Parker Adey October 30, 2025

The Mathematics of Braiding

Name: ______________________________

Q1. Take your three piece of string and braid the "standard 3-strand braid".
If you get stuck, please ask Parker or the volunteers for help!

Instructions: Let's call the three strands Left, Center, and Right.

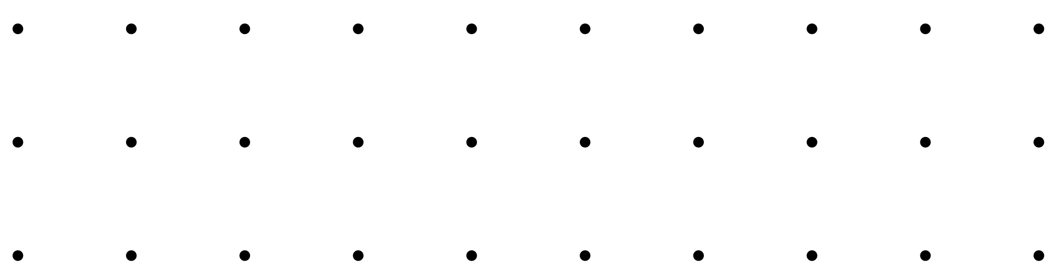
1. Pass the left strand over the center and leave it there.
2. Pass the right strand over the center and leave it there.
3. Repeat from Step 1.

Once you've completed your braid, answer the following:

(a) What do you notice?

(b) What do you wonder?

Q2. Draw a picture of your braid.



Mathematicians have a special language for describing braids. It was invented in the 1920s by Emil Artin, an Austrian-Armenian mathematician who worked at Princeton. In this language, a braid is written down crossing-by-crossing. The "standard 3-strand braid" becomes $(\sigma_1\sigma_2^{-1})^n$.

> For the rest of the workshop, we'll use computers to simulate braids. To log in to your computer, enter the following username: and password:

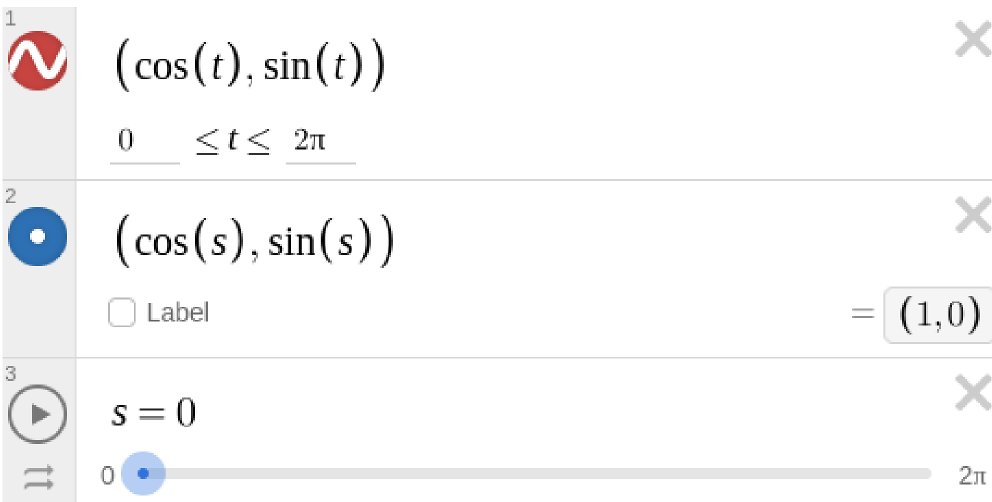
Q3. Open a browser, navigate to `https://www.desmos.com/` and graph the unit circle by entering:

$$(\cos(t), \sin(t))$$

Desmos will automatically create a parameter $0 \le t \le 1$. Adjust its bounds to $0 \le t \le 2\,\texttt{pi}$.

1. Add a slider s by typing `s` in a blank equation.
2. Set the slider bounds to $0 \le s \le 2\,\texttt{pi}$.
3. In a new equation, create a sliding point $(\cos(s), \sin(s))$.
4. Set the slider to "Repeat in One Direction".

At this point, your Desmos panel should look like this:



If you click the play button, this will show a point gliding around the unit circle.
We add a bit more structure.

