol. 110 (1934), pp. 571-592]. To that end, we consider a regular representation of a given "real" string-position in $\mathbb{R}^{(3)}$, and begin by "shading-in" the whole infinite region outside the finite, closed central configuration which represents the given string-position in the plane. By way of example, we shall illustrate the procedure for (the schema for) Osage Diamonds, Fig. 28 (page 56 of these notes):

---
* all crossings are simple.



<u>Fig. B.4</u>: Seifert Surface for Osage Diamonds, I.

We continue by similarly shading all those simple, closed interior regions of the central configuration (if any) which touch the outer, shaded region at crossings only.



<u>Fig. B.5</u>: Seifert Surface for Osage Diamonds, II.

We continue in this manner -- shading any further reegion connected to an already-shaded region at creossings only -- until the whole diagram is separated into shaded and unshaded regions.

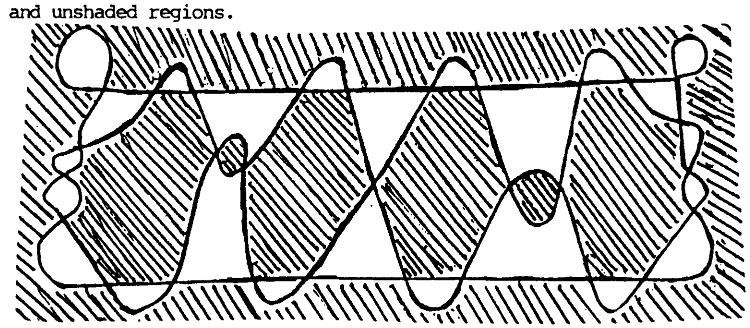


Fig. B.6: Seifert Surface for Osage Diamonds, III.

For example, Fig. B.6, above, is the Seifert Surface for the particular regular representation of Osage Diamonds chosen in Fig. 28.

Some remarks are in order: The Seifert Surface associated to the "simple" string-position of Osage Diamonds is clearly well-defined and unique, with a surprising "regularity" which one would have no reason to suppose would be enjoyed by a much more complicated, highly-convoluted string-position. Note, for example, that we never encounter a situation like the following

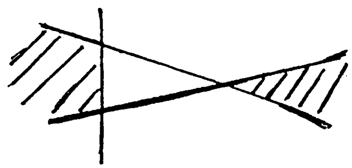


Fig. B.7: A barred Seifert configuration.

wherein one has question as to whether or not the central triangle should be shaded. In fact, the regularity observed in the above example has two aspects:

§.1. Along any arc of the configuration, a shaded and an unshaded region are always contiguous.

Note that we cannot say which of the two possibilities, below, pertains:

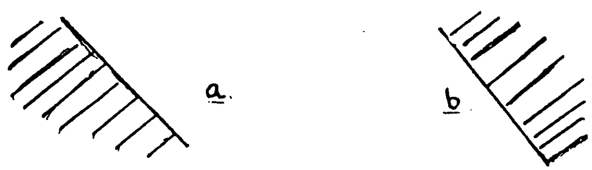


Fig. B.8: The two arc-specific shadings.

But inspection verifies that, in every case, one of them is correct for the Seifert Surface associated to the given regular representation of Osage Diamonds (Fig. B.6).

§.2. At every crossing, two shaded and two unshaded regions meet, and these are always diagonally opposite one another.

Again, we cannot say which of the two possibilities, below, pertains:

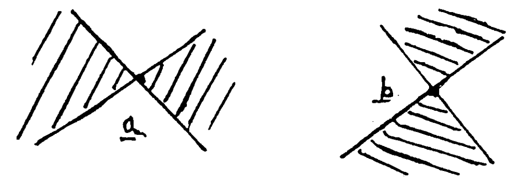


Fig. B.9: The two crossing-specific shadings.

But, for the above example, we verify that one of these is always correct. The surprising fact is that the Osage Diamonds is entirely representative [redacted] of the general situation in these regards; this is the content of the following theorem [D. Rolfsen: Knots and Links, Publish or Perish Press, Berkeley (1976). p. 120, "Seifert Surfaces, Existence Theorem".]

Theorem B.1: Each regular representation of a "real" string-position in $\mathbb{R}^{(3)}$ has an uniquely associated Seifert Surface which enjoys the regularity properties §1, §2, above.

One of the consequences of this remarkable theorem is that the elementary gross-schemata associated to a "real" string-position in $\mathbb{R}^{(3)}$ -- from which application of the Seifert "shading"-algorithm produces the associated Seifert Surface -- exhibits a uniform "regularity" only dimly apprehended heretofore. Of course, distinct regular representations of a "real" string-position in $\mathbb{R}^{(3)}$ will, in general, result in distinct elementary gross-schemata thereof -- see, for example, Fig's. 67.IV and IV' (pages 153,155) in the Brown Bears sequence -- and the ultimate question is: How are these related? Before addressing this question, however, we make a short digression to discuss an alternate method of recovery of the fine-structure in these elementary gross-schemata.

We recall that, since the representation under discussion is regular, by assumption, at each (simple) crossing we find exactly two arcs; one lying above, the other below. After the Seifert shading-algorithm, then, we must be considering exactly one of the following four possibilities:

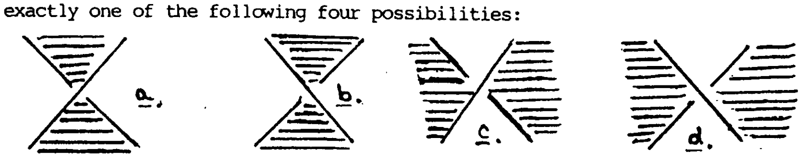


Fig. B.10: Fine-structure recovery a la Seifert.

Now consider a single such crossing. Here the upper string of the crossing may be rotated about the crossing into the lower string in either of two distinct ways: (1) so that it passes only through shaded regions of the configuration (i.e. on the Seifert Surface), or (2) so that it passes only through unshaded regions -- during this rotation. We reject the latter possibility in every case, and consider the former with a little more particularity. If the upper arc's rotation into the lower arc across a shaded region is counterclockwise (that is*, Mathematically "positive"), we attach a "+" to the crossing in question; if clockwise (that is, Mathematically "negative"), we attach a "-" to the crossing. Thus, for example, in Fig. B.10 the correct assignments are a. + , b. - , c. - , d. + , respectively. And clearly, the Seifert-shading plus a simple ± -parity-bit appended to each crossing will serve to completely specify the fine-structure associated to a given elementary gross-schema. We illustrate the discussion, below, for the case of Osage Diamonds, considered previously [Fig's. B.4-6].

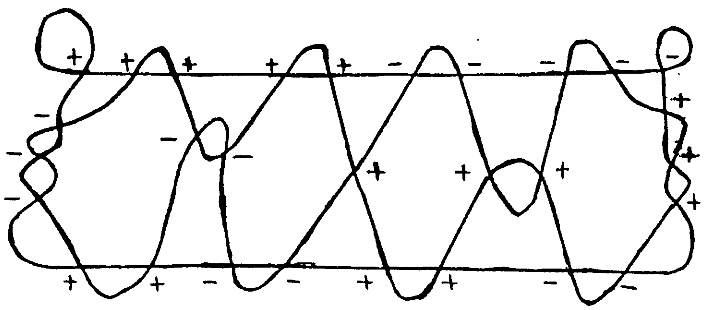


Fig. B.11: Osage Diamonds; fine-structure.

This concludes our "short digression".

We now return to the principal question of the present section, namely: How are two distinct NR-representations of a "real" string-position in $\mathbb{R}^{(3)}$ related to one another? In fact, we shall give a complete answer to the more general question: Given two "real" string-positions in $\mathbb{R}^{(3)}$ that are the "same" -- with respect to their connectivity-type -- how are their NR-representations related to one another?

We say that two distinct "real" string-positions in $\mathbb{R}^{(3)}$ are isotopic if either is continuously deformable into the other. The use of the word "continuously" in this definition is, for example, to disallow deformations in which the

* remember "Polar coordinates"?

string is momentarily broken apart -- and a constituent arc allowed to pass through the breach -- and then reunited at the same break-point. We have the (correct) feeling that any knot could be ultimately dissolved to the "empty loop" if we admitted such deformations. It is easy to see that Isotopy is an equivalence relation on the set of all "real" string-positions in $\mathbb{R}^{(3)}$, the equivalence classes under this relation being termed "Knot-classes" or, more simply, a "Knot". The most distinguished of these equivalence classes, from the present viewpoint, is the set of all "real" string-positions in $\mathbb{R}^{(3)}$ which are isotopic to the "empty loop" -- topologists call this the trivial Knot -- as all the String-Figures of the world are numbered among its members!

We are now in a position to state the Theorem which answers our "ultimate question" of the present section [see K. Reidemeister: Knot Theory, B.C.S. Associates, Moscow, Idaho (1983), page 8].

Theorem B.2: Two "real" string-positions in $\mathbb{R}^{(3)}$ are isotopic if and only if each NR-representation of one of them is Ø-equivalent to every NR-representation of the other.

We remark several immediate implications of this powerful theorem. (1). It now follows, from the definition, that -- if two "real" string-positions in $\mathbb{R}^{(3)}$ are isotopic -- then, of the two NR-representations involved, either is obtainable from the other by a finite sequence of the operations $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ and their inverses; and otherwise this is not the case. (2). Thus, with respect to the isotopy of "real" string-positions in $\mathbb{R}^{(3)}$, there is no loss of information involved in the passage to any NR-representations thereof. (3). The operations $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ and their inverses -- already recognized as fundamental to the theory of String-Figures -- are, in fact complete with respect to the connectivity type of "real" string-positions in $\mathbb{R}^{(3)}$; in that a complete solution to the isotopy problem for such string-positions may be given in terms of these, alone, without recourse to any "new" type of operation. Thus, we shall never encounter a broader definition/type of equivalence than "Ø-equivalence" on such structures. And, finally, (4). To determine whether or not a given "real" string-position in $\mathbb{R}^{(3)}$ dissolves, it suffices to take any NR-representation thereof and to ply this with (usually, obvious) sequences of Ø-type moves. For the string-positions encountered in these notes, it will always be entirely easy to "simplify" these to the empty loop, or to one with an easily-recognizable intrinsic knot.

---- End Appendix B ----

Secondary References: String Tricks

Abbott, P.: ABBOTT'S ENCYCLOPEDIA OF ROPE TRICKS FOR MAGICIANS. Abbott's Magic Novelty Co., Colon, Michigan (1945). pp. 400.

Gayton, A.H.: YOKUTS AND WESTERN MONO ETHNOGRAPHY, I, II
P. 22, "European Cutting Trick" (V.B)

Haddon, A.C.: HEADHUNTERS, BLACK, WHITE AND BROWN
P. 38, "Food" (IV.3)
P. 38, "Mouse" (V.C.2, Var. 4)

Hornell, J.: STRING FIGURES FROM GUJARAT AND KATHIAWAR
P. 149, "Mouse" (V.C.2, Var. 4)
P. 149, "Fly on the Nose" (V.B.6)
P. 149, "Hanging Trick" (V.B.7, Var. 2)

Saville, W.J.V.: IN UNKNOWN NEW GUINEA
P. 102, "The Sugar-Cane" (I.A.6)
P. 102, "Cough" (V.B.7, Var. 2)

Smith, M.W.: THE PUYALLUP-NISQUALLY
P. 227, "Now I Will Show You a Trick" (V.C.2)

Speck, F.G.: NASKAPI: THE SAVAGE HUNTERS OF THE LABRADOR PENINSULA
No. 43, "Seal Stomach" (V.C.2, Var. 4)

Spier, L.: YUMAN TRIBES OF THE GILA RIVER
P. 344, "Cal$_y$api'lk, My Hand is Hot!" (V.C.2)

VII. THE CALCULUS

In this section we return to the central topic under investigation in these notes -- String-Figures -- to discuss the application of the string-figure Calculus directly to the associated linear sequences (cf. pp. 43-45 and pp. 69-72). To date this application is accomplished as follows: Given the schema for a real string-position, we form the associated linear sequence following the conventions of Section I., Systemology. We then operate on this string-position as indicated by an expression from the constructional Calculus to produce a new string-position, to whose schema we associate a linear sequence in the previously-mentioned manner. Thus, a linear sequence and an expression from the Calculus together, produce another linear sequence through an appeal to the geometry of the underlying schemata. The goal of the present section is to free this process from its dependence on the schemata involved in this transition; its successful achievement will entail the "demotion" of the underlying schemata from a "fundamental construct" of the theory to the role of "extremely handy visual adjunct" thereof. Our discussion will parallel the development of Section I., Systemology (pp. 14-28).

After the discussion of the previous section (and, in particular, of Appendix B), we know that the simple schemata are exactly the NR-representations of "real" string-positions (plus frame-nodes) in $\mathbb{R}^{(3)}$, on which the topologically-fundamental transformations are the Ø-operations. The actual transformations we are applying to these string-positions, however, are the manual operations of the Calculus, e.g. $\vec{1}$(5n) #. It therefore remains to "analyze" these manual operations of the string-figure Calculus in terms of the fundamental Ø-operations, directly. For if this be successfully done, the effect of a given operation from the Calculus on a given associated linear sequence will immediately follow, without reference to the relevant schemata involved -- since the effect of an Ø-operation thereon is explicitly known ($Ø_1$, $Ø_2$ -- Lemma 2, p. 11; $Ø_3$ -- p. 259).

A useful analogy for the discussion which follows is that of the "Scientific Programmer" for the Electronic Computer (e.g. the IBM 704) of two decades ago. As a physicist/engineer, when he had to evaluate a complicated numerical expression like

$$\frac{A^B - C\cdot D}{B^2 - \frac{A}{C}},$$

he could avail himself of a "user-friendly" machine language, like Fortran IV, wherein the above computation would have the very similar-appearing expression

((A**B) - (C*D))/((B**2) - (A/C)).

That is, he had essentially only to adopt "Polish notation" to successfully communicate with the computer. But the machine -- with its basic binary command-language -- could make no <u>direct</u> sense of the above string of symbols! Thus the Scientific Programmer, before he could accomplish his computation, had to present his Fortran program to a "Compiler", which transformed it into the binary command-language effecting the above computation in a manner directly acceptable by the machine. The viewpoint of the present section, then, is that the string-figure Calculus is a "user-friendly" language not directly applicable to linear sequences. And, as such, expressions therefrom must be "compiled" into Ø-operations, which <u>are</u> the basic command-language of said linear sequences. Thus, from the perspective of the current analogy, the present section represents an attempt to write a basic Ø-operation compiler for the Calculus of string-figures.

