

Rates of Change: A Visual Exploration

Name: _____

Q1. What are some things that change? Come up with lots of examples.

The **slope** of a line through two points (x_0, y_0) and (x_1, y_1) is given by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}.$$

The symbol Δ is the Greek letter “delta” and it stands for **difference** or change.

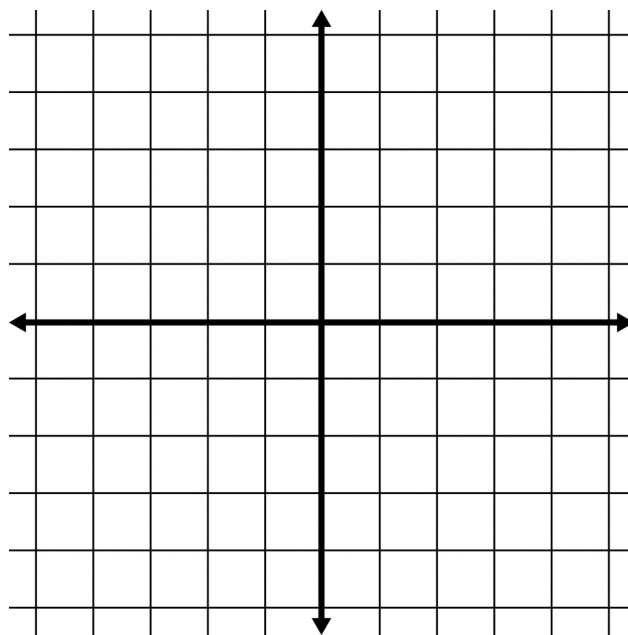
The **point-slope** format of the line through (a, b) with slope m is: $y - b = m(x - a)$.

Q2. Sketch the following lines. For each line, identify the point (a, b) and the slope m .

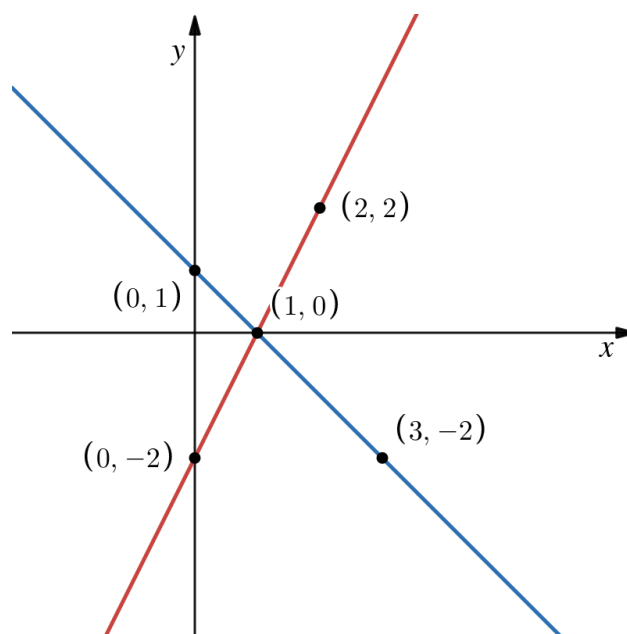
(a) $y - 1 = 2(x - 1)$

(b) $y - 3 = 0(x - 2)$

(c) $y - 2 = -1(x - 3)$



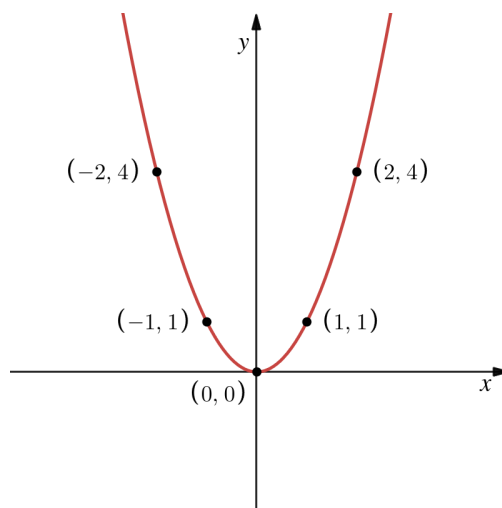
Q3. Consider the following graph which shows two lines.



- (a) What is the point-slope equation of each line?
- (b) How is the sign of the slope related to the direction of the line?
- (c) Does it matter which pairs of points we use to calculate slope?

The notion of slope that we have introduced only works for straight lines which have constant slope. However, in most situations where something is changing, the slope is not constant. The slope varies and fluctuates with time. The major idea of **calculus** is to approximate a quantity that varies by a straight line.

Q4. The following graph shows the parabola $y = x^2$. Notice that the slope changes with the value of x . Looking at the graph, what can you guess about the slopes at the the labelled points?



For this section of the workshop, we'll use computers to explore rate of change.
 To log in to your computer, enter the following username: **abc** and password: **def**.
 Parker and the volunteers will ask you to come up the podium and present your work.

- Q5. Open up Desmos and plot the function $y = x^2$. Add the points $(1, 1)$ and $(2, 4)$ to the graph. Zoom in on the point $(1, 1)$ and try to estimate the slope; click the $+$ button at least ten times. What is the slope at $(1, 1)$? What is the slope at $(2, 4)$?

- Q6. While still zoomed-in, add the following to your graph: $y - 1 = m(x - 1)$. Click "Add a slider m " when prompted and set the range to $-5 \leq m \leq 5$. Play with the value of m until the line $y - 1 = m(x - 1)$ and the curve $y = x^2$ overlap. What value of m works?

- Q7. Repeat the previous question at the point $(2, 4)$. What changes? What's the slope?

- Q8. Complete the following table of values by computing the "slope" m of $y = x^2$ at various values of x .

x	-1	0	1	2
m				

What pattern do you notice? How does the slope m depend on x ?

- Q9. The graph of $y = x^2$ passes through the point (a, a^2) and has slope $m = \underline{\hspace{2cm}}$. To visualize this, create a new graph in Desmos. Add $y = x^2$ and $y - a^2 = m(x - a)$ to your graph and add a slider for a with range $-5 \leq a \leq 5$. (You can zoom in on any point (a, a^2) and your line will overlap the curve $y = x^2$.)

The line that we have just produced is the **tangent line**. In general, for a function $y = f(x)$ and a point $x = a$, the tangent line: passes through $(a, f(a))$ and has the same "slope" as $y = f(x)$.

In Desmos, we explored the “slope” of $y = x^2$ and found that it is $m = 2a$ at $x = a$. The rest of the workshop will be exploring why this fact is true mathematically.

Q10. The definition of slope for a line is as follows.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}.$$

Suppose that we take a small step from $x_0 = a$ to $x_1 = a + h$ along $y = x^2$. What is the resulting expression of the “slope”?

Q11. What happens to the expression for m when h gets very small?

For more information about calculus, see:

1. Thompson, Silvanus P., and Gardner, Martin. *Calculus Made Easy*.
2. Stewart, James. *Calculus: Early Transcendentals*.

Feel free to ask me about mathematics / calculus / university: parker.glynn.adey@utoronto.ca
A quick e-mail: “Hey Parker! I was at your math workshop. I wanted to ask:” goes a long way!