The Art of Braiding Algorithms

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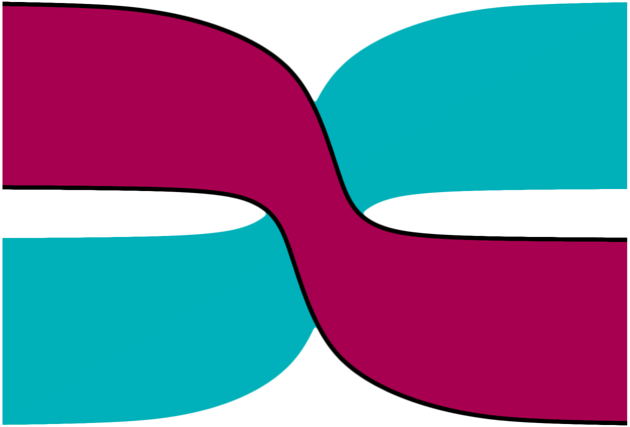
Thanks!

Thanks for coming out to this workshop. This is a test run of a workshop that we'll present at Bridges 2026, a conference on mathematics and art happening at the University of Galway, Ireland, 5–8 August 2026.



Outline

- What are string figures?
- A quick tour of mathematicians and string figures
- A quick tour of anthropologists and string figures
- Eric's accidental relation
- The Fundamental Question: What relations necessarily hold?
- What are braids?
- How do we draw these braids?
- A twist commutation relation

There are two stories here:

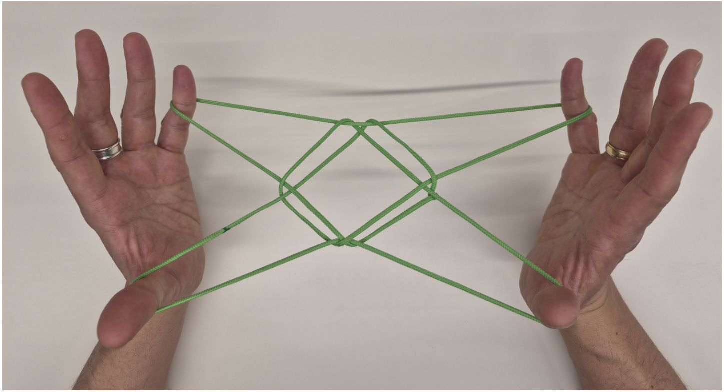
1. the search for a good notation
2. stumbling in to algebraic topology.

Please interrupt us.

Ask lots of questions.
Spark conversation!

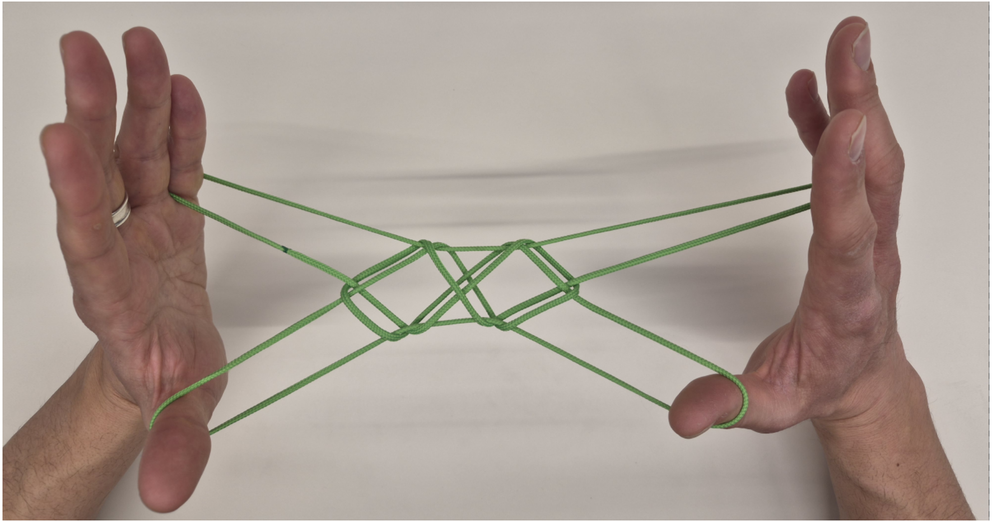
What are string figures?

String figures are culturally universal games played with a simple loop of string. From the Canadian Arctic to the tip of Africa, each culture has its own treasured repertoire of string figures and stories, handed down from generation to generation.