We shall initially restrict ourselves to canonical linear sequences associated to simple schemata, <u>i.e</u>. hands in normal position, no multiple-loops, all crossings simple. And the first question to be addressed is how to identify the constructs fundamental to the string-figure Calculus directly from such a linear sequence. For example, consider the linear sequence associated to Fig. 29.VII (page 59) in the construction of Osage Diamonds:*

|⇒ L2: x1(Ø): x2(Ø): x3(U): x4(Ø): x5(Ø): x6(U): R5: x7(U): x8(Ø): x9(Ø): x1(U): L5: x2(U): x10(Ø): x4(U): x11(U): x6(Ø): x7(Ø): R2: x8(U): x5(U): x11(Ø): x3(Ø): x10(U): x9(U) ■

And let us suppose that we wished to perform the manipulation "$\overrightarrow{\mathrm{R1}}$(<u>R2f</u>) #" on the string-position whose schema has the above linear sequence associated to it. The question we wish to consider here is, "How is the R2f-string to be identified from the above associated linear sequence?" We first note that -- using the "subsequence suppression" convention (see page 69) -- the above linear sequence admits the subsequence

|⇒ ... R2 ... ■

* You can turn back to this figure for a "peek" if you must, but the discussion must not depend on this.

whence R2 <u>is</u> a frame-node for the associated string-position* and, as such, has an unique near string and an unique far string. Further, the subsequence of frame-nodes,

|⇒ L2 ... R5 ... L5 ... R2 ... ■

implies that one of these proceeds from R2 to L5 (ultimately), while the other proceeds from R2 to L2 (<u>cf</u>. page 6). But which is which? In particular, if the R2-string designated by the subsequence

|⇒ ... L5: x2(U): x10(Ø): x4(U): x11(U): x6(Ø): x7(Ø): R2 ... ■

passes to the far side of R2, then the argument of the Calculus manipulation "$\overrightarrow{\mathrm{R1}}$(<u>R2f</u>)#" -- <u>i.e</u>. R2f -- establishes a pointer which is just <u>before</u> (<u>i.e</u>. to the left of) R2 in the linear sequence, <u>viz</u>:

|⇒ ... x7(Ø):↓R2: x8(U) ... ■;

while if the R2-string

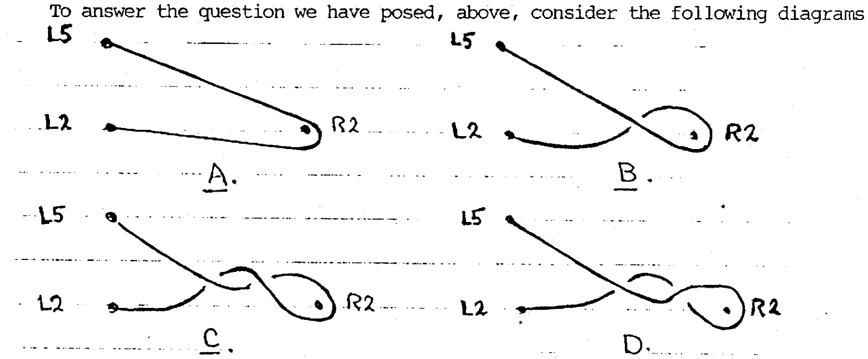
|⇒ L2 ... R2: x8(U): x5(U): x11(Ø): x3(Ø): x10(U): x9(U) ■

passes to the far side of R2, this pointer is established just <u>after</u> (<u>i.e</u>.to the right of) R2 in this sequence, <u>viz</u>:

|⇒ ... x7(Ø): R2↓: x8(U) ... ■

These are two very different situations relative to the modifications of the given linear sequence effected by the manipulation "$\overrightarrow{\mathrm{R1}}$(<u>R2f</u>)#", between which we must be able to discriminate.

To answer the question we have posed, above, consider the following diagrams:



<u>Fig. 150</u>: Argument discrimination.

* Otherwise "$\overrightarrow{\mathrm{R1}}$(<u>R2f</u>)#" would be the empty functor, Φ, by definition.

Since L2 is nearer to you than is L5 (i.e. $2<5$), the only way that the s;R2-L5 can pass to the near side of R2 is if this string crosses the s; R2-L2. In particular, it passes to the far side of R2 if the number of such crossings is even (inclusive of o), and it passes to the near side of R2 if this number is odd. We remark that the example of Fig. 150.D is included to indicate the compatibility of this criterion with the notion of extension-cancellation. Note that it is only the involvement of the s;R2-L5 with the s;R2-L2 that is important for this argument, crossings generated by other arcs of the given string-position being irrelevant thereto (and, hence, they are suppressed in Fig. 150). Thus, to answer our original question, we need only examine the cardinality of the set of crossings shared by s;R2-L5 and s;R2-L2; i.e. referring to the original linear sequence,

$$|\{x2,\ x10,\ x4,\ x11,\ x6,\ x7\}\cap\{x8,\ x5,\ x11,\ x3,\ x10,\ x9\}|$$
$$=|\{x10,\ x11\}|=2,\ \text{even}$$

whence the s;R2-L5 passes to the far side of R2, and the pointer is correctly established just to the left of R2 in the given associated linear sequence. At this point one surely turns to Fig. 29.VII (page 59) to confirm the validity of the above assertions in the context of the present example.

And, for generic functors $F_1 \neq F_2$, the above method will determine the R2f-string (and, hence, the R2n-string) for any canonical linear sequence possessing the distinguished subsequence

$$\models\!\Rightarrow \ldots F_1 \ldots R2 \ldots F_2 \ldots ■$$

as soon as it is determined which of F_1, F_2 is "closer to you with respect to R2". And, of course, since $F_1 \neq F_2$ by assumption, this is always straightforward to determine. Let us define

$$F_1 \underset{(R2)}{<} F_2 \equiv F_1 \text{ is closer to you than } F_2\text{; with respect to R2}$$

Then, for example, we have

$$R1 \underset{(R2)}{<} L1 \underset{(R2)}{<} L2 \underset{(R2)}{<} L3 \underset{(R2)}{<} L4 \underset{(R2)}{<} L5 \underset{(R2)}{<} R5 \underset{(R2)}{<} R4 \underset{(R2)}{<} R3,$$

which successfully effects the above determination by transitivity of the $\underset{(R2)}{<}$ relation. It remains to discuss the case $F_1 = F_2$. Here, the absence of multiple loops implies that the linear sequence involved possesses only the frame-nodes F_1, R2. That is, it admits the distinguished subsequence

$$\models\!\Rightarrow F_1 \ldots R2 \ldots ■$$

which shows all frame-nodes. Also, since this is a canonical associated linear sequence, the initial segment

$$\models\!\Rightarrow \underbrace{F_1 \ldots R2} \ldots ■$$

proceeds from the far F_1-string to R2, while the terminal segment

$$\models\!\Rightarrow \underbrace{F_1} \ldots \underbrace{R2 \ldots} ■$$

proceeds from R2 to the near F_1-string. Further, in this case, every crossing between the R2n- and R2f-strings appears in both the initial and terminal segments of the above linear sequence; hence the R2f-string's determination reduces to merely counting the number of these crossings. For example,

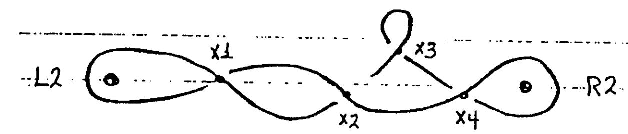


Fig. 151: Example, $F_1 = F_2$.

$$\models\!\Rightarrow L2:\ x1(\emptyset):\ x2(U):\ x3(\emptyset):\ x3(U):\ x4(U):\ R2\overset{\downarrow}{}:\ x4(\emptyset):\ x2(\emptyset):\ x1(U) ■$$

Here

$$|\{x1,\ x2,\ x4\}| = 3,\ \text{odd}$$

whence the R2f-string follows R2 in the above linear sequence (as there indicated)

Thus, for every canonical sequence on frame-node R2 we may easily determine the R2f-string's placement in that sequence. But there is nothing sacred about the choice of R2 for this extended example; the argument is general. In particular, the similar argument for -- say -- the functor L4 is based on the obvious string of inequalities

$$L3 \underset{(L4)}{<} L2 \underset{(L4)}{<} L1 \underset{(L4)}{<} R1 \underset{(L4)}{<} R2 \underset{(L4)}{<} R3 \underset{(L4)}{<} R4 \underset{(L4)}{<} R5 \underset{(L4)}{<} L5,$$

et cetera. The conclusion to be drawn from the above discussion is that the constructs of the string-figure Calculus are identifiable directly from the canonical linear sequences associated to simple schemata, i.e. without reference to the schemata, themselves. [Note also that the parities of the simple crossings encountered in the above argument were completely irrelevant thereto, and could have been suppressed.]

A final question prefacing our discussion of the direct application of the Calculus to canonical linear sequences is, "How important is it that the linear sequences involved in the analysis be in their canonical presentation?", i.e. that they satisfy the conventions Seq. 1 and Seq. 2 (page 6). For example, given that the linear sequence

$\Mapsto$ x1(Ø): x2(U): x3(U): x4(Ø): x5(Ø): R2: x6(U): x7(U): x3(Ø): x8(Ø): x1(U): x9(U): L2: x10(Ø): x11(Ø): x8(U): x2(Ø): x7(Ø): x4(U): R5: x5(U): x6(Ø): x9(Ø): x10(U): L5: x11(U) ■

is associated to a simple schema, determine its canonical presentation. Here, by convention Seq. 1 -- since L2 is the minimal left-hand frame-node appearing in this sequence -- we "rotate" the sequence leftward until L2 appears as its lead entry, viz.

$\Mapsto$ L2: x10(Ø): x11(Ø): x8(U): x2(Ø): x7(Ø): x4(U): R5: x5(U): x6(Ø): x9(Ø): x10(U): L5: x11(U): x1(Ø): x2(U): x3(U): x4(Ø): x5(Ø): R2: x6(U): x7(U): x3(Ø): x8(Ø): x1(U): x9(U) ■

Now the sequence has the right starting-point; does it have the correct orientation? We shall examine the "orientation"-question (convention Seq. 2) in the context of another example. To see that the above linear sequence is correctly oriented, we relabel the individual constituent crossings as follows:

x1 → x10	x7 → x5
x2 → x4	x8 → x3
x3 → x11	x9 → x9
x4 → x6	x10 → x1
x5 → x7	x11 → x2
x6 → x8	

whence the resulting linear sequence is that associated to Fig. 29.VII, considered in our previous example -- it is, thus, correctly oriented. Our conclusion, then, is that if an given linear sequence fails to be in its canonical presentation because of a violation of convention Seq. 1, it may be easily brought thereto by leftward element rotation and uniform sequential relabeling of its constituent crossings.

A similar situation obtains for convention Seq. 2, provided the linear sequence involved lists at least three distinct frame-nodes. For example, given that the linear sequence

$\Mapsto$ ↓L2: x1(U): x2(U): x3(Ø): x4(Ø): x5(U): x6(U): R2: x7(Ø): x8(Ø): x4(U): x9(U): x2(Ø): x10(U): L5: x11(U): x1(Ø): x6(Ø): x7(U): R5: x8(U): x5(Ø): x9(Ø): x3(U): x10(Ø): x11(Ø) ■

is associated to a simple schema (Note that it does satisfy convention Seq. 1) determine whether or not it has the correct orientation (i.e. that it satisfies convention Seq. 2). In this case this may be accomplished by identifying the far string of the starting-point node -- the L2f-string. Here, by our previous method, we recognize a s;L2-R2 and a s;L2-R5 and, since $2 < 5$ and

$$\left| \{x1, x2, x3, x4, x5, x6\} \cap \{x3, x5, x8, x9, x10, x11\} \right| = \left| \{x3, x5\} \right| = 2, \text{ even}$$

we see that the L2f-string establishes a pointer just to the left of L2 in the associated linear sequence (as there indicated), and hence that that sequence has an orientation opposite to that of convention Seq. 2. Thus, the correct presentation for the above linear sequence is given by

$\Mapsto$ L2: x11(Ø): x10(Ø): x3(U): x9(Ø): x5(Ø): x8(U): R5: x7(U): x6(Ø): x1(Ø): x11(U): L5: x10(U): x2(Ø): x9(U): x4(U): x8(Ø): x7(Ø): R2: x6(U): x5(U): x4(Ø): x3(Ø): x2(U): x1(U) ■

And now the sequential relabeling of constituent crossings given by

x1 → x9	x6 → x8
x2 → x10	x7 → x7
x3 → x3	x8 → x6
x4 → x11	x9 → x4
x5 → x5	x10 → x2
	x11 → x1

again produces the linear sequence associated to Fig. 29.VII. The method is general; a linear sequence on at least three distinct frame-nodes which satisfies convention Seq. 1 has an orientation uniquely determined by identifying the far string of its starting-point node. If this orientation violates the convention Seq. 2, the linear sequence may be brought to its canonical presentation by sequence reversal, and subsequent uniform sequential relabeling of its constituent crossings.

The proviso that the linear sequence must contain at least three distinct frame-nodes derives from the previous "string-identification" argument in the case $F_1 = F_2$ (pages 356-357), and is intrinsic. For, in that case, the successful determination of the R2f-string depended on the ability to discriminate between the F_1f and F_1n-strings -- an immediate corollary of the canonical presentation of the given linear sequence. The indispensability of this assumption is made manifest by the following example.

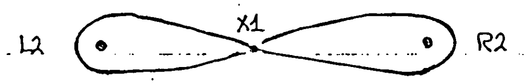


Fig. 152.A: Example, $F_1 = F_2 =$ L2.

⊨ L2: x1(∅): R2↓: x1(U) ■

Given that the above linear sequence is canonical, the R2f-string may be correctly identified by the previous techniques, as indicated above. But, if the above sequence were not in its canonical presentation, then -- by sequence reversal -- we would have

⊨ L2: x1(U): R2↓: x1(∅) ■

for this canonical presentation, which is the linear sequence associated to the schema of Fig. 152.B, below.

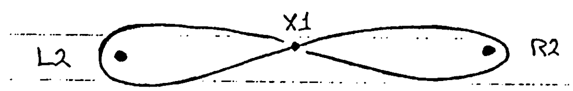


Fig. 152.B: Example, $F_1 = F_2 =$ L2 (Cont.[d]).

This is, again, a legitimate schema -- distinct from the schema of Fig. 152.A (the parities of the crossing x1 in these diagrams are distinct). Thus, in this extremely special case, we cannot decide which presentation of the above linear sequence is canonical -- without a direct appeal to its associated schema. And although the occurrences of this case are relatively rare among the string-positions generated by the Calculus in the constructions of the string-figures of the world, each must be individually handled by special techniques. These will usually be ad hoc and extremely straight-forward, owing to the simplicity of these structures. In fact, the operations to be considered initially will always be explicitly orientation-preserving (e.g. most pick-up or release moves), or orientation-reversing (e.g. shifting from an "Over" to an "Under" perspective, in some cases), whence the orientation of the linear sequence which results from a sequence of such operations will not, for a while, be in doubt.

We are now ready to proceed to the ∅-analysis of the operations of the string-figure Calculus. The first of these, | - extend hands to absorb slack, has already been discussed in the (limited) context of extension-cancellation (cf. pages 10-12). In the current context (of ∅-equivalence) the situation is

entirely analogous. Consider the example

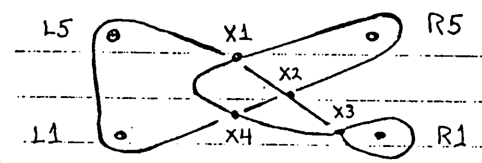


Fig. 153.A: Example, |.

⊨ L1: L5: x1(U): x2(∅): x3(U): R1: x3(∅): x4(∅): x1(∅): R5: x2(U): x4(U) ■

Here, by direct examination (see page 259), the $\emptyset_3$-move "draw s;x1-x4 across x2" is admissible on this linear sequence, and its performance results in the linear sequence

⊨ L1: L5: x2(∅): x1(U): x3(U): R1: x3(∅): x1(∅): x4(∅): R5: x4(U): x2(U) ■

And on this sequence the $\emptyset_2^{-1}$-move "Cancel {x1, x3}" is admissible (by Lemma 2.B). Its performance results in the linear sequence

⊨ L1: L5: x2(∅): R1: x4(∅): R5: x4(U): x2(U) ■

which is rendered canonical under the uniform sequential crossing relabeling

x2 → x1, x4 → x2.

The result is the (canonical) linear sequence

⊨ L1: L5: x1(∅): R1: x2(∅): R5: x2(U): x1(U) ■

And this sequence is uniquely associated to the schema of Fig. 153.B, below:

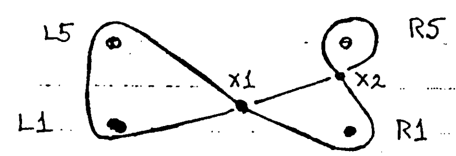


Fig. 153.B: Example, | (Cont.[d]).

The schema of Fig. 153.A is therefore seen to be ∅-equivalent to the simple schema of Fig. 153.B, which is, thus, the result of applying the Calculus manipulation "|" to that previous string-position. Restated: The application of the Calculus manipulation "|" to a canonical linear sequence results in the canonical presentation of a shortest* linear sequence ∅-equivalent to the original sequence

* The determination of such a "shortest" linear sequence is intimately related to "The Word Problem" of Algebraic Group Theory.

We remark that, by definition, the final sequence may always be derived from the initial sequence by a finite sequence of ∅-operations -- each of which is starting-point and orientation-preserving. Thus the resulting sequence at most lacks uniform sequential crossing relabeling of being in its canonical presentation.

