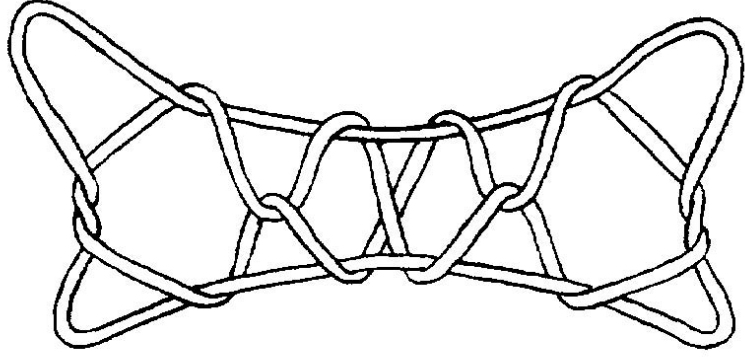


Advice for Students

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1 Introduction

Students often ask me for advice on how to improve in mathematics. I find that I keep recommending the same strategies across a broad range of courses. This document attempts to collect up these strategies. They may or may not work for you. Try them out in whatever order suits you best. Some of the strategies explicitly suggest that you try out other strategies. For example, Watch Your Self Talk suggests that you Set a Timer.

If any of these things help, or you have a strategy that you think should be on here, please let me know.

— Parker Glynn-Adey (parker.glynn.adey@utoronto.ca)

Summary

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2 Be Kind to Yourself

Studying mathematics is hard. Being a student of mathematics is even harder. It is so easy to get bummed out and think that you will never make any progress. We all make errors in mathematics. However, errors lead to learning and progress. From time to time, you should acknowledge that you're doing an awesome thing by learning mathematics. **Be kind to yourself, forgive your own errors, and acknowledge that hard things are difficult.**

3 Read Slowly

When you're reading the textbook, or sources online, read slowly. There is a great quote from Bill Thurston which talks about the rate at which we need to read mathematics.

I was really amazed by my first encounters with serious mathematics textbooks. I was very interested and impressed by the quality of the reasoning, but it was quite hard to stay alert and focused. After a few experiences of reading a few pages only to discover that I really had no idea what I'd just read, I learned to drink lots of coffee, slow way down, and accept that I needed to read these books at 1/10th or 1/50th standard reading speed, pay attention to every single word and backtrack to look up all the obscure numbers of equations and theorems in order to follow the arguments. — Bill Thurston

When you're reading material in mathematics, expect it to take a long time. This is common. You're not a slow reader. Mathematics is just very compact and requires careful attention to detail.



This document is available online at: <https://pgadey.ca/notes/advice-for-students/>

3.1 Read with a Pencil and Paper

To help stay active while reading, keep a pencil and paper handy. I think it is particularly important to work with physical paper and pencil. Paul Halmos has some good advice here.

Reading with pencil and paper on the side is very much better — it is a big step in the right direction. The very best way to read a book, however, with, to be sure, pencil and paper on the side, is to keep the pencil busy on the paper and throw the book away. — Halmos

4 Set a Timer

It is interesting to experiment with spending a small limited amount of time studying. Set a timer on your phone¹, for a period of twenty or thirty minutes, and study without distraction until the timer goes off. When you are studying, it is easy to spend a lot of time and be disappointed with the results. One can get stuck in endless study sessions which switch for studying to hanging out, to looking things up online, to eating, to reading the book, to working on some problems, to continued studying, to catching up on the news, and back to studying, *etc.*

If you pick a small amount of time, say twenty minutes, then you can guarantee that during those twenty minutes you are purely studying. Before starting the timer, turn off anything that could distract you and sink in to the material. During that timed interval, you should be fully present with the material that you're studying. After the timer goes off, you can decide if you want to start up another timer or switch tasks. One way to think about working with a timer is that you're purposefully interrupting yourself to see if you're still on task.

During the timed interval, it is helpful to set a concrete task to work on. You can write this task out before starting the timer, and check it if you feel yourself getting pulled off task. Some tasks that you might consider are:

- solve a particular problem,
- produce examples of a definition,
- read a single page of a book with pencil and paper.