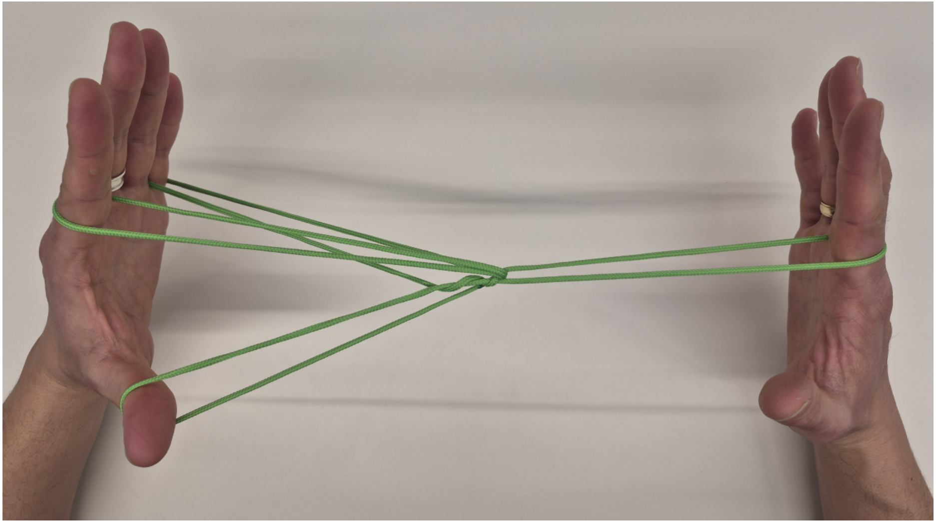
Plinthios Brokhos from 100CE Greece

What are string figures?



Koura (Crayfish) from Andersen's Maori String Figures (1927)

What are string figures?



Fish Spear from Jayne's String Figures (1906)

A Quick Tour of Anthropologists and String Figures

Tylor (1880) Remarks on the Geographical Distribution of Games.

> *Next, as to a game which we consider still more childish. Mr. A. R. Wallace ("Malay Archipelago," p. 88) being one wet day in a Dyak house in Borneo, to amuse the lads took a piece of string and showed them "cat's cradle," when to his surprise he found that they knew more about it than he did, a native boy taking it off his hand and making several new figures which quite puzzled him.*

Rivers and Haddon (1902) A Method of Recording String Figures and Tricks.

> *We are informed that these figures are much more complicated than are ours, and that they represent various natural and artificial objects sometimes in a state of rest, sometimes in a state of motion.*

A Quick Tour of Anthropologists and String Figures

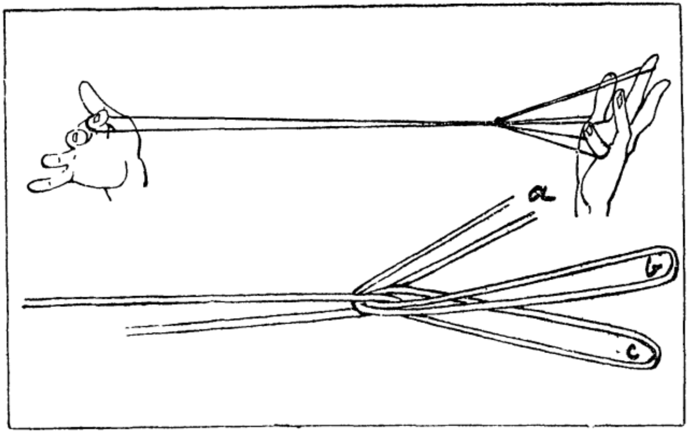


FIG. 1.—BAUR, "fish spear."

a. Thumb. *b.* Middle finger. *c.* Little finger.