The next class of manipulations from the string-figure Calculus to be considered are the "release"-moves; note that, unlike the preceding operation, "|", these all have arguments (as in □ L1, □ R2, etc.). The analysis is simple; let F be a generic functor. To apply the Calculus manipulation "□ F" to a canonical linear sequence which lists F as a frame-node*, simply delete the entry "F" from the given sequence. This operation on linear sequences is clearly orientation-preserving and starting-point preserving providing that F is not the starting-point for the given sequence. For example, consider the string-position resulting from the manipulational string

O.A: >R1∞ |,

whose schema is given in Fig. 154, below.

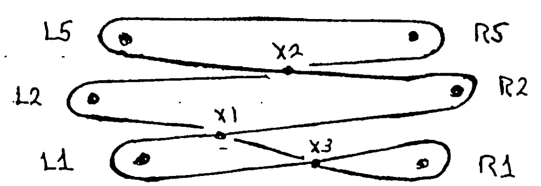


Fig. 154: O.A: >R1∞ |.

|⇒ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): x3(U): R1: x3(∅) ■

Applying the Calculus manipulation "□ L1" to this linear sequence results in the sequence

|⇒ x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): x3(U): R1: x3(∅) ■

which is in violation of convention Seq. 1 and is, hence, not canonical. To bring it to its canonical presentation (cf. page 358) we perform leftward element rotation until the minimal left-hand frame-node appearing in this sequence (i.e. L2) becomes its lead entry, viz.

|⇒ L2: x1(U): x3(U): R1: x3(∅): x1(∅): R2: x2(∅): L5: R5: x2(U) ■

* If F is not a frame-node for the sequence, we consider the operation "□ F" to be Φ, as mentioned earlier.

and then employ the uniform sequential crossing relabeling given by

x1 → x1, x2 → x3, x3 → x2

to produce the sequentially-labeled sequence

|⇒ ↓L2: x1(U): x2(U): R1: x2(∅): x1(∅): R2: x3(∅): L5: R5: x3(U) ■,

satisfying convention Seq. 1. Next we check convention Seq. 2, using the previous method to identify the L2f-string, which here establishes a pointer just to the left of L2 in the above linear sequence (as there indicated); the sequence has the orientation opposite to canonical. As before, the sequence is now brough to its canonical presentation by sequence-reversal, and subsequent uniform sequer tial relabeling of its constituent crossings. The result is

|⇒ L2: x1(U): R5: L5: x1(∅): R2: x2(∅): x3(∅): R1: x3(U): x2(U) ■ ,

on which we recognize the extension-cancellable crossing-pair {x2, x3}, by Lemma 2.B. We should not, of course, perform this cancellation, unless the subsequent Calculus-manipulation were "|".

From the above example, we see that if the functor F is the starting-point of a linear sequence, then the operation "□ F" applied to that sequence produces a derivative sequence which always violates convention Seq. 1, and which may violate convention Seq. 2 as well. The situation is, perhaps, most effectively handled ab initio by identifying the secondmost minimal left-hand node of the origina sequence, and directly (i.e. in that context) determining its far string; and, when this precedes the element in the linear sequence, performing immediate sequence-reversal, followed by leftward element rotation, etc. Explicitly:

Problem: apply "□ L1" to the sequence

|⇒ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): ↓L2: x1(U): x3(U): R1: x3(∅) ■

Solution: 1. Since L1 is the minimal left-hand node contained in the sequence, we are in the "special" case for this operation.

2. Identify the secondmost minimal left-hand node contained in the sequence (i.e. the starting-point for the derived sequence). This is seen to be L2.

3. Determine the L2f-string (indicated above). The derived sequence will be oriented opposite from the canonical.

4. Delete L1, reverse the sequence, and rotate leftward until the new starting-point becomes the lead element. This gives

|⇒ L2: x2(U): R5: L5: x2(∅): R2: x1(∅): x3(∅): R1: x3(U): x1(U) ■

<u>5</u>. Uniform sequential crossing relabeling

x1 → x2, x2 → x1, x3 → x3.

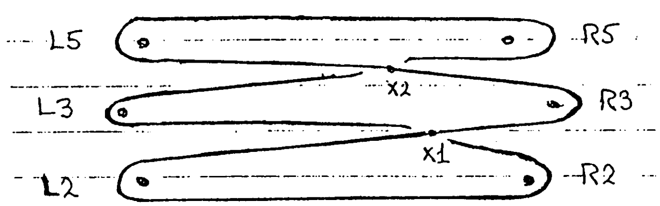
This gives the previous result, which is canonical.

We remark that, often, a "release"-move is immediately followed by an "extension". In this case, it is usually much more economical to perform both operations completely on the given linear sequence, bypassing all intermediate uniform sequential crossing relabelings, and only effecting such a relabeling as the final step in bringing the derivative sequence to its canonical presentation.

The next class of manipulations to be considered are the "pick-up" moves; again, these will all have arguments (as in $\overrightarrow{L1}$(<u>L5n</u>), $\overrightarrow{L2}$((<u>Rp</u>), <u>etc</u>.). Recall that the context, still, is that of simple schemata (in particular, no multiple loops), whence the functor involved will have no representative node in the linear sequence under discussion. As a first example, consider the Calculus operation "$\overrightarrow{L1}$(<u>L5n</u>)" applied to the string-position resulting from the manipulational string

<u>O</u>.A: $\overrightarrow{2\infty}$ → 3: $\overrightarrow{1\infty}$ → 2 |,

whose schema is given in Fig. 155.A, below.



<u>Fig. 155.A</u>: Example (1), Pick-up moves.

|⇒ L2: x1(Ø): R3: x2(Ø):↓L5: R5: x2(U): L3: x1(U): R2 ■

We confirm that the functor L1 does not appear (as an entry) in the linear sequence, and that the argument-node, L5, does; hence the given Calculus-operation is well-defined on the above linear sequence. Next we identify the argument (arc) L5n -- using the previously developed methods -- and find that this string establishes a pointer just before the entry L5 in the given linear sequence (as there indicated). At this point it is tempting -- and egregiously incorrect -- to naively insert the entry "L1" just before the "L5" in the original sequence, <u>viz</u>.

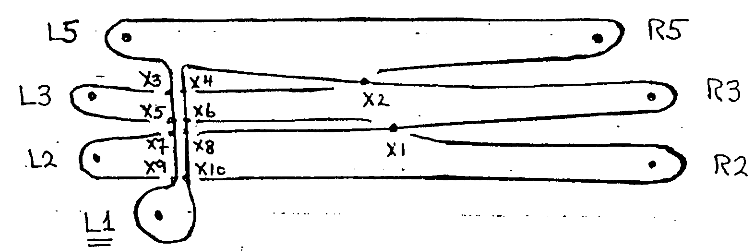
|⇒ L2: x1(Ø): R3: x2(Ø): <u>L1</u>: L5: R5: x2(U): L3: x1(U): R2 ■,

to produce the derived sequence. It should be recalled that -- to date -- the theory requires that the hands be in # before a linear sequence may be associated to the string-position held thereon; and here, **L** -- with L1 lying somewhere between L3 and L5 -- is certainly <u>not</u> in normal position! What the above "mistake" points up, however, is that -- from the standpoint of linear-sequence modification by the string-figure Calculus -- the original problem currently under consideration is ill-posed, and should be restated: Consider the Calculus operation "$\overrightarrow{L1}$(<u>L5n</u>) #" applied to the string-position resulting from the manipulational strin

<u>O</u>.A: $\overrightarrow{2\infty}$ → 3: $\overrightarrow{1\infty}$ → 2 |.

Those complex, multiple-pick-up moves in which a constituent entry is <u>not</u> followec by # will be treated separately, as they arise in context. Thus, for the present discussion, we shall restrict ourselves to pick-up moves that are immediately followed by "#", and hence to the amended problem just enunciated.

The method of solution which we shall employ is that of pulling the object string, L5n, across all intermediary strings (here, towards you) as a long, thin, "spike" to the indicated functor, L1 -- creating two new sequentially ordered crossings for each intermediate string so encountered by said object string.* This is illustrated for the present example by Fig. 155.B, below.



<u>Fig. 155.B</u>: Example (2), Pick-up moves.

|⇒ L1: x9(Ø): x7(Ø): x5(Ø): x3(Ø): L5: R5: x2(U): x4(U): x3(U): L3: x5(U): x6(U): x1(U): R2: x10(U): x9(U): L2: x7(U): x8(U): x1(Ø): R3: x2(Ø): x4(Ø): x6(Ø): x8(Ø): x10(Ø) ■

This linear sequence lacks only uniform sequential relabeling of its constituent crossings from being in its canonical presentation. We remark, at this point,

* A sequence of $Ø_2$-operations.

that we shall adopt the convention that each intermediary loop across which the object string ("spike") is drawn shall intercept the spike as a "pure" loop, i.e. before it suffers involvement (via crossings) with any other strings of the figure. That is, the Calculus operation "$\overrightarrow{L1}$(L5f) #" applied to the string-position of Fig. 155.C, below,

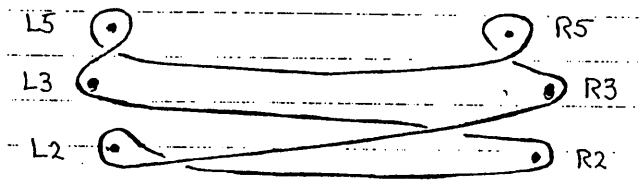


Fig. 155.C: Example (3A), Pick-up moves

is to be considered, by the convention, to produce the string-position

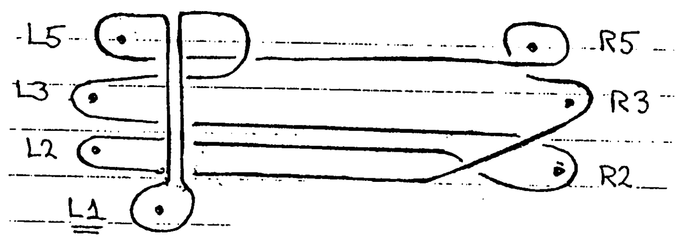


Fig. 155.D: Example (3B), Pick-up moves

rather than, say, one in which the crossing between the L2n and L2f-strings occurs to the L2-side of the L5f-spike.

Returning to the original example --applying "$\overrightarrow{L1}$(L5n) #" to the string-position of Fig. 155.A -- we see that the linear sequence associated to that schema admits the node-subsequence

|⇒ L2 ... R3 ... L5 ... R5 ... L3 ... R2 ■

in which the nodes intermediate to the functor, L1, and the argument, L5n, are precisely L2, L3; whence there are precisely four intermediate strings: L3f, L3n, L2f, L2n -- the first being nearest the argument, the last being nearest the functor (for definiteness). Identifying these in the original linear sequence, we find

L2n L2f   L5n   L3f L3n
|⇒ ↓L2↓: x1(∅): R3: x2(∅):↓L5: R5: x2(U):↓L3↓: x1(U): R2 ■ ,

the pointer for the L5n-string having been previously established. We now create two new (sequentially ordered) crossings on each of these four strings -- to describe the spike's passage across them -- the numerically smallest being closest to the functor to which the given string belongs (again, a convention). Since x2 is the highest-labeled crossing appearing in the original linear sequence, these will be, respectively,

L3f → x4(U): x3(U):
L3n → x5(U): x6(U)
L2f → x7(U): x8(U)
L2n → x10(U): x9(U).

Note that all the above crossing-parities are "Under" by virtue of the arrow above the indicated Calculus manipulation "$\overrightarrow{L1}$(L5n) #". Finally, concerning the spike, itself; since L5n establishes a pointer before L5 in the original linear sequence, we shall have

L5n → x4(∅): x6(∅): x8(∅): x10(∅): L1: x9(∅): x7(∅): x5(∅): x3(∅) ,

the numerically smallest crossing being closest to the functor (L5) to which the string belongs. Thus, had L5n established a pointer just after L5 in the origina linear sequence, we would have found

L5n → x3(∅): x5(∅): x7(∅): x9(∅): L1: x10(∅): x8(∅): x6(∅): x4(∅) ,

again placing the numerically smallest crossing closest to the functor (L5).

We have now accounted for all (new) crossings induced by the Calculus manipulation "$\overrightarrow{L1}$(L5n) #" on the linear sequence associated to the string-position of Fig. 155.A and, inserting the above "crossing-phrases" into this sequence at the indicated arrows, we generate the linear sequence

|⇒ x10(U): x9(U): L2: x7(U): x8(U): x1(∅):R3: x2(∅):
(L2n)   (L2f)
x4(∅): x6(∅): x8(∅): x10(∅): L1: x9(∅): x7(∅): x5(∅): x3(∅): L5:
L5n = the Spike
R5: x2(U): x4(U): x3(U): L3: x5(U): x6(U): x1(U): R2 ■
(L3f)   (L3n)

This linear sequence, when the starting-point node has been brought to lead position by sequence-rotation, will be found to be identical to that associated to Fig. 155.B and is, thus, the derived sequence (when brought to its canonical representation by uniform sequential crossing relabeling). We remark that, in the present example, the derived sequence enjoys the canonical orientation; that this is not always the case will be clear from a consideration of the application of the Calculus manipulation "$\overrightarrow{L1}$(L5f) #", wherein the derived sequence will not have the canonical orientation. In general, the convention Seq. 2 must be "checked" in all cases of a pick-up move involving a "starting-point" functor, when bringing the derived linear sequence to its canonical presentation.

Let us now return to the original problem, with string on hands, and perform the Calculus manipulation "$\overrightarrow{L1}$(L5n) #" on the string-position whose schema is given in Fig. 155.A. The string-position which results has the schema of Fig. 155.E, below.

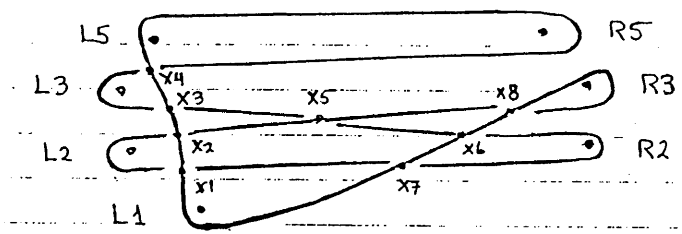


Fig. 155.E: Example (4), Pick-up moves.

|⇒ L1: x1(∅): x2(∅): x3(∅): x4(∅): L5: R5: x4(U): L3: x3(U): x5(U): x6(U): R2: x7(U): x1(U): L2: x2(U): x5(∅): x8(U): R3: x8(∅): x6(∅): x7(∅) ■

This is to be compared to the schema of Fig. 155.B, which also purports to be the result of the above experiment. And while the two schemata involved are not identical, they are ∅-equivalent.* Thus, a shortest linear sequence for either is a shortest linear sequence for both (since they belong to the same ∅-equivalence class) –– and so, at a subsequent "|"-operation, the distinctions between these two distinct experimental outcomes will disappear. Thus our "spike"-implementation of the Calculus manipulation "$\overrightarrow{L1}$(L5n) #" is seen to be entirely general with respect to the ∅-equivalent classes involved.

* To see this ∅-equivalence, begin with Fig. 155.B and perform the ∅-operations

1). $\emptyset_2^{-1}$: $\{x2, x4\} \to \emptyset$,

2). $\emptyset_3$: Draw s;x6–x8 across x1.

The resulting string-position, apart from the names for the individual crossings involved, has the schema of Fig. 155.E, above.