At the top of another blank piece of paper write:

How would I like to spend the next five years?

Again, free write for two minutes. This question will help frame and specialize some of the topics that came up in the first question. Once you've completed your two minutes of free writing, take another two minutes to revise and add to your short-term goals list.

At the top of another blank piece of paper write:

If I knew now that I was going to suddenly die in six months, how would I live until then?

A hidden hypothesis of this question is that your family will not be majorly impacted by your death, and you don't need to make any preparations for it. You can imagine that you suddenly de-materialize and *poof* out of existence while your family and friends lovingly continue on their merry way. The question seeks to determine how you would live if you had a dramatically reduced lifespan. Are there things you'd prioritize differently? Again, free write for two minutes. Once you've completed your two minutes of free writing, take another two minutes to revise and add to your six-month goals list.

Finish the exercise by reviewing and revising your answers to all three questions. You now have a written account of what you want to do with your life at a variety of time scales. This is a very useful thing to have.

¹Alternatively, you can find a timer online: <https://www.online-timer.net/>.

Meditation can help with test anxiety; emotional self-regulation, and a whole host of other issues. You don't need to do much of it for the effects to be present. As little as five minutes a day can have significant positive effects.

10.3 Write a Journal: Gratitude

Get a cheap notebook and a pen, write down the date, and then write a bit about your life. Do this as often as you like. Taking time to reflect on your life, acknowledge the hardships, and write about the things that bring you joy is powerful. I've been doing this every other day for about twenty five years. Journaling is a core part of my life.

Your journal is a place where you can do anything. You can write until you are out of ideas, and then you'll find that you have even more ideas beyond the usual ones. You can re-visit an experience in writing, and gain a new perspective on it. You can write a list of all the things that you're thankful for. You can write about your life goals. Revisiting your life goals in writing can give a strong sense of purpose.

Eventually, if you keep on writing, you'll find the life has an incredibly rich and varied texture. You will start to appreciate things more and more by paying attention to them. And, rather subtly, you'll be more grateful.

10.4 Life Goals

And my final piece of advice is this: Write a concrete description of what you want to do with your life. So many things click in to place once you've thought for a bit about what you want the purpose of your life to be.

This final section explains a short writing exercise called *The Alan Lakem Life Goals Exercise*. You need three pieces of a paper, a pen, a timer, and about fifteen minutes. I recommend doing the exercise on paper, instead of electronically, because I find that I engage more fully and write slower on paper. Here is an outline of the life goals exercise.

What are my lifetime goals?

Write freely for two minutes, without censoring yourself at all. You are not committing to these goals, you're just writing them down. Dream big. Perhaps you want to visit the moon. Run a major company. Dream small. You could go for a country drive with your family. Bake more bread. Try to list as much as possible. Once you've completed your two minutes of free writing, take another two minutes to revise and add to your life goals list. Perhaps there is a theme to your life goals that you only noticed once you were done the first draft. Add that theme.

5 Watch Your Self Talk

We all talk to ourselves almost constantly. I call this self-talk. It generally comes in two varieties: neutral self-talk and negative self-talk. Most of time, self-talk is just mindless chatter about our surroundings or experiences. "Oh, that person has a nice hat. I am *really* early for class. I wonder what that noise is."

When we are working on something new to us, self-talk can become counter-productive or harmful. "I'm so dumb. I have no idea what is going on. I'll never make POST." These examples of negative self-talk probably seem quite contrived. Negative self-talk is often nasty, clever, and attempts to undermine our efforts. I want to make it clear that we *all* experience negative self-talk. Here are some real examples of counter-productive self-talk that came up for me, Parker Glynn-Adey, as I was writing this note about advice for students:

- "All of this stuff is obvious and stupid."
- "Writing this is a waste of time, no-one will actually do any of this."
- "I'm not a counsellor or a guru. Who am I to give this advice to anyone?"