The first figure recorded by Rivers and Haddon (1902) from Mer Island in the Torres Strait

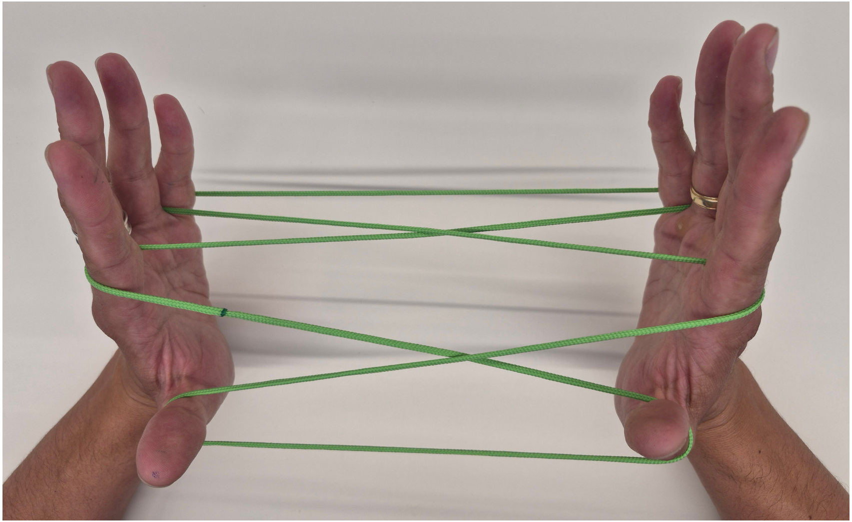
A Quick Tour of Mathematicians and String Figures

Ball (1920) An Introduction to String Figures: An Amusement for Everybody

> *It is a truism, and in fact a truth as well, that all sensible people have hobbies. I am not alone in finding that the collection of string figures is an agreeable hobby, and it may be added a very cheap one, ... moreover the figures are easy to weave, they have a history, and they are capable of many varieties.*

> *A remarkable feature ... is that a large number of the figures begin in one way. In this the tips of the thumbs and little-fingers of each hand are put together, and then from below into the loop of string; next the digits are separated, and the hands drawn apart (this is called the First Position); and, lastly, the palmar loop on each hand is picked up by the back of the index-finger; this is known as Opening A.*

Opening A



A Quick Tour of Mathematicians and String Figures

Amir-Moez (1965) Mathematics and String Figures

Opening A	. . .	oA
Opening B	. . .	oB,
Right interchange	. . .	ri,
Left interchange	. . .	li,
Out-twist	. . .	ot,
In-twist	. . .	it,
Triangle holding	. . .	th,
Spread the figure	. . .	sf,
Pick a string	. . .	p,
Release	. . .	r,
Straighten hands	. . .	sh.

$$oA, T : r, T + s1 + s2 + s3 - s4, T.ps4, sh, RI : it, RT : it, ri, li, th, sf$$

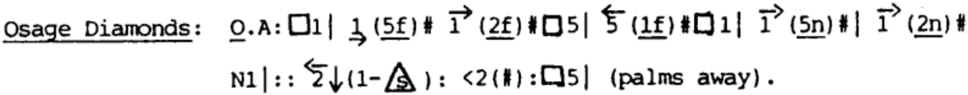
As we see here our notations are only adequate for figures with lozenges. We shall not deprive the reader from the enjoyment of discovery of more adequate notation.

A Quick Tour of Mathematicians and String Figures

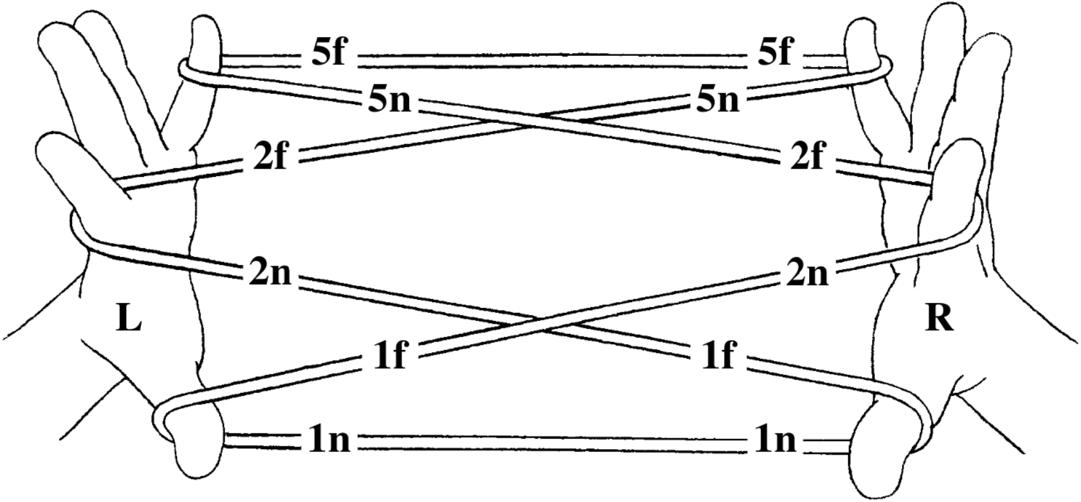
International String Figure Association (ISFA) was founded in 1978 by Hiroshi Noguchi, a Japanese mathematician, and Philip Noble, an Anglican missionary stationed in Papua New Guinea.