We now consider the application of the Calculus manipulation "$\underrightarrow{L1}$(L5n) #" to the string-position of Fig. 155.A as a minor variation on the above discussion. The accompanying schema will be identical to that of Fig. 155.B, except that now the "spike" will run beneath the (four) transverse strings rather than above them; i.e. the parities of the crossings x3, x4, x5, x6, x7, x8, x9, x10 in that figure will be reversed. Specifically, we shall generate the derived linear sequence

|⇒ $\underbrace{\text{x10(∅): x9(∅)}}_{\text{L2n}}$: L2: $\underbrace{\text{x7(∅): x8(∅)}}_{\text{L2f}}$: x1(∅): R3: x2(∅):

$\underbrace{\text{x4(U): x6(U): x8(U): x10(U): L1: x9(U): x7(U): x5(U): x3(U)}}_{\text{L5n = "Spike"}}$: L5:

R5: x2(U): $\underbrace{\text{x4(∅): x3(∅)}}_{\text{L3f}}$: L3: $\underbrace{\text{x5(∅): x6(∅)}}_{\text{L3n}}$: x1(U): R2 ■

by direct analogy with the case of the previous example. Similarly, applying the complex pick-up move "$\overrightarrow{L1}$(L2f): $\underrightarrow{L1}$(L5n) #" to this string-position (of Fig. 155.A) will generate the analogous derived linear sequence

|⇒ $\underbrace{\text{x10(U): x9(U)}}_{\text{L2n}}$: L2: $\underbrace{\text{x7(U): x8(U)}}_{\text{L2f}}$: x1(∅): R3: x2(∅):

$\underbrace{\text{x4(U): x6(U): x8(∅): x10(∅): L1: x9(∅): x7(∅): x5(U): x3(U)}}_{\text{L5n = "Spike"}}$: L5:

R5: x2(U): $\underbrace{\text{x4(∅): x3(∅)}}_{\text{L3f}}$: L3: $\underbrace{\text{x5(∅): x6(∅)}}_{\text{L3n}}$: x1(U): R2 ■

Et cetera.

The case in which a functor, F_1, passes away from you to pick up the far string of a distinct functor, F_2, is handled in entirely similar fashion –– only, now, the F_2n-string is also an intermediary string (the first, in the ordering) of the process. For definiteness, we consider the application of the Calculus manipulation "$\overrightarrow{L1}$(L5f) #" to the string-position of Fig. 155.A. The derivative linear sequence so generated will be

|⇒ $\underbrace{\text{x12(U): x11(U)}}_{\text{L2n}}$: L2: $\underbrace{\text{x9(U): x10(U)}}_{\text{L2f}}$: x1(∅): R3: x2(∅): $\underbrace{\text{x4(U): x3(U)}}_{\text{L5n}}$: L5:

$\underbrace{\text{x3(∅): x5(∅): x7(∅): x9(∅): x11(∅): L1: x12(∅): x10(∅): x8(∅): x6(∅): x4(∅)}}_{\text{L5f = "Spike}}$:

R5: x2(U): $\underbrace{\text{x6(U): x5(U)}}_{\text{L3f}}$: L3: $\underbrace{\text{x7(U): x8(U)}}_{\text{L3n}}$: x1(U): R2 ■

We remark that this linear sequence has an orientation which is <u>opposite</u> to the canonical one. <u>Et cetera</u>.

By bilateral symmetry, we may discuss the application of the Calculus manipulation "$\overrightarrow{R1}$(<u>R5n</u>) #" to the schema of Fig. 155.A. Explicitly, we will generate the derived linear sequence

$$\models\!\Rightarrow \text{L2: x1}(\emptyset)\text{: }\underbrace{\text{x6(U): x5(U)}}_{\text{R3n}}\text{: R3: }\underbrace{\text{x3(U): x4(U)}}_{\text{R3f}}\text{: x2}(\emptyset)\text{:}$$

$$\text{L5: R5: }\underbrace{\text{x3}(\emptyset)\text{: x5}(\emptyset)\text{: x7}(\emptyset)\text{: x9}(\emptyset)\text{: R1: x10}(\emptyset)\text{: x8}(\emptyset)\text{: x6}(\emptyset)\text{: x4}(\emptyset)\text{:}}_{\text{R5n = "Spike"}}$$

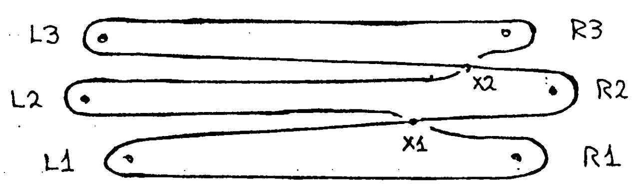
$$\text{x2(U): L3: x1(U): }\underbrace{\text{x8(U): x7(U)}}_{\text{R2f}}\text{: R2: }\underbrace{\text{x9(U): x10(U)}}_{\text{R2n}} \blacksquare$$

which is established by "rote" manipulation of the parent linear sequence (associated to the schema of Fig. 155.A) according to the algorithm established above; a direct construction confirms the validity of the process for this example. <u>Et cetera</u>.

Finally, appealing to the symmetry of the "far-side" viewpoint of string-figure schemata (see pages 35-36 of these notes), we may discuss pick-up moves in which the functor moves towards you in its approach to the object string (argument). To that end, consider the Calculus operation "$\overleftarrow{L5}$(<u>L1f</u>) #" applied to the string-position

$\underline{O}$.A: $\overleftarrow{5}\infty \rightarrow 3\,|$,

whose schema is given in Fig. 155.F, below.



<u>Fig. 155.F</u>: Example (5A), Pick-up moves.

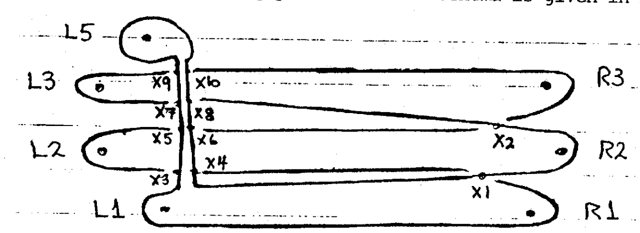
Note that this <u>is</u> Fig. 155.A, under the frame-node relabeling

<u>Fig. 155.A</u>		<u>Fig. 155.B</u>	<u>Fig. 155.A</u>		<u>Fig. 155.B</u>
L2	→	L1	R2	→	R1
L3	→	L2	R3	→	R2
L5	→	L3	R5	→	R3

<u>i.e</u>. under 5 → 3 → 2 → 1. The associated linear sequence is

$\models\!\Rightarrow$ L1: x1(∅): R2: x2(∅): L3: R3: x2(U): L2: x1(U): R1 ■

<u>i.e</u>. this is the linear sequence associated to the schema of Fig. 155.A under the above frame-node relabeling. The "spike"-implementation of the manipulation "$\overleftarrow{L5}$(<u>L1f</u>)#" results in a string-position whose schema is given in Fig. 155.G, below.



<u>Fig. 155.G</u>: Example (5B), Pick-up moves.

$\models\!\Rightarrow$ L1: x3(∅): x5(∅): x7(∅): x9(∅): L5: x10(∅): x8(∅): x6(∅): x4(∅): x1(∅):
R2: x2(∅): x8(U): x7(U): L3: x9(U): x10(U): R3: x2(U): x6(U): x5(U):
L2: x3(U): x4(U): x1(U): R1 ■

Here, the frame-nodes intermediate to the functor, L5, and argument, L1f are precisely L2, L3; whence there are precisely four intermediate strings L2n, L2f, L3n, L3f -- the first being nearest the argument, the last being nearest the functor. Two sequentially-ordered crossings on each of these four strings describe the spike's passage across them, the numerically smallest in each pair of said crossings being closest to the functor to which the given string belongs. The derived linear sequence, directly produced by our algorithm, is thus seen to be

$$\models\!\Rightarrow \text{L1: }\underbrace{\text{x3}(\emptyset)\text{: x5}(\emptyset)\text{: x7}(\emptyset)\text{: x9}(\emptyset)\text{: L5: x10}(\emptyset)\text{: x8}(\emptyset)\text{: x6}(\emptyset)\text{: x4}(\emptyset)\text{:}}_{\text{L1f = "Spike"}}$$

$$\text{x1}(\emptyset)\text{: R2: x2}(\emptyset)\text{: }\underbrace{\text{x8(U): x7(U)}}_{\text{L3n}}\text{: L3: }\underbrace{\text{x9(U): x10(U)}}_{\text{L3f}}\text{: R3: x2(U):}$$

$$\underbrace{\text{x6(U): x5(U)}}_{\text{L2f}}\text{: L2: }\underbrace{\text{x3(U): x4(U)}}_{\text{L2n}}\text{: x1(U): R1} \blacksquare$$

This is identical to the linear sequence associated to Fig. 155.G directly. <u>Et cetera</u>.

This completes our ∅-analysis of the bilaterally specific (<u>i.e</u>. "single-arrow") pick-up moves.* A handy visualization of this process is afforded by

---

* We shall return to the case of "pick-up from above" subsequent to the discussion of "loop-twists", below.

considering Fig. 155.A in the context of the operation "$\overrightarrow{L1}$(L5n)#". Basically, this move produces the Ø-equivalent string-position of Fig. 155.B with L1-node deleted -- created from Fig. 155.A by four $Ø_2$-operations -- into which diagram the frame-node L1 is inserted as indicated (cf. the "naive insertion of page 364). We remark that the wholistic perspective of

$$\vec{I}(\underline{5n})\# \equiv \begin{Bmatrix} \overrightarrow{L1}(\underline{L5n})\# \\ \overrightarrow{R1}(\underline{R5n})\# \end{Bmatrix} \equiv \begin{Bmatrix} \overrightarrow{L1}(\underline{L5n}) \\ \overrightarrow{R1}(\underline{R5n}) \end{Bmatrix}\#$$

now follows from the simultaneous application of the above algorithm to each of the bilateral halves of the parent configuration (see pages 18-19). All the above analyses are entirely general with respect to the Ø-equivalence classes involved.

It remains to discuss the purely wholistic ("double-arrow") pick-up moves typified by, say, "$\overrightarrow{\overrightarrow{LF}}$(Rs)#", where F is the generic finger, s the generic string -- to which matter we now turn our attention; again, we shall initially restrict ourselves to the case of "pick-up from below", deferring the corresponding case of "pick-up from above" until after the discussion of "loop-twists". Our aim is to analyze the present situation via extrapolation of the previous algorithm; the following example will illustrate the ideas involved. Consider the string-position resulting from the manipulational string

O.1: >> R15∞: O.JA |,

whose associated schema is given in Fig. 156.A, below.

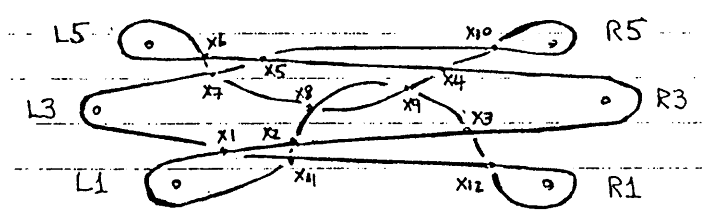


Fig. 156.A: Example (6A), Pick-up moves.

|⇒ L1: x1(Ø): x2(Ø): x3(Ø): R3: x4(Ø): x5(Ø): x6(Ø): L5: x6(U): x7(U): x8(U): x9(Ø): x4(U): x10(U): R5: x10(Ø): x5(U): x7(Ø): L3: x1(U): x11(Ø): x12(Ø): R1: x12(U): x3(U): x9(U): x8(Ø): x2(U): x11(U) ∎

And let us consider the application of the pick-up move "$\overrightarrow{\overrightarrow{L2}}$(R5n)#" to this string-position. On the surface, at first glance, this appears to be a far more formidable problem in linear sequence crossing analysis than was, say, the similar-appearing application of "$\overrightarrow{L2}$(L5n)#" to this string-position -- owing to the crossings induced by the central tangle of strings between the hands, over which the object string, R5n, must somehow pass in this new situation. However, as will be seen below, the "spike - implementation" method of the previous examples may be profitably extrapolated to the present situation. Diagrammatically, we proceed

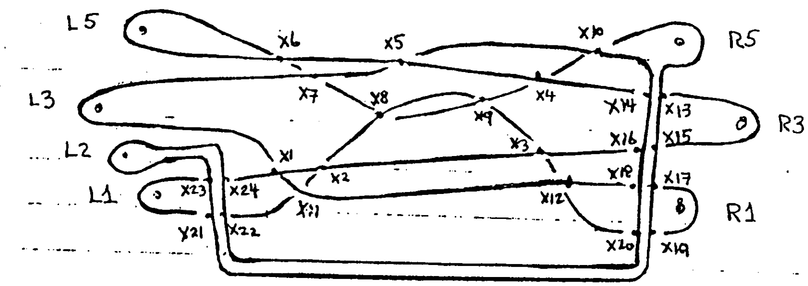


Fig. 156.B: Example (6B), Pick-up moves.

|⇒ x22(U): x21(U): L1: x23(U): x24(U): x1(Ø): x2(Ø): x3(Ø):
(x22(U): x21(U) = L1n; x23(U): x24(U) = L1f)

x16(U): x15(U): R3: x13(U): x14(U): x4(Ø): x5(Ø): x6(Ø): L5:
(x16(U): x15(U) = R3n; x13(U): x14(U) = R3f)

x6(U): x7(U): x8(U): x9(Ø): x4(U): x10(U): R5:

R5n "Spike" { x13(Ø): x15(Ø): x17(Ø): x19(Ø): x21(Ø): x23(Ø): L2:
x24(Ø): x22(Ø): x20(Ø): x18(Ø): x16(Ø): x14(Ø): }

x10(Ø): x5(U): x7(Ø): L3: x1(U): x11(Ø): x12(Ø):

x18(U): x17(U): R1: x19(U): x20(U): x12(U): x3(U): x9(U):
(x18(U): x17(U) = R1f; x19(U): x20(U) = R1n)

x8(Ø): x2(U): x11(U) ∎

Thus we create an Ø-equivalent of the final position which avoids the central tangle. As before, a subsequent "|" will elide any distinctions between members of the same Ø-equivalence class. We remark that, since the functor, L2, is nearer to you than is the argument, the R5n-string (i.e. 2 < 5), the spike must proceed first towards you, then to the left, and away from you to the indicated functor in order to constitute a correct rendering of the manipulation "$\overrightarrow{\overrightarrow{L2}}$(R5n)#" as applied to this string-position. And, of course, the manipulation "$\underset{\Rrightarrow}{\text{L2}}$(R5n)#" will be spike-implemented exactly as in Fig. 156.B, except that the parities of all spike-crossings, x13 through x24, must be reversed. Et cetera.

Compare the previous situation with that of applying the Calculus manipulation "$\overset{\Rrightarrow}{\text{L4}}$(R1f)#" to the string-position of Fig. 156.A. Diagrammatically,

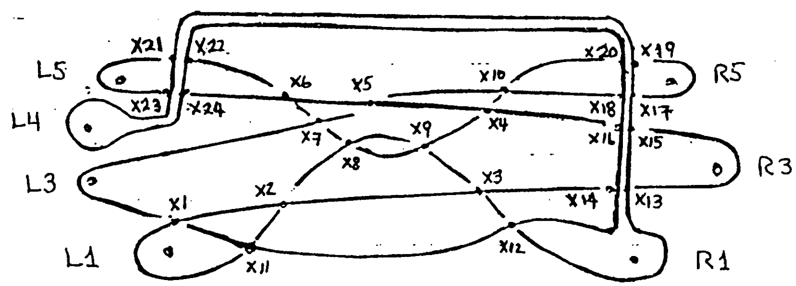


Fig. 156.C: Example (6C), Pick-up moves.

|⇒ L1: x1(∅): x2(∅): x3(∅): $\underbrace{\text{x14(U): x13(U)}}_{\text{R3n}}$: R3: $\underbrace{\text{x15(U): x16(U)}}_{\text{R3f}}$: x4(∅):

x5(∅): x6(∅): $\underbrace{\text{x24(U): x23(U)}}_{\text{L5n}}$: L5: $\underbrace{\text{x21(U): x22(U)}}_{\text{L5f}}$: x6(U): x7(U):

x8(U): x9(∅): x4(U): x10(U): $\underbrace{\text{x20(U): x19(U)}}_{\text{R5f}}$: R5: $\underbrace{\text{x17(U): x18(U)}}_{\text{R5n}}$:

x10(∅): x5(U): x7(∅): L3: x1(U): x11(∅): x12(∅):

R1f "Spike" { x14(∅): x16(∅): x18(∅): x20(∅): x22(∅): x24(∅): L4: x23(∅): x21(∅): x19(∅): x17(∅): x15(∅): x13(∅):

R1: x12(U): x3(U): x9(U): x8(∅): x2(U): x11(U)■

In this case, since the functor, L4, is further away from you than is the argument, the R1f-string (i.e. 4 > 1), the spike must proceed first away from you, then left, and towards you to the indicated functor in order to give a correct rendering of the manipulation "$\overset{\Rrightarrow}{\text{L4}}$(R1f)#" as applied to this string-position. Et cetera.