5. Create a list:

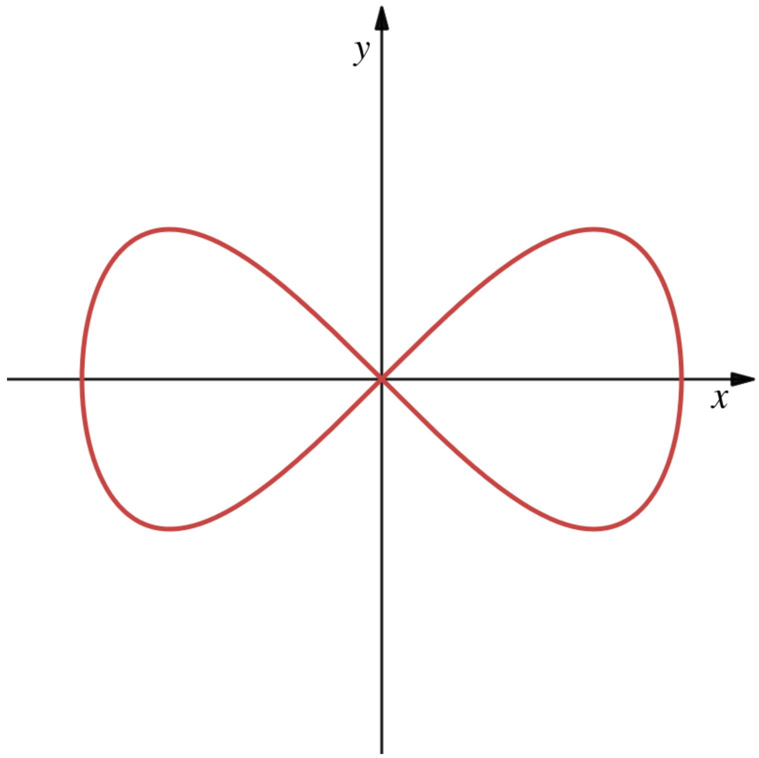
$$N = \left[0, \frac{2\ \texttt{pi}}{3}, \frac{4\ \texttt{pi}}{3}\right]$$

6. Modify your sliding point to $(\cos(s+N), \sin(s+N))$.

At this point, you should have three points spinning around a circle.

https://www.desmos.com/calculator/vsoenyfxnu

Q4. And now, a bit of a math question. Our original equation $(\cos(t), \sin(t))$ describes a unit circle. Try to modify the y-coordinate so that you get the following beautiful infinity sign.



Your equation will be of the following format: $(\cos(t), \underline{\hspace{2cm}}\sin(t))$.
Modify your Desmos equations $(\cos(t), \sin(t))$ and $(\cos(s+N), \sin(s+N))$ accordingly.

Q5. We'll now make a 3D braid. In a new tab, navigate to: `https://c3d.libretexts.org/CalcPlot3D`

1. Delete the default surface by clicking X.
2. Click "Add to graph" $\longrightarrow$ "Space curve $r(t)$."
3. Set $x(t) = \cos(t)$, $y(t) = \cos(t)\sin(t)$, $z = t/3$.
4. Repeat steps (2) and (3) with $x(t) = \cos(t + 2\texttt{pi}/3)$, $y(t) = \cos(t + 2\texttt{pi}/3)\sin(t + 2\texttt{pi}/3)$, $z = t/3$.
5. Repeat steps (2) and (3) with $x(t) = \cos(t + 4\texttt{pi}/3)$, $y(t) = \cos(t + 4\texttt{pi}/3)\sin(t + 4\texttt{pi}/3)$, $z = t/3$.

You can play with the colouring of the paths by clicking the gear icon.

For more information about braiding, see:

1. Speiser, N. (1983). *The manual of braiding.*
2. Birman, J. S., & Brendle, T. E. (2005). Braids: a survey. In *Handbook of knot theory* (pp. 19-103).

Feel free to ask me about mathematics / braiding / university: `parker.glynn.adey@utoronto.ca`
A quick e-mail: "Hey Parker! I was at your braiding workshop. I wanted to ask: ...." goes a long way!