Storer (1988) String-Figures

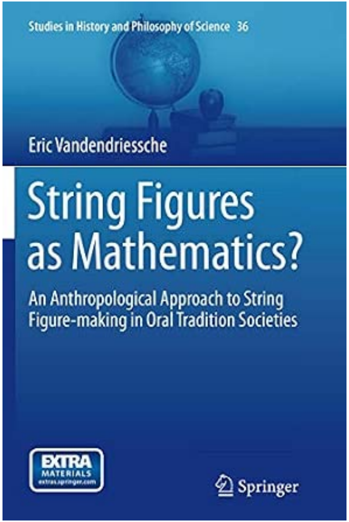
> *The wordy ramblings of collectors were too imprecise to satisfy, and topological Knot-Theorists apparently dismissed the entirety of the string-figures of the world as "trivial." And, although i learned a great deal from both groups of writers, i hungered for an approach that was neither too weak to be effective, nor so powerful that it identified (as "trivial" at that) all the objects of my insatiable interest.*



A Quick Tour of Mathematicians and String Figures



String Figures as Mathematics? (2016)

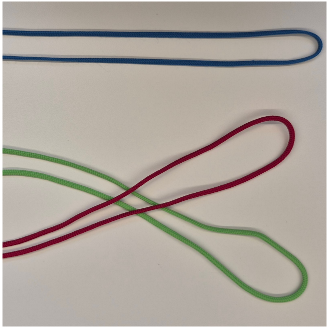


Loops and Loop-Based Notation

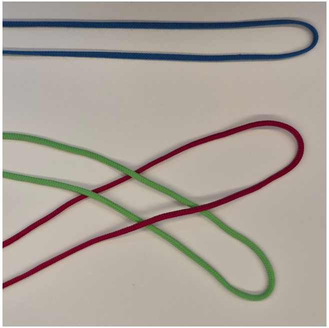
We can get insight into string figures by looking at the net motion of the loops.
Loops can twist towards ($<$) or away from ($>$) the player.
When a loop moves relative to another loop, we need to ask three questions:

1. Is the loop moving towards ($\leftarrow$) or away ($\rightarrow$) from us?
2. Is the loop going $\overrightarrow{\text{over}}$ or $\underrightarrow{\text{under}}$ its neighbour?
3. Is the loop moving $\underrightarrow{\text{up}}\uparrow$ or $\overrightarrow{\text{down}}\downarrow$ through its neighbour in the process?

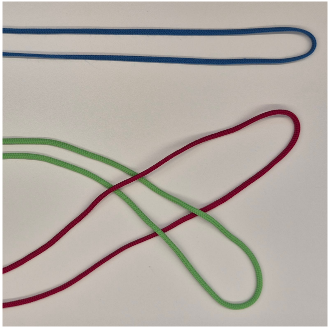
Loop Manipulations



$\underleftarrow{2}$

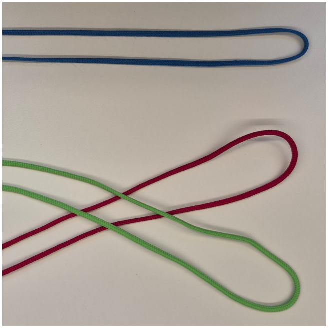


$\overleftarrow{2}\downarrow$



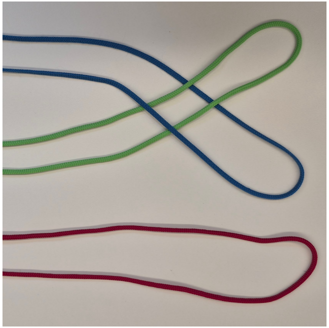
$\underleftarrow{2}\uparrow$

Loop Manipulations



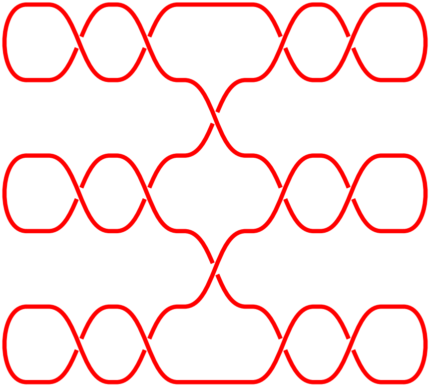
$\overleftarrow{2}$

$\overrightarrow{2}\downarrow$



$\underrightarrow{2}\uparrow$

Three Notations for One Opening



Rivers and Haddon Form Opening A. Twist all the loops a full turn towards you.