It remains to discuss what is, perhaps, the principal application of the "double-arrow" pick-up moves, in which the functor picks up a palmar string of the opposite hand. And here, in the corresponding spike-implementation of this class of moves, the direction of the spike is immaterial; we shall always choose "towards you" for this direction, for definiteness. Again, we proceed by example: Consider the string-position resulting from the manipulational string

O.1: >> R15∞: $\overset{\Lleftarrow}{\text{R2}}$(Lp)# |

whose associated schema is given in Fig. 156.D, below.

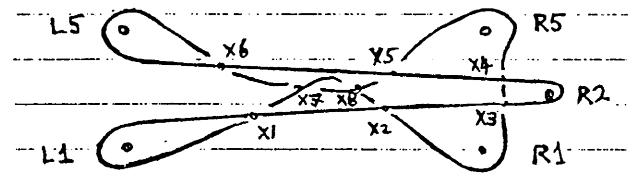


Fig. 156.D: Example (7A), Pick-up moves.

|⇒ L1: x1(∅): x2(∅): x3(∅): R2: x4(∅): x5(∅): x6(∅): L5: x6(U): x7(U): x8(∅): x5(U): R5: x4(U): x3(U): R1: x2(U): x8(U): x7(∅): x1(U)■

Let us apply the complex pick-up move "$\overset{\Rrightarrow}{\text{L2}}$↓(R2∞): $\underset{\Rrightarrow}{\text{L2}}$(Rp)#" to this string-position. Here, the spike-implementation of this manipulation produces the schema

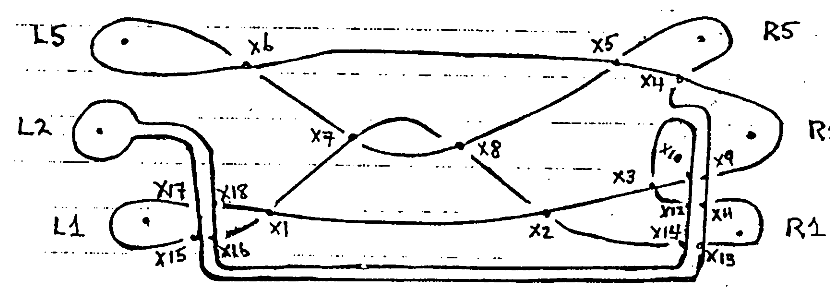


Fig. 156.E: Example (7B), Pick-up moves,

where the initial direction for the spike (towards you) is an arbitrary convention. For the associated linear sequence, we have

|⇒ $\underbrace{\text{x16(U): x15(U)}}_{\text{L1n}}$: L1: $\underbrace{\text{x17(U): x18(U)}}_{\text{L1f}}$: x1(∅): x2(∅): x3(∅): $\underbrace{\text{x10(U): x9(U)}}_{\text{R2n}}$:

R2: x4(∅): x5(∅): x6(∅): L5: x6(U): x7(U): x8(∅): x5(U): R5: x4(U):

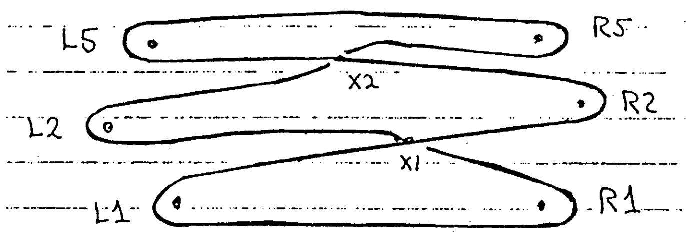
Rp = "Spike" { x9(∅): x11(∅): x13(∅): x15(∅): x17(∅): L2: x18(∅): x16(∅): x14(∅): x12(∅): x10(∅):

x3(U): $\underbrace{\text{x12(U): x11(U)}}_{\text{R1f}}$: R1: $\underbrace{\text{x13(U): x14(U)}}_{\text{R1n}}$: x2(U): x8(U): x7(∅): x1(U)■

The Calculus manipulation "$\underset{\Rrightarrow}{\text{L2}}$(Rp)#" will be spike-implemented exactly as in

Fig. 156.E, <u>except</u> that the parities of all spike-crossings, x9 through x18, must be reversed. <u>Et cetera</u>.

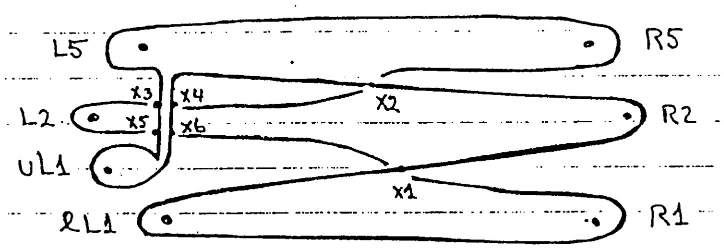
This completes our Ø-analysis of the Calculus manipulations described by "pick-up from below" moves in the absence of multiple loops. In every case the above algorithm produces the outcome of such an operation by rote manipulation of the parent associated linear sequence, <u>without</u> appeal to the schema involved therewith; the operations from the string-figure Calculus thus far discussed are therefore seen to be independent of the underlying schemata associated to them. And the extension of this analysis to the case of string-positions with distinct multiple loops is straightforward and immediate, <u>via</u> introduction of the split-node schemata of Section, I., Systemology (pages 20-23). In particular, on the string-position <u>O</u>.A,



<u>Fig. 157.A</u>: Example (8A), Pick-up moves,

|⇒ L1: x1(Ø): R2: x2(Ø): L5: R5: x2(U): L2: x1(U): R1 ■

if we now wish to apply the Calculus manipulation "$\overrightarrow{L1}$(<u>L5n</u>)#" -- thus producing two (distinct) loops, ℓL1∞, uL1∞, on L1 -- we first replace the linear-sequence entry "L1" by "ℓL1", and then proceed <u>via</u> the algorithm to effect the indicated pick-up on the (loopless) functor "uL1" lying between ℓL1 and L2 in the relevant frame-diagram. This results in the string-position of Fig. 157.B, below.



<u>Fig. 157.B</u>: Example (8B), Pick-up moves.

$$\mapsto \ell L1:\ x1(\emptyset):\ R2:\ x2(\emptyset):\ \underbrace{x4(\emptyset):\ x6(\emptyset):\ uL1:\ x5(\emptyset):\ x3(\emptyset)}_{L5n\ =\ \text{"Spike"}}:\ L5:\ R5:$$

$$x2(U):\ \underbrace{x4(U):\ x3(U)}_{L2f}:\ L2:\ \underbrace{x5(U):\ x6(U)}_{L2n}:\ x1(U):\ R1\ \blacksquare$$

It should be apparent that the above analysis is entirely independent of the number of distinct nodes in the underlying frame-diagram, and relies only on their relative positions (see <u>e.g</u>. Fig. 16, page 23). <u>Et cetera</u>.

The last collection of string-figure Calculus manipulations for Ø-analysis in the present section is that of the loop-specific operations, the first of which -- release -- was discussed previously in the context of singleton loops. The corresponding case for distinct multiple loops in the split-node schema is entirely similar, the sole added (easy!) complication being the occasional need of renaming the loops remaining on a given functor after a loop has been released therefrom (see pages 22-23 for a discussion of "loop demotion", and the "alias" viewpoint of loop-manipulations in general). For example, consider the linear sequence associated to Fig. 29.V (page 58) in the construction of Osage Diamonds:

|⇒ ℓL1: x1(U): x2(U): x3(Ø): uR1: x4(Ø): R2: x4(U): x3(U): x2(Ø): x5(Ø): x6(U): x7(U): L2: x7(Ø): uL1: x6(Ø): x5(U): x1(Ø): ℓR1 ■

And suppose, now, that we wished to apply the Calculus manipulation "□uL1" to this string-position. We proceed, as before indicated, by deleting the entry "uL1" from the above linear sequence, and then renaming all L1-nodes (<u>i.e</u>. loops remaining on L1) -- if any -- after the usual convention (page 21). In the present case, since only one loop remains on L1 after the indicated manipulation, this is accomplished by replacement of the entry "ℓL1" in the above linear sequence by the symbols "L1" -- or, by deletion of the initial symbol of "ℓL1", if you prefer. The derived linear sequence in this case is thus given by

$$\mapsto L1:\ x1(U):\ x2(U):\ x3(\emptyset):\ uR1:\ x4(\emptyset):\ R2:\ x4(U):\ x3(U):\ x2(\emptyset):\ x5(\emptyset):$$

$$\underbrace{x6(U):\ x7(U)}:\ L2:\ \underbrace{x7(\emptyset):\ x6(\emptyset)}:\ x5(U):\ x1(\emptyset):\ \ell R1\ \blacksquare$$

Here we recognize the $\emptyset_2^{-1}$-cancellable crossing-pair x6, x7, which would disappear under a subsequent "|"-operation. <u>Et cetera</u>.

Next we discuss the distinguished loop-specific manipulation "Navaho"; let F be a generic finger, occurring in a given string-postion, which is incident with exactly two distinct loops -- ℓF∞, uF∞. We shall diagram the operation

"NF" as follows:

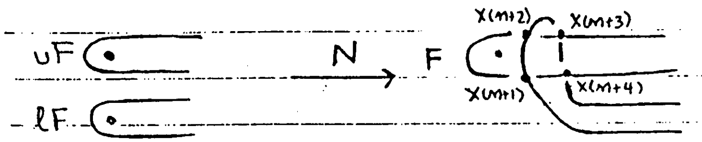


Fig. 158.A: Example (1), Navaho.

Here, xn is the numerically maximal-labeled crossing in the schema associated to the given string-position prior to the application of the manipulation "NF" thereto. The "Navaho" move introduces four new crossings into this schema; x(n+1), x(n+2), x(n+3), x(n+4); as illustrated in Fig. 158.A, above. Also, the functor nodes ℓF, uF have been replaced by the singleton node F. [The corresponding diagram in which F is a right functor will be the mirror image of that in Fig. 158.A.] Thus, the linear sequence associated to the original string-position shows ℓF, uF as (the only) F-entries; to produce the derived sequence we must identify the ℓFf- and uFf-strings' position in the original linear sequence. There are two substitutions to be made:

N.1. If ⊨ ...$\overset{uFf}{\downarrow}$uF...■ , replace the entry "uF" in the (original) linear sequence by

x(n+3)(∅): x(n+2)(U): F: x(n+1)(U): x(n+4)(∅);

otherwise, make the above substitution with the reversed sequence, i.e.

x(n+4)(∅): x(n+1)(U): F: x(n+2)(U): x(n+3(∅).

N.2. If ⊨ ...$\overset{ℓFf}{\downarrow}$ℓF...■, replace the entry "ℓF" in the linear sequence by

x(n+4)(U): x(n+3)(U): x(n+2)(∅): x(n+1)(∅);

otherwise, make the above substitution with the reversed sequence.

Note that the above substitutions may be directly read-off from Fig. 158.A by following the constituent loops in the directions indicated by the placement of their "far"-strings in the associated linear sequence. Further, the entry "uF" is deleted from the original linear sequence by N.1, while "ℓF" is deleted by N.2. The entry "F" is inserted into the derived sequence by N.1 and -- in every case -- each of the four newly created crossings occurs exactly twice in the de-

rived sequence, the two occurrences being marked by opposite parities. We remark that, when F is the starting-point node of the original linear sequence, we will always be in the subcase

⊨ ... $\overset{ℓFf}{ℓF\downarrow}$...■

of the substitution N.2 -- by the convention Seq. 2. Thus, for example, to apply the manipulation "N(L1)" to the string-position obtained from

O.A: $\overrightarrow{L1}$(L5n)# |,

illustrated below,

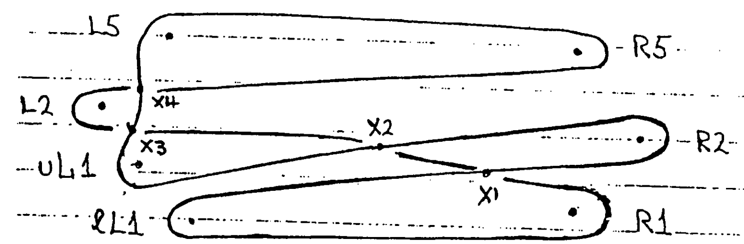


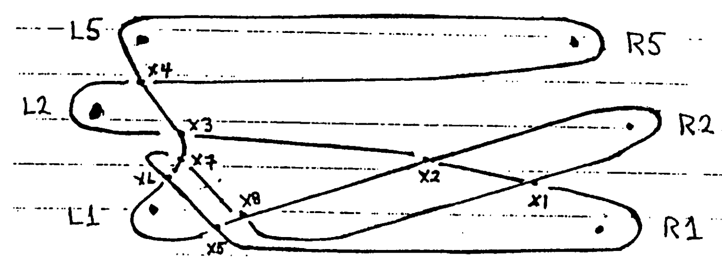
Fig. 158.B: Example (2A), Navaho,

⊨ ℓL1↓: x1(∅): R2: x2(∅): uL1↓: x3(∅): x4(∅): L5: R5: x4(U): L2: x3(U): x2(U): x1(U): R1■

we note that, here, n=4; and both ℓL1f- and uL1f-strings follow their respective nodes in the above linear sequence. Thus, by the algorithm, the derived linear sequence is

⊨ x5(∅): x6(∅): x7(U): x8(U): x1(∅): R2: x2(∅):
x8(∅): x5(U): L1: x6(U): x7(∅): x3(∅): x4(∅):
L5: R5: x4(U): L2: x3(U): x2((U): x1(U): R1■

The accompanying schema is given in Fig. 158.C, below.



And we directly verify (i.e. by construction) that the above schema is associated to the string-position resulting from the manipulational sequence

$$\underline{O}.A: \overrightarrow{L1}(\underline{L5n})\#\ |: N(L1).$$

We remark that the above derived linear sequence may be brought to its canonical presentation by leftward rotation and uniform sequential crossing relabeling; i.e. it is correctly oriented.*

We close our discussion of the Calculus operation "Navaho" with a brief examination of another example of the four possible cases of the above substitution algorithm. The string-position resulting from the manipulational sequence

$$\underline{O}.A: <L1\infty:\ >R2\infty:\ \overleftarrow{L5}(\underline{L1n})\#\ |$$

is illustrated in the schema of Fig. 158.D, below.

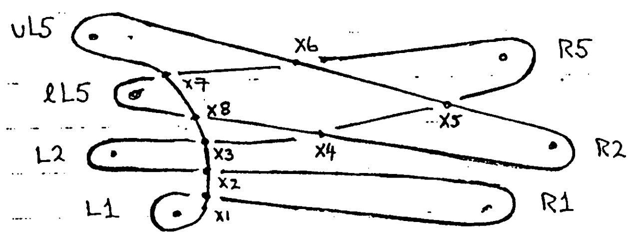


Fig. 158.D: Example (3A), Navaho.

|⇒ L1: x1(U): R1: x2(U): L2: x3(U): x4(U): x5(U): R5: x6(U): x7(U):
↓ℓL5: x8(U): x4(∅): R2: x5(∅): x6(∅): ↓uL5: x7(∅): x8(∅): x3(∅):
x2(∅): x1(∅)■

Here n=8, and both ℓL5f- and uL5f-strings precede their respective nodes in the above linear sequence. Thus, by the substitution algorithm, application of the Calculus operation "N(L5)" to the above string-position will produce the derived sequence

|⇒ L1: x1(U): R1: x2(U): L2: x3(U): x4(U): x5(U): R5: x6(U): x7(U):
x12(U): x11(U): x10(∅): x9(∅): x8(U): x4(∅): R2: x5(∅): x6(∅):
x11(∅): x10(U): L5: x9(U): x12(∅): x7(∅): x8(∅): x3(∅): x2(∅): x1(∅)■

The accompanying schema is given in Fig. 158.E, below, for the purposes of comparison.

