Virtual Ring Routing

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The Set-Up

Multi-hop routing algorithms describes a scheme for communication between devices.

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Main Concerns When Routing:

- Memory capacity constraints
- Over-reliance on specific nodes in the network
- Scalability
- Length of routing path

The Premise of VRR

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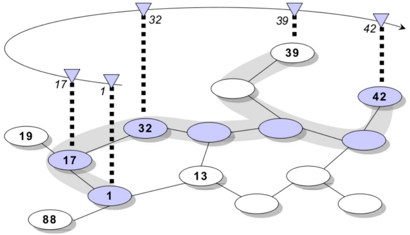

Figure: The underlying physical network and the virtual overlay (image from Wikipedia)

The Physical Network

An undirected graph $G = (V; E)$

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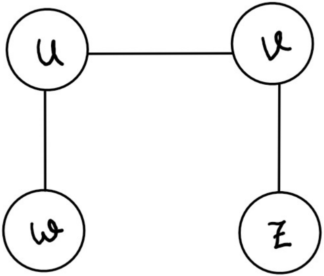

$(u; v)\ 2\ E$ is a physical link

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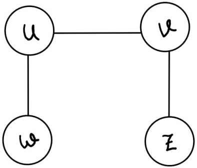

$hu; zi\ 2\ E_v$ is a virtual link, where $hu; zi = vl((u; v); (v; z))$

Identifier Space

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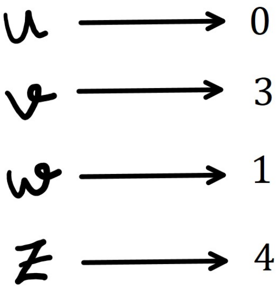
- well-ordered set
- distance function *d*

Also need a function that assigns each node to a unique *id 2* Ω
In VRR, we always take Ω N and assume the function assigns *id* randomly

Node to Id Mapping



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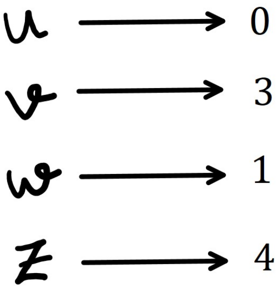

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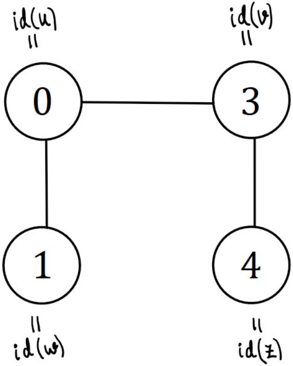

We often use v or $id(v)$ to refer to the same node

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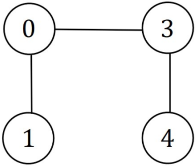
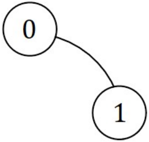
Note: It is always possible to create a virtual link between any two nodes in V since we require G to be connected

For Example...



Physical Network



$\langle 0; 1 \rangle = vl(0; 1)$

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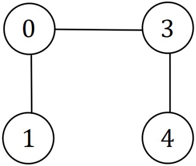

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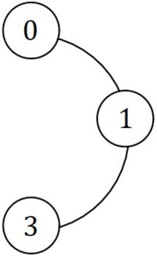

$$h0; 1i = vl(0; 1)$$

$$h1; 3i = vl((1; 0); (0; 3))$$

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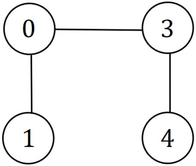

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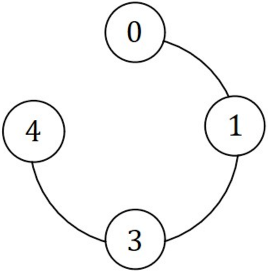

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$$\langle 3; 4 \rangle = vl(3; 4)$$

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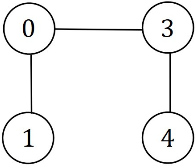

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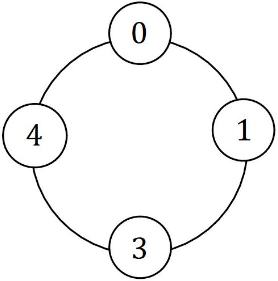
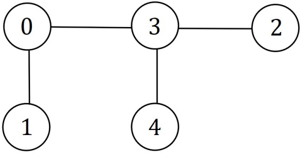


Virtual Network

$$h0;1i = vl(0;1)$$
$$h1;3i = vl((1;0);(0;3))$$
$$h3;4i = vl(3;4)$$
$$h4;0i = vl((4;3);(3;0))$$

Another Node?