So, what can we do about negative self-talk? We can Be Kind to Ourselves! We can fight it with positive self-talk! If you notice that your self-talk is getting out of hand and starting to sound negative, you can fight it with some positive self-talk. You'll need to play with this technique to find your own tone for positive self-talk. It helps to rehearse it out loud a couple times. "I'm so dumb. Oh yeah? I am learning. I'm studying manifolds. This is the opposite of dumb." Or something like, "I'll never pass this course. I'm doomed. Well, I'm able to devote the next half-hour to studying. If I study for the next half hour, then I will improve my odds of passing the course."

In this last example of positive self-talk, I included in another learning strategy. If we're working with a timer then we can use the timer to limit the scope of our negative self-talk. We can say: "Yeah, this problem is hard. I don't understand it right now. But, in fifteen minutes, once my timer goes off, I can leave it behind."

6 Identify The First Tricky Thing

Getting completely lost is common when learning a new mathematical concept by reading a textbook, or reviewing your lecture notes. As you read along, a haziness or confusion starts to develop imperceptibly. Things make less and less sense as you read. Eventually, you get so confused that you

have no idea what anything means. At this point, your negative self-talk can take over and force you away from the new material.

One strategy for getting out of this situation without letting negative self-talk crush us, is to backtrack to the beginning of the material and identify the first tricky thing. By reading carefully, with pencil and paper, we can usually find a specific sentence where the material got tricky.

Here is an analogy for this process. We can imagine the material that we want to learn as a road at night. The road is lit by streetlamps, which represent sentences in a reading. At the beginning of the road, everything is bright and comprehensible. At the end of the road, everything is dark and incomprehensible. When we noticed that we've come to a dark part of the road, we can go back to the beginning, and pay attention to the individual street lamps. There will be some first street lamp that is not turned on. That's the "first tricky thing" and we want to pay attention to it.

Once we know where the first tricky thing is, we can set about trying to understand it. With any luck, we can figure it out and make some progress. That street lamp turns on, and the road becomes a little bit brighter. Knowing the location of the first tricky part can help us ask a precise question.

7 Learn to Ask Precise Questions

Mathematics is a language for asking and answering extremely precise questions. By getting better at asking questions, we can improve our ability to answer questions. Often, when a student comes to asking for direct one-on-one help with a course, I encourage them to come back the next week with three questions about the course material. Usually, their first batch of questions is quite vague. I answer the vague questions, send them back, and ask for more precise questions. A week later, the questions are much better. Asking questions is a skill that improves with practice.

As you're reading a piece of mathematics, it is helpful to identify the first tricky thing. Once you know its location, you can ask specific questions about it. By formulating a precise question about the tricky thing, you can hopefully figure it out. At the very least, you have a question to ask someone.

Suppose that you were reviewing your lecture notes about rotations. At some point, the notes assert:

$\det(R_\theta) \neq 0$ and so rotations are invertible.

If you wanted to understand this point better, you might formulate a question about it. Consider the following questions which get more and more specific.

If you try this prescription out immediately before studying, then I am certain that you will feel the increase of energy and awareness. Once you get in a daily groove of exercise, continue to experiment and explore. If you're interested in learning more about exercise, then I would be glad to suggest additional resources.

10.2 Do Nothing: Meditate

The act of sitting down, breathing deeply, and clearing your mind is much harder than you would expect. I recommend that everyone try meditation, at least a few times, to appreciate how difficult it is.

As an exercise, try the following simple meditation technique:

- Find somewhere that you can sit comfortably without interruption.
- Set a timer for two minutes.
- Pay attention to your breath:
 - Notice the sensation of breathing in.
 - Notice the sensation of breathing out.
 - When you get distracted, acknowledge that you are distracted, and return to focusing your attention on your breath.
 - When the two minute timer goes off, you are done meditating.

If you try this exercise, you will *immediately* get distracted⁶. It happens instantaneously. As soon as you want to sit and breathe, you'll start thinking about all sorts of random things. Once you are distracted, acknowledge it and think to yourself "Wow — I'm distracted." Turn your attention back to your breath. Keep gently returning your awareness back to your breath until the timer goes off.

The point of this sort of meditation is to build up the ability to direct your thoughts. Gently returning your attention to your breath is a means of learning of this ability. Once you've tried meditating a few times, you'll become more aware of what it feels like to be distracted. If you try to do short bursts of focused work, as I recommend in Set a Timer, then meditation will help you realize when you are distracted.