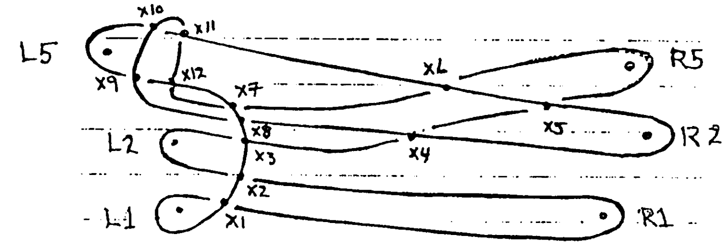


Fig. 158.E: Example (3B), Navaho.

Here we recognize the $\emptyset_2^{-1}$-cancellable crossing-pair x7, x12 -- and subsequent pair x6, x11 -- which would disappear under a later "|"-operation. Et cetera.

This concludes our discussion of the Calculus operation "Navaho". We remark that, as usual, the wholistic perspective of

$$NF \equiv \begin{Bmatrix} N(LF) \\ N(RF) \end{Bmatrix}$$

now follows from the simultaneous application of the above algorithm to each of the bilateral halves of the parent configuration. All of the above analyses are entirely general with respect to the ∅-equivalence classes involved.

The next loop-specific operation to be considered is the twist. Let F be a generic functor, occurring in a given string-position*, which is incident with an unique loop, F∞. We shall diagram the "twist" operations as follows:

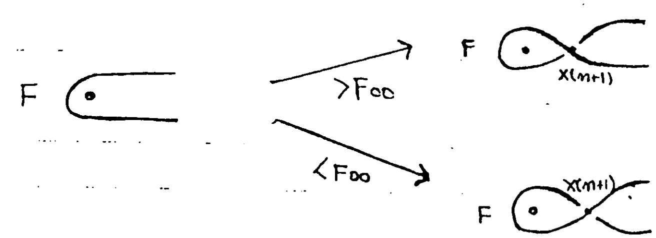


Fig. 159.A: Example (1), Twists.

Here, xn is the numerically maximal-labeled crossing in the schema associated to

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* This need not be true, in general.

* Otherwise, > F∞ = Φ.

the given string-position prior to the application of a twist operation thereto; either of the moves "> F∞, < F∞" introduces exactly one new crossing, x(n+1), into this schema, as illustrated in Fig. 159.A, above. [The corresponding diagram in which F is a right functor will be the mirror image of that in Fig. 159.A.] The modification to the linear sequence associated to the original string-position induced by the operations "> F∞, < F∞" consists of a single substitution, determined (in either case) by the position of the Ff-string in the original linear sequence. In particular,

"Twist": If ⊨ ... $\overset{Ff}{\downarrow}$F ... ■, replace the entry "F" in the (original) linear sequence by

1). > F∞:

F —> x(n+1)(U): F: x(n+1)(∅),

2). < F∞:

F —> x(n+1)(∅): F: x(n+1)(U),

respectively; otherwise, make the above substitution with the right-hand side sequence reversed.

Note that the above substitutions may be directly read-off from Fig. 159.A by following the constituent loop in the direction indicated by the placement of its "far" string in the original linear sequence.

And we may treat multiple loop-twists, e.g. ">> F∞", by concatenating singleton twists, viz. "> F∞: > F∞". Let us treat this situation, completely, in the case

⊨ ... $\overset{Ff}{\downarrow}$F ... ■

as a final "exercise" before leaving the present subsection. By the above algorithm, the Calculus operation "> F∞" modifies the original linear sequence to

⊨ ... x(n+1)(U): $\overset{Ff}{F^{\downarrow}}$: x(n+1)(∅) ... ■

in which the Ff-string now immediately follows F in the derived linear sequence. Hence, applying the algorithm for the second "> F∞" operation to this new linear sequence, we produce the second derived linear sequence

⊨ ... x(n+1)(U): x(n+2)(∅): F: x(n+2)(U): x(n+1)(∅) ... ■

The corresponding operation, ">> F∞", thus -- in this case -- may be presented diagrammatically by

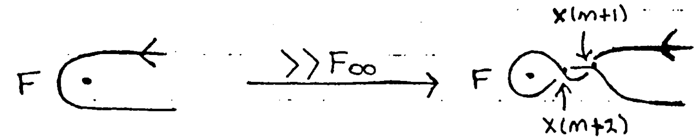

Fig. 159.B Example (2), Twists,

which is directly verified by performing this experiment in the string. Et cetera

Using the above discussion of loop-twists, we may now complete our discussion of pick-up moves, initiated earlier (pages 364-377). We have yet to discuss the moves "pick-up from above", which we now realize via the equivalent concatenated pair, "pick-up from below" followed by the appropriate "180°-twist" (see page 17 for this discussion). For example,

$\overrightarrow{L1}(\overline{L5n})\# \equiv \overrightarrow{L1}(\underline{L5n})\#$: > L1∞,

$\overleftarrow{L4}(\overline{L1n})\# \equiv \overleftarrow{L4}(\underline{L1n})\#$: < L4∞,

$\underrightarrow{L2}(\overline{R5n})\# \equiv \underrightarrow{L2}(\underline{R5n})\#$: > L2∞,

et cetera. Since the right-hand sides of all such expressions have been completel ∅-analyzed, we may consider the corresponding left-hand sides to be similarly complete in these regards.

The last of the loop-specific operations to be considered in the present section is that of the translation of loops; there are several subtopics. The first of these to which we turn our attention is the matter of inter-digit loop-transfer The discussion which follows will be strongly reminiscent of the spike-implementation procedure, introduced earlier in the context of the pick-up moves. Form the string-position, O.A, whose schema is given in Fig. 160.A, below.

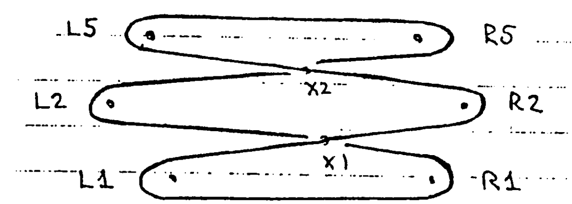

Fig. 160.A: Example (1A), Loop-transfer.

⊨ L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■

We shall apply the Calculus operation "$\overrightarrow{L1\infty}$ —> L4" to this string-position by pulling the L1∞ away from you across all intermediary strings as a long, thin spike to the indicated functor, L4 -- thus creating two new sequentially ordered crossings for each intermediate string so encountered by the traveling spike. The result is illustrated in Fig. 160.B, below.

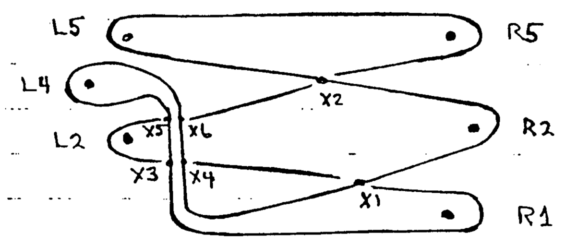

Fig. 160.B: Example (1B), Loop-transfer.

|⇒ L2: x5(U): x6(U): x2(U): R5: L5: x2(∅): R2: x1(∅): x4(∅): x6(∅): L4: x5(∅): x3(∅): R1: x1(U): x4(U): x3(U) ■

Algorithmically, we identify the intermediary strings over which the spike must pass by an examination of the frame-node subsequence of the original linear sequence; that is

|⇒ L1 ... R2 ... L5 ... L2 ... ■

Whence there is exactly one frame-node, L2, lying between L1 and L4; thus there are precisely two intermediate strings: L2n, L2f -- ordered from loop to functor, for definiteness. Identifying these in the original linear sequence, we find

$$\models\!\!\Rightarrow \text{L1 ... R2 ... L5 ...}\overset{\text{L2f}}{\downarrow}\text{L2}\overset{\text{L2n}}{\downarrow}\text{...} \blacksquare$$

We next create two new (sequentially ordered) crossings on each of these two intermediate strings -- to describe the spike's passage across them -- the numerically smallest crossing being closest to the functor to which the given string belongs (a convention). Since x2 is the numerically highest-labeled crossing appearing in the original linear sequence, these will be, respectively,

L2n → x3(U): x4(U)
L2f → x6(U): x5(U).

Of course, all the above crossing-parities are "Under" by virtue of the arrow above the indicated operation "$\overrightarrow{\text{L1}\infty}$ → L4". About the spike, itself -- since L4

is further away from you than is L1, if L1n establishes a pointer before L1 in the original linear sequence, we shall have

L1 → x3(∅): x5(∅): L4: x6(∅): x4(∅),

while if L1n establishes a pointer after L1, we shall have the reversed substitution

L1 → x4(∅): x6(∅): L4: x5(∅): x3(∅).

Here, of course L1 is the starting-point node for the canonical linear sequence, whence it is the first substitution that is pertinent.

We have now accounted for all (new) crossings induced by the operation "$\overrightarrow{\text{L1}\infty}$ → L4" on the linear sequence associated to the string-position of Fig. 160.A and, inserting the above "crossing-phrases" into this sequence at the indicated arrows, we generate the derived linear sequence

$$\models\!\!\Rightarrow \underbrace{\text{x3(∅): x5(∅): L4: x6(∅): x4(∅)}}_{\text{L1}}\text{: x1(∅): R2: x2(∅): L5:}$$
$$\text{R5: x2(U): }\underbrace{\text{x6(U): x5(U)}}_{\text{L2f}}\text{: L2: }\underbrace{\text{x3(U): x4(U)}}_{\text{L2n}}\text{: x1(U): R1}\blacksquare$$

Note that, since L2f precedes the starting-point entry, L2, in this derived sequence, it fails to be canonically oriented. We may check that leftward sequence rotation -- to bring L2 to lead position -- followed by sequence reversal, produces the linear sequence directly associated to the string-position schematized in Fig. 160.B; it thus lacks only uniform sequential relabeling of its crossings of being in its canonical presentation.

In exactly like manner, the Calculus operations

$\underrightarrow{\text{L1}\varphi}$ → L4,
$\overrightarrow{\text{L1}\infty}$↓(L2∞): $\underrightarrow{\text{L1}\varphi}$ → L4,
$\overleftarrow{\text{L5}\infty}$ → L1,
$\overrightarrow{\text{R1}\infty}$ → R3,

et cetera may be reduced to an algorithmic analysis of the linear sequences associated to the underlying string-positions -- without appeal to the schemata, themselves. The arguments involved parallel the corresponding cases in the analysis of the (bilaterally specific) pick-up moves, and will be omitted.

The next loop-specific operation to be discussed is that of intra-digit loop-transfer, i.e. permutations of the loops on a single functor according to well-defined rules as to their under/over passages. And, since any such permutation is decomposable into a product of transpositions* of adjacent pairs, it suffices to treat the case in which the generic functor, F, has exactly two distinct loops -- ℓF∞, uF∞ -- i.e. A. ℓF∞ → uF∞, and B. uF∞ → ℓF∞ (see page 27 for these definitions). As usual, we assume a canonical linear sequence whose numerically maximum-labeled crossing is xn.

Case A: ℓF∞ → uF∞

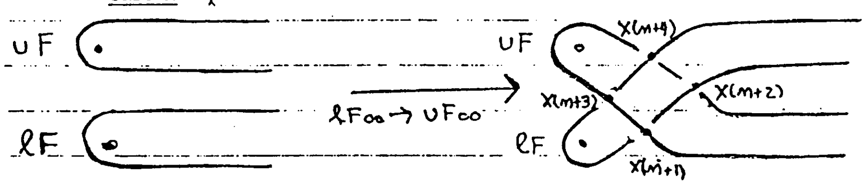

Fig. 161.A: ℓF∞ → uF∞.

The operation "ℓF∞ → uF∞" introduces four new crossings -- x(n+1), x(n+2), x(n+3), x(n+4) -- into the derived linear sequence, as illustrated in Fig. 161.A above. There are two substitutions to be made.

(ℓF∞ → uF∞). 1: If $\models\!\!\Rightarrow \ldots\ uF\overset{\ell Ff}{\downarrow}\ldots\blacksquare$, then
ℓF → x(n+1)(∅): x(n+3)(∅): uF: x(n+4)(U): x(n+2)(U).

Otherwise, make the above substitution with the right-hand side sequence reversed.

(ℓF∞ → uF∞). 2: If $\models\!\!\Rightarrow \ldots uF\overset{uFf}{\downarrow}\ldots\blacksquare$, then
uF → x(n+2)(∅): x(n+1)(U): ℓF: x(n+3)(U): x(n+4)(∅).

Otherwise, make the above substitution with the right-hand side sequence reversed.

Here, in the derived sequence, ℓF and uF have exchanged places from the original linear sequence, each "packed" on either side with two of the four new crossings.

* Hall, M. Jr.: THE THEORY OF GROUPS. The Macmillan Company, New York (1959). p. 60.

Case B: uF∞ → ℓF∞

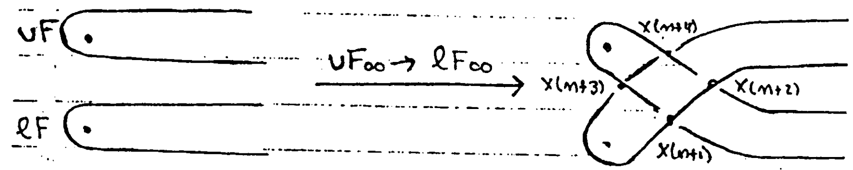

Fig. 161.B: uF∞ → ℓF∞.

Again, the operation "uF∞ → ℓF∞" introduces four new crossings -- x(n+1), x(n+2), x(n+3), x(n+4) --as illustrated in Fig. 161.B. And, again, there are two substitutions to be made.

(uF∞ → ℓF∞). 1: If $\models\!\!\Rightarrow \ldots\ F\overset{\ell Ff}{\downarrow}\ldots\blacksquare$, then
ℓF → x(n+1)(U): x(n+3)(∅): uF: x(n+4)(∅): x(n+2)(U).

Otherwise, make the above substitution with the right-hand side sequence reversed.

(uF∞ → ℓF∞). 2: If $\models\!\!\Rightarrow \ldots uF\overset{uFf}{\downarrow}\ldots\blacksquare$, then
uF → x(n+2)(∅): x(n+1)(∅): ℓF: x(n+3)(U): x(n+4)(U).

Otherwise, make the above substitution with the right-hand side sequence reversed.

It should be noted that, in the derived sequences in both this and the preceding cases, each of the newly-introduced crossings appears exactly twice, the distinct occurrences being marked by opposite parities.

As an exercise, let us solve the following

Problem: Derive the linear-sequence substitutions corresponding to the manipulational phrase

ℓF∞ → uF∞: □ uF∞

in the case where the far strings of both ℓF, uF precede them in the original linear sequence.

Solution: Referring to Case A, above, for this situation we find that the appropriate substitutions for the phrase "ℓF∞ → uF∞" are, respectively,

{ ℓF → x(n+2)(U): x(n+4)(U): uF: x(n+3)(∅): x(n+1)(∅),
{ uF → x(n+4)(∅): x(n+3)(U): ℓF: x(n+1)(U): x(n+2)(∅).

Next, to the sequence so derived, we apply the second entry in the given manipulational phrase, "□ uF∞" (see page 377). This will produce the second derived sequence -- i.e. the solution to the problem -- determined by the substitutions

$$\begin{cases} \ell F \to x(n+2)(U):\ x(n+4)(U):\ x(n+3)(\emptyset):\ x(n+1)(\emptyset), \\ uF \to x(n+4)(\emptyset):\ x(n+3)(U):\ F:\ x(n+1)(U):\ x(n+2)(\emptyset). \end{cases}$$

The above solution is more recognizable if we uniformly relabel the four constituent crossings as follows:

$$x(n+1) \to x(n+1), \qquad x(n+3) \to x(n+2),$$
$$x(n+2) \to x(n+4), \qquad x(n+4) \to x(n+3).$$

Under this relabeling, the above linear sequence substitutions corresponding to the phrase

$$\ell F\infty \to uF\infty:\Box\, uF\infty$$

become

$$\begin{cases} \ell F \to x(n+4)(U):\ x(n+3)(U):\ x(n+2)(\emptyset):\ x(n+1)(\emptyset), \\ uF \to x(n+3)(\emptyset):\ x(n+2)(U):\ F:\ x(n+1)(U):\ x(n+4)(\emptyset) \end{cases}$$

which are N2 and N1, respectively (see page 378). That is, under the stated conditions,

$$NF \equiv \ell F\infty \to uF\infty:\Box\, uF\infty,$$

a valid assertion from our earlier discussion of these moves. The present discussion, of course, takes place in a "schema-free" context.