If we add another node then



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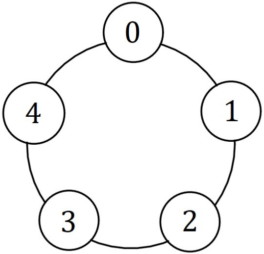

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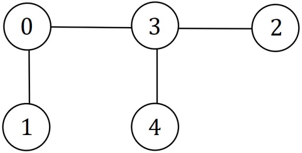

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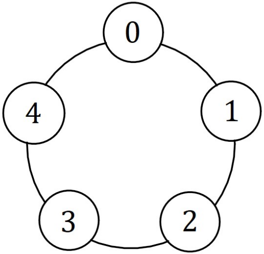

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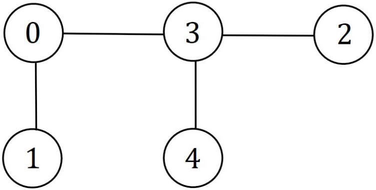
$$h1;2i = vl(h1;3i;(3;2))$$
$$h2;3i = vl(2;3)$$

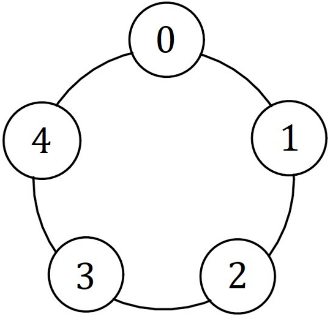
Physical and Virtual Neighbours



Physical Network

$P_3 = f0; 4; 2g$

$V_3 = f4; 2g$



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Routing Table

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The **routing table** at node $u\ 2\ V$ is a collection of 4-tuples (from, to, next hop, prev. hop) from $V \quad V \quad V \quad V$ of routing paths that go through node u.

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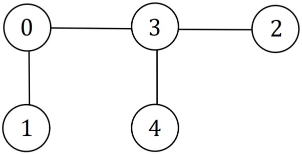
- End_u denotes the set of all nodes in the "to" or "from" coordinate in an entry on the routing table of u other than u itself

  i.e. all the nodes we know how to travel to from u

For Example...



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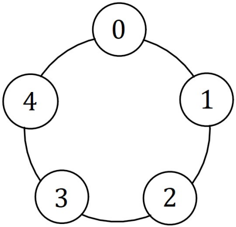

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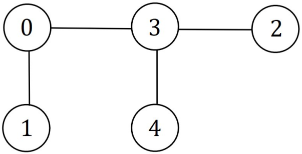

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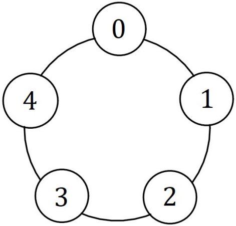

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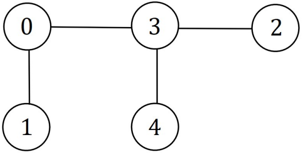

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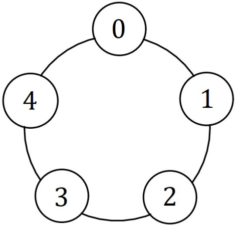

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Once the physical network is fixed, the routing table is fixed

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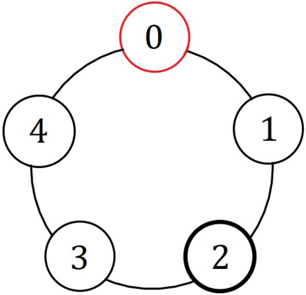
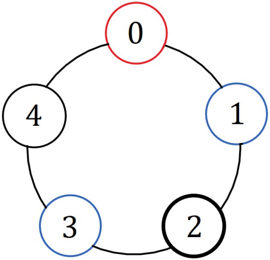


Virtual Network

First Attempt

$V_2 = f3; 1g$



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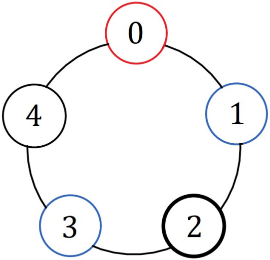

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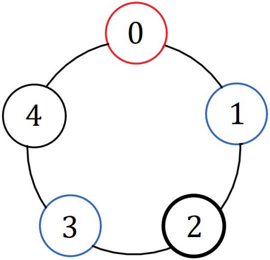

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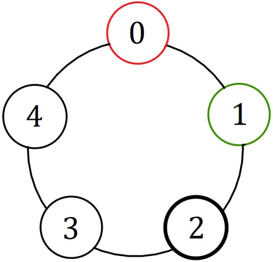

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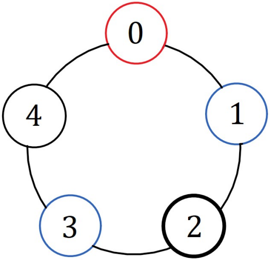

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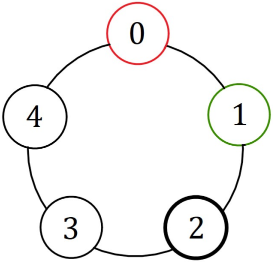
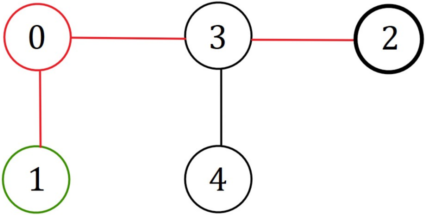
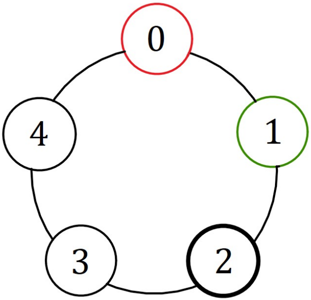




Second Attempt?