⁶As I write this, I tried out the meditation exercise myself. Immediately, I got distracted because I wasn't sure if the timer that I set worked. (The timer that I used doesn't display any count down and I haven't used that particular timer software in a long time.) I got distracted thinking about about: whether the baby would cry, whether my runny nose would become a problem, a friend of mine has a young daughter who is having trouble sleeping at night, by the fridge turning on, etc. I got distracted thinking about all the things that I got distracted by. I got distracted by wondering if I should try to keep a count of the number of things that I got distracted by. The list of distractions just *keeps on going*.

This sub-section contains some general advice which is not explicitly mathematical or related to study skills. I hesitated to include the following material in this page. However, my life has been dramatically improved by practicing a little bit of each of these things. So, I've included a little bit about exercise, meditation, and journaling.

10.1 Do Something: Exercise

Warning: I am not a medical doctor. Even though the amount of exercise that I am going to recommend is minimal, I don't know your physical condition or any issues that you might have. It is always worthwhile to talk with a doctor before beginning a new exercise program. Talk to your doctor, and listen to your body.

Try the following experiment, if your circumstances permit it⁴. Ask

yourself how energetic and aware do you feel right now? Mentally make a note of how you are feeling; assign yourself a score from one (barely awake) to ten (hyper energetic and aware). Get up from your computer, put this booklet away, or put down your phone, and try out the following. Jump up and down twenty times in a row. And once you're done, ask yourself again, how energetic and aware you feel. In my experience, the little bit of exercise caused a noticeable and beneficial boost in my energy level. After jumping around a bit, I have more energy and a greater sense of awareness and focus. As you've just seen, by doing the experiment, a little bit of exercise can change your energy level and ability to focus.

I recommend doing some amount of exercise, even an itty-bitty little bit, every day. Just walking a half-hour per day is known to have significant medical benefits. You can choose your own exercises to do. The exercise program that you choose does not need to be complicated and you don't need to go the gym⁵.

I recommend the following simple exercises as a good place to start: walking, jumping jacks, and squats. None of these require any equipment, and they can be done anywhere. A minimal prescription for daily exercise, might be something like this:

1. 10 minutes walking
2. 3 sets of 25 jumping jacks
3. 3 sets of 10 squats

⁴As I write this, I tried out the experiment myself. So, you can be sure that at least one person somewhere has done this. It's 08:40 in the morning, I'm sitting by the bank of the River Lea near Three Mills in London, England. Initially, I rated my awareness and energy level 5/10. After jumping twenty times, I feel closer to 7/10.
⁵As a UTSC student, you have free access to Toronto Pan Am Sports Center. If you want to get in to serious gym stuff, like weight-lifting or team sports, they have a tonne of great resources.

1. I don't understand invertibility.

2. Why are rotations invertible?

3. Why do the rotation matrices always have non-zero determinant?

The first "question" (which is really a statement) is so broad as to be unapproachable. If you asked a professor or TA this "question" then they would probably try to answer it. However, they would spend a lot of time talking about things other than your main concern: rotations. The second question is more specific. It addresses your concern about rotations and invertibility; however, it could be addressed geometrically. The second question is still not specific enough to help address your point. And so, we formulate the third question which is very precise. It is precise enough to address exactly the part of the notes that is unclear to you.

7.1 Know The Definitions

In mathematics, there are lots and lots of definitions. Everything means something precise. One way to boost your grades is to memorize the precise definition of everything. If you know, at a glance, what all the terms mean, you'll be well on your way to understanding.

Consider the following complicated definition from MAT A29.

The Riemann sum is:

$$\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} f(x_k^*) \Delta x_k$$

This definition has a lot of sub-parts. We name them now:

- The **end-points** are a sequence x_k such that:

$$a = x_0 < x_1 < \dots < x_n = b$$

- Δx_k is the **length** of the interval $[x_k, x_{k+1}]$.
- x_k^* is a **sample point** in the interval $[x_k, x_{k+1}]$.

If this limit exists and is independent of the choice of sample points and end-points, then we say that the function is **Riemann integrable**.