Storer $\underline{O}.A : \ll 1\infty : \ll 2\infty \ : \ll 5\infty.$

Loop Braids $\underline{O}.A : (< 1)(< 1)(< 2)(< 2)(< 3)(< 3)$

Eric's Accidental Relation

$$\underline{O}.A : \overrightarrow{1\infty}\downarrow(2\infty) : \underleftarrow{1\infty} \to 1 \equiv \underline{O}.A : \ll 1\infty$$

Consider the following manipulation sequence,

1. Form Opening A: $\underline{O}.A$.
2. Pass the thumb loop down through the index loop: $\overrightarrow{1\infty}\downarrow(2\infty)$.
3. Return the former thumb loop to the thumbs: $\underleftarrow{1\infty} \to 1$.

This is equivalent to,

1. Form Opening A: $\underline{O}.A$.
2. Rotate the thumb loops a full twist toward you: $\ll 1\infty$.

The Fundamental Question

What relations always hold among loop manipulations?

Group Theory

Definition

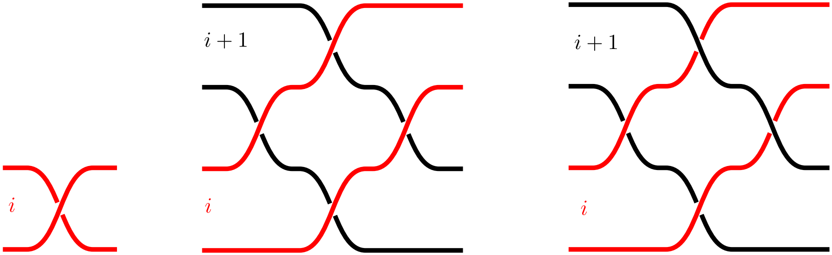
A group is an algebraic structure $(G, \cdot)$ where $G \times G \to G$ satisfies:

- There is an identity element $e \in G$ such that $ge = eg = g$.
- For every $g \in G$ there is g^{-1} such that $gg^{-1} = g^{-1}g = e$.
- The product is associative: $g(hk) = (gh)k$.

Heuristically, groups models any situation where operations can be "done and undone." Think of Rubik's cubes, addition and subtraction of numbers, or symmetry operations.

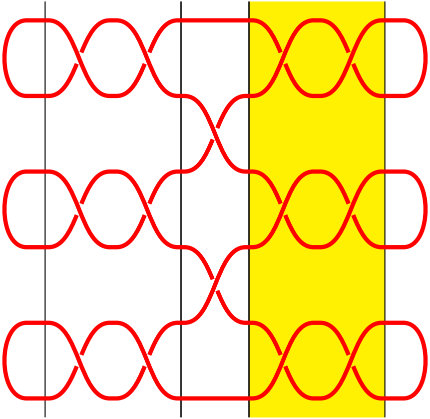
Loop Braids

Operation	**Loop Braid Notation**
Half-twist loop i clockwise.	$> i$
Cross loop i over loop $(i+1)$.	$\overrightarrow{i}$
Insert loop i over and down through loop $(i+1)$.	$\overrightarrow{i}\downarrow$



Twisting $(> i)$, crossing $(\overrightarrow{i})$, and inserting $(\overrightarrow{i}\downarrow)$.

From String Figures to Braids



Definition

The braid region of a string figure is formed by loop manipulations near the fingers.