The penultimate Calculus manipulation to be discussed in the present section is the (direct) cross-hand loop-transfer. To that end, let F_1, F_2 be generic functors, and suppose that a given string-position entails a single loop on RF_1, while LF_2 is incident with no loop. We wish to transfer the $RF_1\infty$ to LF_2 as follows:

$$RF_1\infty \Rightarrow LF_2 \equiv \overset{\Rightarrow}{LF}_2 \downarrow (RF_1\infty):\Box RF_1\#\,|.$$

We shall realize this manipulation by the method of spike-implementation -- by now thoroughly understood -- illuminating only the new "twist" that arises in this context.

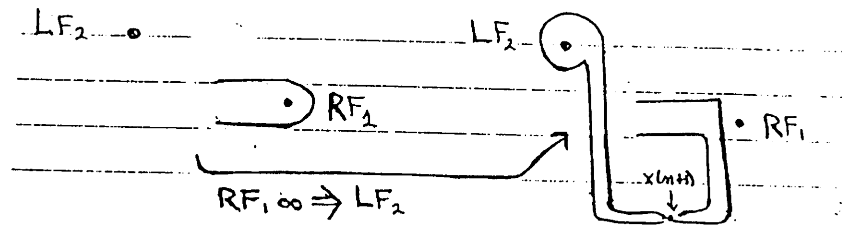

Fig. 162: $RF_1\infty \Rightarrow LF_2$.

Here, the $RF_1\infty$ has been released, and then drawn as a long, thin spike to the near side of the figure (without twisting) over all intermediate strings (if any). Thence across to the left side of the figure with a single 180°-twist away in the process; finally the spike is drawn directly away to LF_2 over all intermediate strings (if any). The spike is, ultimately, place on LF_2 as a simple loop.

Clearly, this process is schema-independent, as the linear-sequence specific machinery which accomodates the $RF_1\infty$'s passage (as a spike) has already been firmly established. The sole novelty of the present analysis is the minimal one of inserting a 180°-twist away from you between the spike's passage towards you over the near-side strings of $\mathbb{R}$, and its passage away from you over the near-side strings of $\mathbb{L}$; and this is trivial. We remark that the symmetric move, $LF_1\infty \Rightarrow RF$ has accompanying diagram which is the mirror-image* of Fig. 162 -- with L and R frame-nodes suitably relabeled; the spike-analysis is, of course, symmetric.

The final loop-specific manipulation from the string-figure Calculus to be discussed in the present section is the interhand loop "exchange" (see pages 27-28 for the definition). That is, given a string-position in which, for the generic functor, F, there is an unique $LF\infty$ and an unique $RF\infty$, we wish to discuss the operations "XF(L)" and "XF(R)". The method of the previous topic, the direct cross-hand transfer via spike-implementation, will be employed herein, and only the novelties of the present case will be discussed. An illustrative example will clarify the (schema-independent) method: Consider the string-position O.A, whose schema is given in Fig. 163.A, below.

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* In particular, the singleton 180°-twist is still in the direction "away from you", as it should be.

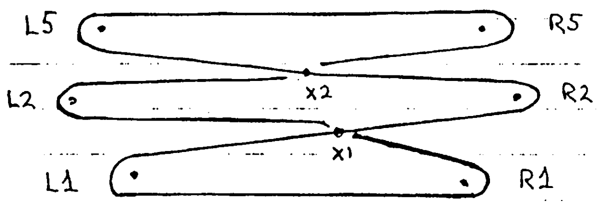

Fig. 163.A: Example (1A), loop-exchange.

⊨> L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■

We shall apply the Calculus manipulation "X2(L)" to this string-position according to the schema-diagram of Fig. 163.B, below.

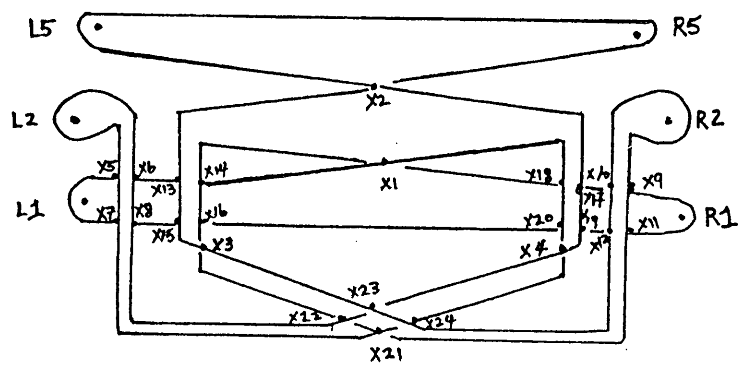

Fig. 163.B: Example (1B), loop-exchange; X2(L).

⊨> L1: x5(U): 6(U): x13(U): x14(U): x1(∅): x18(∅): x20(∅): x4(U): x24(U): x21(∅): x7(∅): x5(∅): L2: x6(∅): x8(∅): x22(∅): x23(U): x4(∅): x19(∅): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x15(∅): x3(∅): x23(∅): x24(∅): x12(∅): x10(∅): R2: x9(∅): x11(∅): x21(U): x22(U): x3(U): x16(∅): x14(∅): x1(U): x18(U): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x20(U): x16(U): x15(U): x8(U): x7(U) ■

We remark that the diagram corresponding to the operation "X2(R)" is the same as that in Fig. 163.B except that the parities of crossings x21, x22, x23, x24 are reversed.

Now, Fig. 163.B, above, consists of O.A, modified for the simultaneous cross-hand loop-transfers L2∞ ⇒ R2, R2∞ ⇒ L2 -- consisting of two spikes with

their cross-hand crossings x3, x4, respectively -- together with the four loop-interaction crossings x21, x22, x23, x24, specifying the R2∞'s passage through (i.e. inside of) the L2∞; note that these latter four crossings are entirely reminiscent of the crossings x(n+1), x(n+2), x(n+3), x(n+4) of Fig's. 161.A and B, whose analysis was likewise schema-independent. Rather than pursue the lengthy statement of the interhand loop "exchange" algorithm (all elements of which have been extensively treated in the foregoing) at this time,* we shall instead content ourselves with the apparent observation that all operations from Section I., Systemology have now been shown to yield completely to an ∅-analysis in every case and, hence, that the schemata of our former slavish dependence are, in reality, entirely dispensable with respect to the foundational theory.

Now let us perform the gedanken-experiment of the schema-independent construction of a string-figure. We begin with an opening -- say, O.A -- to which is associated a "short" canonical linear sequence, or "word":

⊨> L1: x1(∅): R2: x2(∅): L5: R5: x2(U): L2: x1(U): R1 ■

We may consider each entry of this linear sequence to be a child's letter-block -- or a "Scrabble"-tile -- laid out in the above array, with the blocks representing a frame-node being distinguished from the crossing-blocks by a distinctive color; the runs of crossing-blocks between consecutive frame-blocks represent the strings of the figure under discussion, and are "compartmentalized" by their boundary frame-blocks. And, a pick-up or loop-transfer move inserts well-defined sequences of blocks in a compartment-specific way, according to the precepts of the governing algorithm, to produce a (sometimes much) longer word -- the derived linear sequence of the transformed string-position. For definiteness, suppose the loop-transfer move "X2(L)" is the operation in question, applied to the linear sequence for O.A. The derived linear sequence, as we've seen, is the lengthy one associate to Fig. 163.B, above (which lacks uniform sequential relabeling of its crossings of being in its canonical presentation). If the subsequent manipulation is "|", this linear sequence collapses to the short, simple,**

⊨> L1: x1(∅): x2(U): L2: x3(U): x4(∅): L5: R5: x4(U): x3(∅): R2: x2(∅): x1(U): R1 ■

* See Appendix C, pages 398-401.

** See Appendix C, pages 392-397 , for this lengthy computation.

New pick-up or loop-transfer moves applied to this linear sequence will, again, produce longer, more complicated derived linear sequences, until a "release, extend" pair produces a shortest word (through Ø-operation cancellation). The process may be repeated several times in the execution of the string-figure in question. Thus viewed through the schema-independent eyes of the model, the construction of a given string-figure gives rise to a "pulsation" in the ordered set of associated linear sequences: they grow and collapse, grow and collapse ... until, at the final collapse (‖), the figure has emerged. The shortest words resulting from the intermediate collapses (|) are the distinguished points on the path to the end result, whose crisp, clean sub-constructs comprise the arguments of the Calculus manipulations acting upon them; i.e. after the collapse these string-positions must be searched for the constituent arc or loop pertinent to the next operation. These must be reasonably easy to identify, whence the corresponding word should be short. We remark that, in the majority of string-figures, the short words corresponding to the distinguished intermediate string-position, themselves, tend to grow in length in a regular way, until they attain the length of the derived linear sequence associated to the final design.

------- End Calculus Discussion -------

Appendix C.1: The Operation "|" Applied to the String-Position of Fig. 163.B.

The string-position of Fig. 163.B had the (non-canonical) associated linear sequence

① |⇒ L1: x5(U): x6(U): x13(U): x14(U): x1(Ø): x18(Ø): x20(Ø): x4(U): x24(U): x21(Ø): x7(Ø): x5(Ø): L2: x6(Ø): x8(Ø): x22(Ø): x23(U): x4(Ø): x19(Ø): x17(Ø): x2(Ø): L5: R5: x2(U): x13(Ø): x15(Ø): x3(Ø): x23(Ø): x24(Ø): x12(Ø): x10(Ø): R2: x9(Ø): x11(Ø): x21(U): x22(U): x3(U): x16(Ø): x14(Ø): x1(U): x18(U): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x20(U): x16(U): x15(U): x8(U): x7(U) ■

application of the Calculus operation "|" to which, it was asserted on page 390, would produce the (canonical) derived linear sequence

|⇒ L1: x1(Ø): x2(U): L2: x3(U): x4(Ø): L5: R5: x4(U): x3(Ø): R2: x2(Ø): x1(U): R1 ■

We now turn our attention to this lengthy derivation, as an exercise in the application of the Ø-operations. We shall adopt the notation

$$\emptyset_1^{-1}(xn) \equiv \{xn\} \to \phi \quad \text{(Lemma 2.A)}$$

$$\emptyset_2^{-1}(xm,\ xn) \equiv \{xm,\ xn\} \to \phi \quad \text{(Lemma 2.B)}$$

$$\emptyset_3(xm,\ xn;\ xk) \equiv \text{draw s;xm-xn across xk.}$$

Beginning with the linear sequence ①, above, apply $\emptyset_2^{-1}$(x1, x18) to produce the derived linear sequence ②, below.

② |⇒ L1: x5(U): x6(U): x13(U): x14(U): x20(Ø): x4(U): x24(U): x21(Ø): x7(Ø): x5(Ø): L2: x6(Ø): x8(Ø): x22(Ø): x23(U): x4(Ø): x19(Ø): x17(Ø): x2(Ø): L5: R5: x2(U): x13(Ø): x15(Ø): x3(Ø): x23(Ø): x24(Ø): x12(Ø): x10(Ø): R2: x9(Ø): x11(Ø): x21(U): x22(U): x3(U): x16(Ø): x14(Ø): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x20(U): x16(U): x15(U): x8(U): x7(U) ■

To ② we apply $\emptyset_3$(x23, x24;x4) and $\emptyset_3$(x3, x15; x16) simultaneously -- since they are disjoint -- to produce ③, below.

(3) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x20(∅): x24(U): x4(U): x21(∅): x7(∅): x5(∅): L2: x6(∅): x8(∅): x22(∅): x4(∅): x23(U): x19(∅): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x15(∅): x24(∅): x23(∅): x12(∅): x10(∅): R2: x9(∅): x11(∅): x21(U): x22(U): x16(∅): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x20(U): x15(U): x16(U): x8(U): x7(U) ■

To (3) we apply $\emptyset_3$(x15, x24; x20), to produce (4), below.

(4) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x20(∅): x4(U): x21(∅): x7(∅): x5(∅): L2: x6(∅): x8(∅): x22(∅): x4(∅): x23(U): x19(∅): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x15(∅): x23(∅): x12(∅): x10(∅): R2: x9(∅): x11(∅): x21(U): x22(U): x16(∅): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x15(U): x20(U): x16(U): x8(U): x7(U) ■

To this we apply $\emptyset_3$(x15, x23; x19), to produce (5), below.

(5) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x20(∅): x4(U): x21(∅): x7(∅): x5(∅): L2: x6(∅): x8(∅): x22(∅): x4(∅): x19(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x15(∅): x12(∅): x10(∅): R2: x9(∅): x11(∅): x21(U): x22(U): x16(∅): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x15(U): x19(U): x20(U): x16(U): x8(U): x7(U) ■

To (5) we may apply both $\emptyset_2^{-1}$(x12, x15) and $\emptyset_3$(x4, x22; x21), since these are disjoint, to produce (6), below.

(6) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x20(∅): x21(∅): x4(U): x7(∅): x5(∅): L2: x6(∅): x8(∅): x4(∅): x22(∅): x19(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x11(∅): x22(U): x21(U): x16(∅): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x19(U): x20(U): x16(U): x8(U): x7(U) ■

To (6), we apply $\emptyset_3$(x20, x21; x16), to produce (7), below.

(7) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x21(∅): x20(∅): x4(U): x7(∅): x5(∅): L2: x6(∅): x8(∅): x4(∅): x22(∅): x19(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x11(∅): x22(U): x16(∅): x21(U): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x19(U): x16(U): x20(U): x8(U): x7(U) ■

To (7), we apply $\emptyset_3$(x19, x22; x16), to produce (8), below.

(8) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x21(∅): x20(∅): x4(U): x7(∅): x5(∅): L2: x6(∅): x8(∅): x4(∅): x19(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x11(∅): x16(∅): x22(U): x21(U): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x11(U): x16(U): x19(U): x20(U): x8(U): x7(U) ■

To (8), we may apply both $\emptyset_2^{-1}$(x11, x16) and $\emptyset_3$(x4, x19; x20), since these are disjoint, to produce (9), below.

(9) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x21(∅): x4(U): x20(∅): x7(∅): x5(∅): L2: x6(∅): x8(∅): x19(∅): x4(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x22(U) x21(U): x3(U): x14(∅): x17(U): x10(U): x9(U): R1: x20(U): x19(U): x8(U): x7(U) ■

To (9), we apply first $\emptyset_2^{-1}$(x8, x19) -- followed by $\emptyset_2^{-1}$(x7, x20) on the derived sequence -- to produce the second derived sequence (10), below.

(10) ⊨> L1: x5(U): x6(U): x13(U): x14(U): x24(U): x21(∅): x4(U): x5(∅): L2: x6(∅): x4(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x3(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x22(U): x21(U): x3(U): x14(∅): x17(U): x10(U): x9(U): R1 ■

We note, at this point, that the s;L5-R5 and s;L1-R1 strings are crossing-free, <u>i.e</u>. the central tangle has been retracted back between them. We continue, applying $\emptyset_3$(x3, x13; x14) to (10) to produce (11), below.

(11) ⊨> L1: x5(U): x6(U): x14(U): x13(U): x24(U): x21(∅): x4(U): x5(∅): L2: x6(∅): x4(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x3(∅): x13(∅): x24(∅): x23(∅): x10(∅): R2: x9(∅): x22(U): x21(U): x14(∅): x3(U): x17(U): x10(U): x9(U): R1 ■

To (11) we may apply both $\emptyset_2^{-1}$(x13, x24) and $\emptyset_3$(x4, x6; x5), since these are disjoint, to produce (12), below.