Sometimes weeks later, after we've been using this a long while, someone will ask: "What is a sample point?" And the answer is: "Exactly what it says in the box. It's a point in $[x_k, x_{k+1}]$."

The student then asks a follow-up question: “Well, what’s $[x_k, x_{k+1}]$?” And the answer is the same: “Exactly what it says in the box. It’s an interval between end-points.” This kind of conversation can go on and on. The point is this: A mathematical object is always precisely defined. If you know the definition, or know where to look for it, then you can save yourself a lot of confusion and trouble.

On the other hand, there is a distinction between what a definition says on paper and what it *means*. This deeper level of meaning is hard to get at and learning to extract meaning from definitions is an important skill to develop. These meanings are often so personal that they are hard to communicate in writing. Different people, with different life experiences, will find different meanings in the same mathematical objects. A simple and widely recognized example would be that the Riemann sum really *means*: “slice up the area in to thin rectangles and add up their areas.”

This meaning can help contextualize and remember parts of the definition. Looking at it again, we see that the products $f(x_k^*)\Delta x_k$ are areas of rectangles. The term Δx_k is supposed to be small or thin. These heuristics are not written in to the definition, but they help remember it.

And so, definitions have two important aspects: the definition in writing, and its meaning. It is, especially in the beginning, important to know the precise definitions as close verbatim as possible. One cannot use these meanings, or heuristic understandings, to solve homework problems.

8 Teach Other People (or Rubber Duckies)

The best way to learn a subject is to teach it.
— Everybody Ever

Humans are social creatures. We do our best work *with* other people. For this reason, some people do their best thinking only when talking out loud to an audience. If you can convince a friend or room mate to help you study, then ask them to patiently listen while you explain stuff. Promise that you will return the favour and listen to them explain stuff after. It is especially helpful if they don’t know the material which you want to learn.

A patient audience, who will listen to you endlessly, is a rare thing. Finding an audience to teach is the hard part of this approach. Luckily, there is a simple solution. Get a rubber ducky². Explain the problem the subject that you’re learning to the rubber ducky. Be sure to talk out loud and clearly explaining everything to the ducky. Initially, it will feel a little bit crazy. After a while though, you’ll get comfortable with the process.

²It doesn’t have to be an actual rubber ducky, but it should be lively and friendly. Our brains process visual images containing eyes and mouths differently from other visual stimuli. My rubber ducky is a little three-eyed monster. I would advise against explaining stuff out loud to an empty room or a pet rock. It can be done, but it’s not as effective as explaining to a happy little rubber duck.

9 Keep Seeking Help

There are so many resources available at UTSC to help you succeed in mathematics.

1. The Math Help Center IA 3100
2. CTL’s Math and Stats Support
3. Facilitated Study Groups
4. TAs’ office hours
5. Professors’ office hours
6. Upper year students

However, it is easy to avoid all these resources and feel totally isolated. We feel that seeking help is a waste of time or that it diminishes us. The truth is that you don’t need to face your courses alone. When I was growing up, my instinct was to avoid seeking help and figure things out on my own. Eventually, I learned to combat this instinct and actively seek help.

Seeking help is a skill. You can get better at it through practice. Part of seeking help is learning to asking precise questions that will benefit you. Another part of seeking help is to learning to identify who³ can help you and when.

Initially, asking for help makes us feel vulnerable or exposed. Society makes us feel that ignorance or not-knowing is a bad. It is worthwhile to pay attention to your self-talk surrounding help seeking. With practice, we can get very comfortable seeking help. Initially, it helps to be mindful of the fact that the university is an institution of learning. Its role in society is to produce and disseminate new knowledge. By seeking help, by seeking learning, you are actively furthering that mission.

Professors love to help and discuss ideas. Often students ask if they can ask a “quick” question. Ask deep, murky, confusing questions! We love to talk about ideas. It’s the best part of the job.

³Identifying who can help you is a major accomplishment once you reach the research level of learning. After you complete a PhD, there might only be a handful of people on Earth who know your field of study as well as you. It is helpful to be on good terms with these people. I had several research breakthroughs that consisted *entirely* of finding someone else that was better prepared to answer than me.