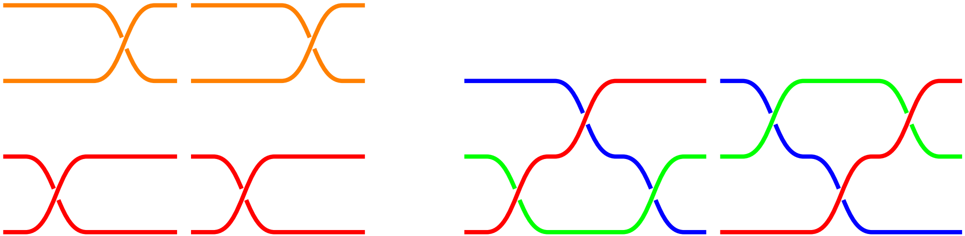
Braid Theory

Theorem (Artin 1920s)

The braid group on n strands B_n has generators σ_i for $1 \leq i \leq n-1$.
The generators satisfy the following relations:

Commutivity If $|i-j| > 1$ then $\sigma_i \sigma_j = \sigma_j \sigma_i$.

Braid Relation For each i, we have: $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$.



The commutivity relation (left) and the braid relation (right).

Back To Eric's Accidental Relation

$$\underline{O}.A : \overrightarrow{1\infty}\downarrow(2\infty) : \underleftarrow{1\infty} \to 1 \equiv \underline{O}.A : \ll 1\infty$$

Consider the following manipulation sequence,

1. Form Opening A: $\underline{O}.A$.
2. Pass the thumb loop down through the index loop: $\overrightarrow{1\infty}\downarrow(2\infty)$.
3. Return the former thumb loop to the thumbs: $\underleftarrow{1\infty} \to 1$.

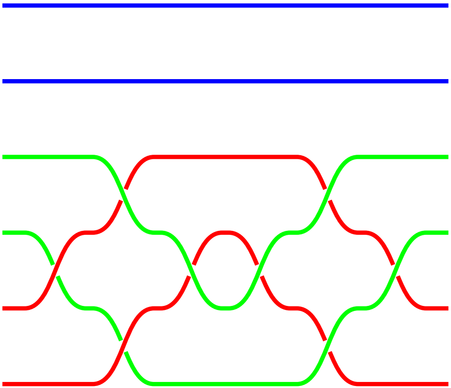
This is equivalent to,

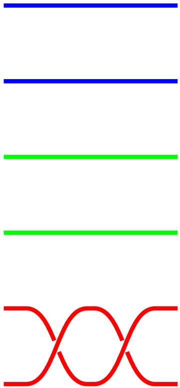
1. Form Opening A: $\underline{O}.A$.
2. Rotate the thumb loops a full twist toward you: $\ll 1\infty$.

Eric's Accidental Relation

Theorem

In the loop braid group $(\overrightarrow{1}\downarrow)(\underleftarrow{2})$ *is not equivalent to* $(<1)(<1)$.