(12) ⊨> L1: x6(U): x5(U): x14(U): x21(∅): x5(∅): x4(U): L2: x4(∅): x6(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x3(∅): x23(∅): x10(∅): R2: x9(∅): x22(U): x21(U): x14(∅): x3(U): x17(U): x10(U): x9(U): R1 ■

To (12) we apply $\emptyset_3$(x5, x21; x14), to produce (13), below.

(13) ⊨> L1: x6(U): x14(U): x5(U): x5(∅): x21(∅): x4(U): L2: x4(∅): x6(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x3(∅): x23(∅): x10(∅): R2: x9(∅): x22(U): x14(∅): x21(U): x3(U): x17(U): x10(U): x9(U): R1 ■

And to (13) we may apply the disjoint ϕ_1^{-1}(x5) and ϕ_3(x6, x22; x14) to produce (14), below.

(14) ⊨> L1: x14(U): x6(U): x21(∅): x4(U): L2: x4(∅): x22(∅): x6(∅): x23(U): x17(∅): x2(∅): L5:R5: x2(U): x3(∅): x23(∅): x10(∅): R2: x9(∅): x14(∅): x22(U): x21(U): x3(U): x17(U): x10(U): x9(U): R1 ■

To (14) we apply ϕ_3(x6, x22; x21), to produce (15), below.

(15) ⊨> L1: x14(U): x21(∅): x6(U): x4(U): L2: x4(∅): x6(∅): x22(∅): x23(U): x17(∅): x2(∅): L5: R5: x2(U): x3(∅): x23(∅): x10(∅): R2: x9(∅): x14(∅): x21(U): x22(U): x3(U): x17(U): x10(U): x9(U): R1 ■

To (15) we may apply the disjoint ϕ_2^{-1}(x4, x6) and ϕ_3(x3, x23; x22), to produce (16), below.

(16) ⊨> L1: x14(U): x21(∅): L2: x23(U): x22(∅): x17(∅): x2(∅): L5: R5: x2(U): x23(∅): x3(∅): x10(∅): R2: x9(∅): x14(∅): x21(U) x3(U): x22(U): x17(U): x10(U): x9(U): R1 ■

To (16), we apply first ϕ_2^{-1}(x17, x22) -- followed by ϕ_2^{-1}(x3, x10) on the derived sequence -- to produce the second derived sequence (17), below.

(17) ⊨> L1: x14(U): x21(∅): L2: x23(U): x2(∅): L5: R5: x2(U): x23(∅): R2: X9(∅): x14(∅): x21(U): x9(U): R1 ■

To (17) we apply ϕ_3(x9, x14; x21), to produce (18), below.

(18) ⊨> L1: x21(∅): x14(U): L2: x23(U): x2(∅): L5: R5: x2(U): x23(∅): R2: x14(∅): x9(∅): x9(U): x21(U): R1 ■

Now, application of ϕ_1^{-1}(x9) to (18) produces the final cancellation, and (19), below.

(19) ⊨> L1: x21(∅): x14(U): L2: x23(U): x2(∅): L5: R5: x2(U): x23(∅): R2: x14(∅): x21(U): R1 ■

The linear sequence (19), just derived, satisfies the conventions Seq. 1 and Seq. 2 and, hence, lacks only uniform sequential crossing relabeling of being in its canonical presentation. Thus, we set

x21 → x1　　x23 → x3
x14 → x2　　x2 → x4

in (19), above, to produce the ultimate derived linear sequence,

(20) ⊨> L1: x1(∅): x2(U): L2: x3(U): x4(∅): L5: R5: x4(U): x3(∅): R2: x2(∅): x1(U): R1 ■

as earlier promised. Notice that (20) is the canonical linear sequence associated to the schema of Fig. 163.C, below.

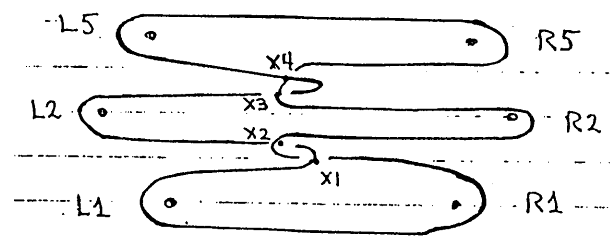

Fig. 163.C: Example (1C); O.A: X2(L) |.

As an exercise, the interested reader may wish* to pursue the application of the operation "|" to the linear sequence.

⊨>L1: x5(U): x6(U): x13(U): x14(U): x1(∅): x18(∅): x20(∅): x4(U): x24(∅): x21(U): x7(∅): x5(∅): L2: x6(∅): x8(∅): x22(U): x23(∅): x4(∅): x19(∅): x17(∅): x2(∅): L5: R5: x2(U): x13(∅): x15(∅): x3(∅): x23(U): x24(U): x12(∅): x10(∅): R2: x9(∅): x11(∅): x21(∅): x22(∅): x3(U): x16(∅): x14(∅): x1(U): x18(U): x17(U): x10(U): x9(U): R1: x11(U): x12(U): x19(U): x20(U): x16(U): x15(U): x8(U): x7(U) ■

which is associated to the spike-implemented string-position resulting from the manipulational sequence

O.A: X2(R).

Here, the ultimate derived linear sequence is

⊨> L1: L2: L5: R5: R2: R1 ■

as is directly verifiable (cf. pages 27-28 of these notes). The schema corresponding to this linear sequence is, apparently, given in Fig. 163.D, below.

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* Then again, he may not.

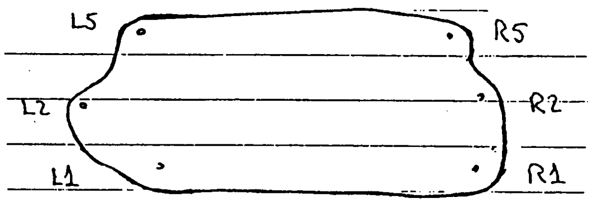

Fig. 163.D: Example (1D); O.A: X2(R) |.

Appendix C.2 (Cont^{d}.)

Here we supply the missing details for the schema-independent analysis of the Calculus operation "cross-hand loop exchange", discussed on pages 388-390 of the present section. This is, largely, an exercise in "bookkeeping", which is -- at best -- tedious; we include the details as both an example and an exercise, for completeness of the discussion; the reason for its relegation to the Appendix will become clear momentarily, if it is not already so at present.

For definiteness, we consider the application of the Calculus manipulation "XF(L)" to a given canonical linear sequence whose maximal crossing label is n. Then

1. Verify that the linear sequence contains both LF and RF as frame-nodes.*
2. Identify the left-frame-nodes to the near side of LF. Label these

$$LF_1, LF_2, \ldots, LF_k$$

respectively, ordered from LF to the near side of the figure (i.e. LF_1 is the sequence's frame-node nearest LF, and LF_k is the starting-point node for the sequence). Similarly identify the right-frame-nodes to the near side of RF_1 and label them

$$RF_1, RF_2, \ldots, RF_\ell,$$

ordered from RF to the near side of the figure.

3. Identify (via pointers) the near (and, hence, far) strings of each of the loops LF, LF_1, LF_2, ..., LF_k; RF, Rf_1, RF_2, ..., RF_ℓ; in the (original) linear sequence.

The diagram below (Fig. 163.E) is a generalization of the text schema, Fig. 163.B, which will clarify the subsequent complicated crossing-labeling.

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* Otherwise, XF(L) = Φ , by definition.

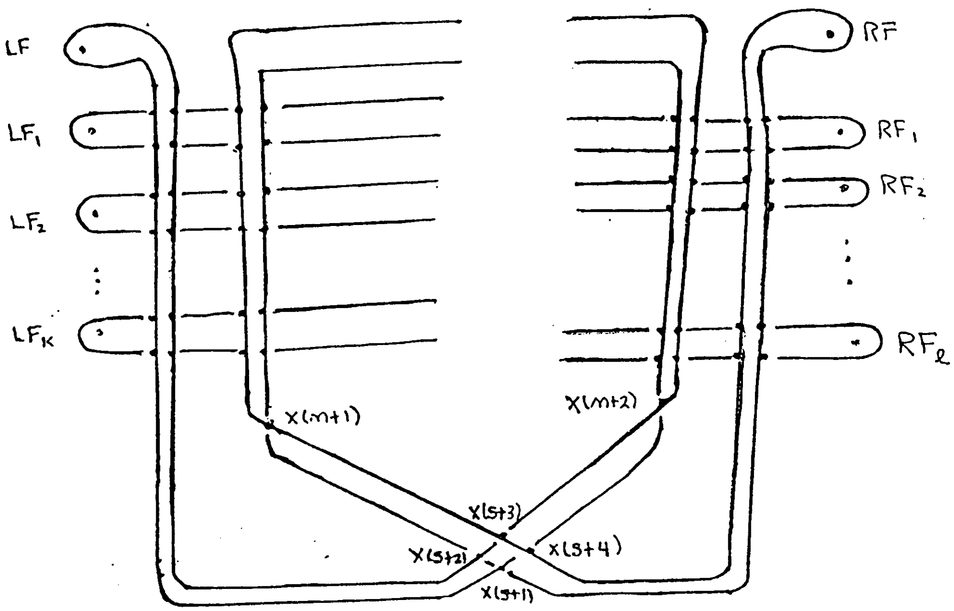

Fig. 163.E: XF(L): The general case.

Of the new crossings introduced by the manipulation, the first two -- x(n+1), x(n+2) -- are assigned the left- and right-spike's self-intersections in passage across the design, while the last four -- x(s+1), x(s+2), x(s+3), x(s+4), with $s = (n+2)+8(k+\ell)$ -- are reserved for the spikes' mutual interaction (here, the right spike's passage through the left spike). The remainder of the crossings are given in terms of the strings incident with the left and right fingers, beginning at the functor and proceeding away therefrom along the indicated string;

4.A. For i = 1, 2, 3, ..., k;

$LF_i f \longrightarrow x[(n+2)+(4i-3)] : x[(n+2)+(4i-2)] : x[(n+2)+4(k+\ell)+(4i-3)] :$
$x[(n+2)+4(k+\ell)+(4i-2)]$,

$LF_i n \longrightarrow x[(n+2)+(4i-1)] : x[(n+2)+4i] : x[(n+2)+4(k+\ell)+(4i-1)] :$
$x[(n+2)+4(k+\ell)+4i]$.

B. For j = 1, 2, 3, ..., ℓ;

$RF_j f \longrightarrow x[(n+2)+4k+(4j-3)] : x[(n+2)+4k+(4j-2)] : x[(n+2)+4(2k+\ell)+(4j-3)] :$
$x[(n+2)+4(2k+\ell)+(4j-2)]$,

4.B. (cont.[d])

$RF_j n \longrightarrow x[(n+2)+4k+(4j-1)] : x[(n+2)+4k+4j] : x[(n+2)+4(2k+\ell)+(4j-1)] :$
$x[(n+2)+4(2k+\ell)+4j]$.

Of course, as we proceed along any of the indicated strings above, in either direction, every crossing involved has "Under" parity, by virtue of the convention that the spikes' passage is to be over all intermediary strings which it encounters.

We are now able to give the first of the two explicit substitutions into the original, given linear sequence specifying the Calculus manipulation "XF(L)":

XF(L). 1. A. For i = 1, 2, 3, ..., k; if $\models\!\!\Rightarrow \ldots \overset{LF_i n}{\downarrow}LF_i \ldots$ ∎, then

$LF_i \longrightarrow x[(n+2)+4(k+\ell)+4i]\,(U) : x[(n+2)+4(k+\ell)+(4i-1)]\,(U) : x[(n+2)+4i]\,(U) :$
$x[(n+2)+(4i-1)]\,(U) : LF_i : x[(n+2)+(4i-3)](U) : x[(n+2)+(4i-2)]\,(U) :$
$x[(n+2)+4(k+\ell)+(4i-3)]\,(U) : x[(n+2)+4(k+\ell)+(4i-2)]\,(U)$;

otherwise, make the above substitution with the right-hand side sequence reversed.

B. For j = 1, 2, 3, ..., ℓ ; if $\models\!\!\Rightarrow \ldots \overset{RF_j n}{\downarrow}RF_j \ldots$ ∎, then

$RF_j \longrightarrow x[(n+2)+4(2k+\ell)+4j]\,(U) : x[(n+2)+4(2k+\ell)+(4j-1)]\,(U) :$
$x[(n+2)+4k+4j]\,(U) : x[(n+2)+4k+(4j-1)]\,(U) : RF_j :$
$x[(n+2)+4k+(4j-3)]\,(U) : x[(n+2)+4k+(4j-2)]\,(U) :$
$x[(n+2)+4(2k+\ell)+(4j-3)]\,(U) : x[(n+2)+4(2k+\ell)+(4j-2)]\,(U)$;

otherwise, make the above substitution with the right-hand side sequence reversed.

It remains to discuss the substitutions into the original linear sequence corresponding to the two individual spikes, themselves. It will prove convenient, in this regard, to introduce the following analogue of the "Summation Notation":

For $n \geq 1$,

$$\overset{n}{\underset{i=1}{\odot}} xi \equiv x1 : x2 : x3 : \ldots\ x(n-1) : x(n) .$$

For n = 0, we shall consider the right-hand side of the above expression to be the empty set, ∅.

XF(L). 2: A. If $\models\!\Rightarrow \ldots \overset{LFn}{Lf\!\downarrow}\ldots \blacksquare$, then

$$\begin{aligned}
LF \longrightarrow\ & \left\{\bigodot_{i=1}^{2k} x[(n+2)+4(k+\ell)+(2i-1)]\,(\emptyset)\right\} : x(n+1)\,(\emptyset) : x[(n+2)+8(k+\ell)+3]\,(\emptyset) : \\
& x[(n+2)+8(k+\ell)+4]\,(\emptyset) : \left\{\bigodot_{j=1}^{2\ell} x[(n+2)+4k+(4\ell-2j+2)]\,(\emptyset)\right\} : RF : \\
& \left\{\bigodot_{j=1}^{2\ell} x[(n+2)+4k+(2j-1)]\,(\emptyset)\right\} : x[(n+2)+8(k+\ell)+1]\,(U) : \\
& x[(n+2)+8(k+\ell)+2]\,(U) : x(n+1)\,(U) : \left\{\bigodot_{i=1}^{2k} x[(n+2)+4(k+\ell)+(4k-2i+2)]\,(\emptyset)\right\};
\end{aligned}$$

otherwise, make the above substitution with the right-hand side sequence reversed.

B. If $\models\!\Rightarrow \ldots \overset{RFn}{RF\!\downarrow}\ldots \blacksquare$, then

$$\begin{aligned}
RF \longrightarrow\ & \left\{\bigodot_{j=1}^{2\ell} x[(n+2)+4(2k+\ell)+(2j-1)]\,(\emptyset)\right\} : x(n+2)\,(\emptyset) : x[(n+2)+8(k+\ell)+3]\,(U) : \\
& x[(n+2)+8(k+\ell)+2]\,(\emptyset) : \left\{\bigodot_{i=1}^{2k} x[(n+2)+(4k-2i+2)]\,(\emptyset)\right\} : LF : \\
& \left\{\bigodot_{i=1}^{2k} x[(n+2)+(2i-1)]\,(\emptyset)\right\} : x[(n+2)+8(k+\ell)+1]\,(\emptyset) : x[(n+2)+8(k+\ell)+4]\,(U) : \\
& x(n+2)\,(U) : \left\{\bigodot_{j=1}^{2\ell} x[(n+2)+4(2k+\ell)+(4\ell-2j+2)]\,(\emptyset)\right\};
\end{aligned}$$

otherwise, make the above substitution with the right-hand side sequence reversed.

The linear sequence resulting from making the substitutions of XF(L).1 and 2, above, in the original linear sequence will -- when brought to its canonical presentation -- be the one associated to the spike-implementation of the Calculus manipulation "XF(L)" applied to the schema of the original sequence; i.e. it will be a derived linear sequence.

As before mentioned, the substitutions for the manipulation "XF(R)" may be obtained from XF(L).1 and 2 by reversing the parities of the crossings x(s+1), x(s+2), x(s+3), x(s+4) -- where $s = (n+2)+8(k+\ell)$ -- in each of these.

INDEX

(cont.[d])

(cont^d)

(cont.d)