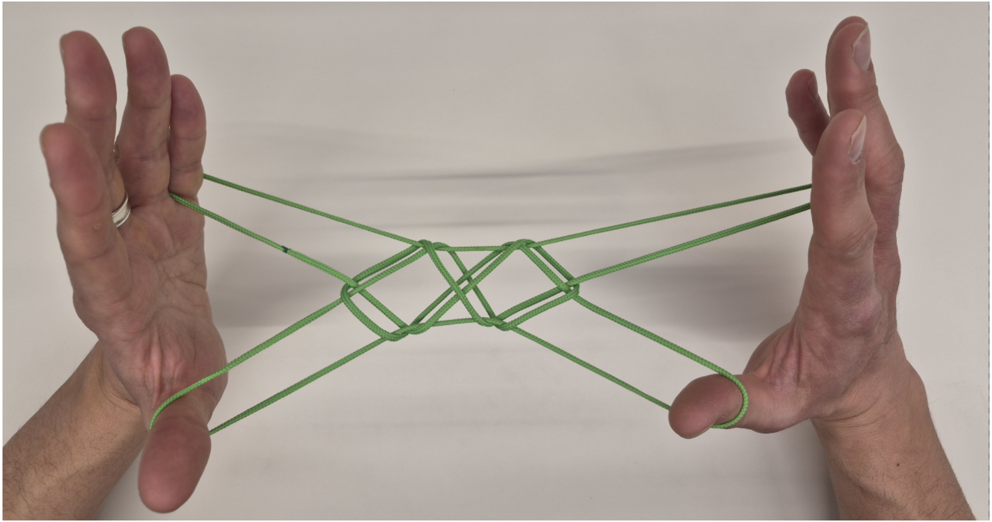




The Fun ~~damental~~ Question

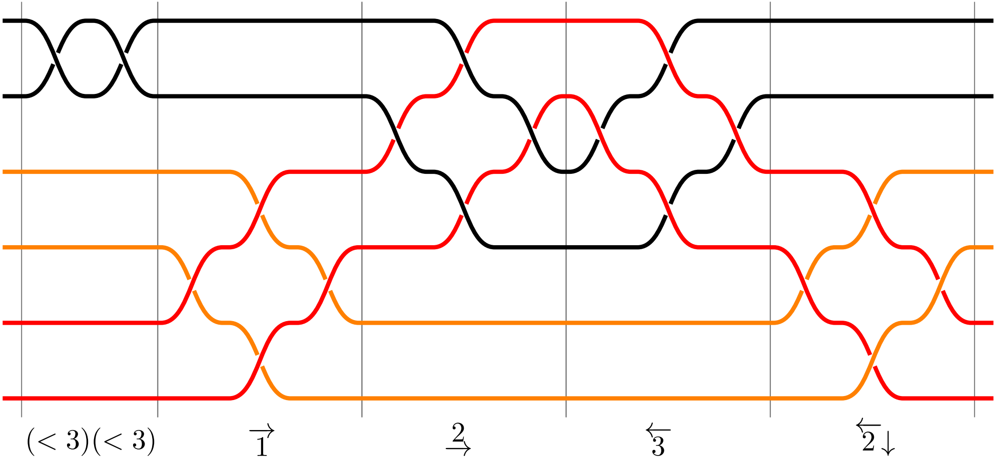
How do we verify loop braid algorithms in practice?

Koura



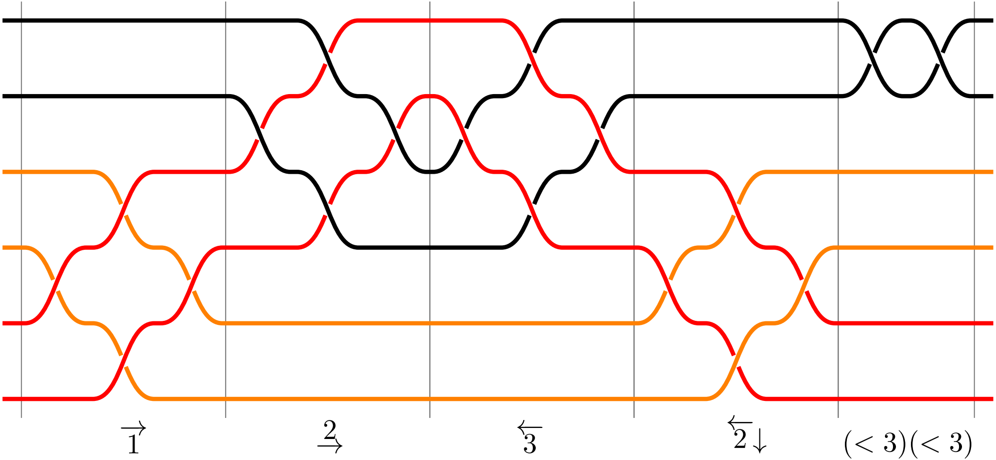
Koura (Crayfish) from Andersen's Maori String Figures (1927)

Koura As A Loop Braid



If we stare hard at this, we notice that the twists $(< 3)(< 3)$ commute!

Koura As A Loop Braid (Commuted)

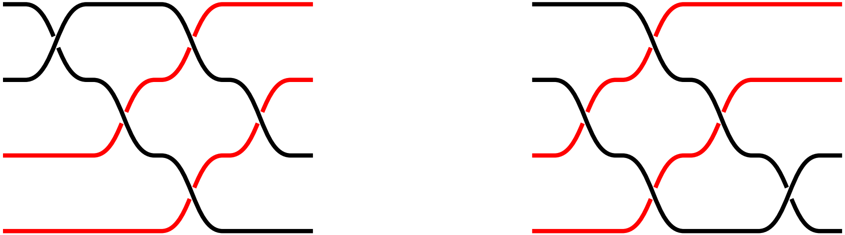


And, indeed, one can commute that twist along.

A Twist Relation

Theorem

In the loop braid group $L_n \subset B_{2n}$ we have: $(< n)(\overleftarrow{n}) = (\overleftarrow{n})(< n-1)$.



To our great surprise, the loop braid group is well studied in the literature.
An explicit group presentation with ~ 30 relations is known.

Damiani, Celeste. "A journey through loop braid groups."
Expositiones Mathematicæ 35.3 (2017): 252-285.

Thanks for playing with us today.

Questions? Comments? Ideas?