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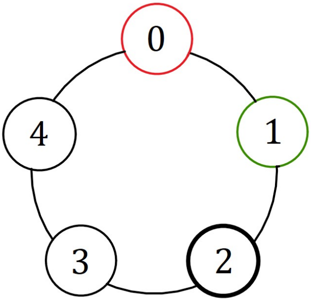
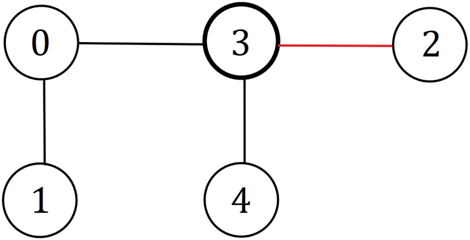
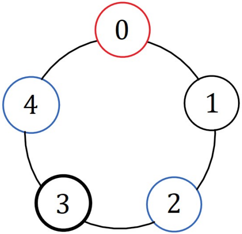




Comparing at Each Node

$V_3 = f4; 1g$



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Third (And Hopefully Last) Attempt

Using the Routing Table

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Using the Routing Table

$End_3 = \{0; 1; 2; 4\}$

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The Actual Routing Path

Routing Table Entry for 0

from	to	next hop	prev. hop
4	0	0	4

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Greedy Transition Vs. Non-Greedy Transition

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Routing Table at 2

from	to	next hop	prev. hop
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$V_2 = f\,1; 3g,\ T^0 = 1$

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2	3	3	-
2	1	3	-

$V_2 = \{1, 3\}$, $T' = 1$

$T' \notin V_3$

Greedy Transition

$T' \in V_2$

Non-Greedy Transition

Summarizing The Routing Algorithm

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- If no such x_2 exists in End_u, then we forward the message along to the next hop node towards $T' = x_1$
- Once we reach an intermediate target x_i, we set the intermediate target from End_{x_i}

What exactly is a better node?

x is a better node if $d(x; T) < d(T'; T)$, where T is the target, T' is the intermediate target

Note: We want to choose the "best" possible x, i.e. the one that gets us closest to the target

Some Edge Cases

Some Edge Cases

Remarks About the Intermediate Target(s)

- Each new intermediate target T' is always closer to the target T than the previous one.
- At any node u, the current intermediate target is closer to the target than u or equidistant
- Once we set $T' = T$, there are no more new intermediate targets

Properties of This Routing Algorithm

Proposition 1

VRR schemes using greedy routing exhibit no loops

Proof: If we have a loop, we must visit the same node twice, say v.

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At time 1

At time 2

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Bene ts of VRR

- Each node only stores local info; less memory used relative to increase in size of physical network
- Flexible since adding/ removing a node only a ects nodes that had that node in their routing table

Limitations of VRR

Routing path may not be the shortest path

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De nition 3

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Consider the following example

Stretch

Suppose again that we are routing $\langle 2, 0 \rangle$. Our ring:

The Actual Routing Path

from	to	next hop	prev. hop
2	3	3	-
2	1	3	-

$d(1;0) < d(3;0)$

=) $T^0 = 1$

The Actual Routing Path

from	to	next hop	prev. hop
3	5	5	-
3	4	4	-
3	2	2	-
2	1	4	2
4	5	5	4

$d(5;0)$ $d(1;0)$

=) $T^{0} = 1$

The Actual Routing Path

from	to	next hop	prev. hop
4	3	3	-
4	1	2	-
4	5	3	-

$d(5;0) \quad d(1;0)$

$=) \quad T^0 = 1$

Stretch

$L_A = 5$

Stretch

$L_A = 5$

$L_S = 3$

Stretch

$L_A = 5$

$L_S = 3$

Stretch is 5:3

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Current Research: Careful Study of Stretch

The questions:

How can we study stretch?

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Current Research: Careful Study of Stretch

The questions:

- How can we study stretch?
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- Setting up a good model for the physical network
- Computing the expected path length

The virtual network for large N

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Physical Network as a Ring

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Physical Network as a Ring

- When the physical network is also a ring, we call it a physical ring.
- There are only two physical neighbours
- We can consider the physical ring a permutation of the virtual ring

Stretch on a Ring

Proposition 2

If the physical network is a ring of $N > 3$ nodes, then the maximum stretch for any routing path is $\frac{N-2}{2}$

Proof of Prop'n 2

Proof: Consider source-target pair S and T. Suppose that, on the physical ring, $d(S;T) = k \quad 2$

Figure: The Physical Ring

Proof of Prop'n 2

Proof: Consider source-target pair S and T. Suppose that, on the physical ring, $d(S; T) = k \quad 2$

Figure: The Physical Ring

By prop'n 1, the actual routing path either travels the minor arc or the major arc.

Proof Cont'd

If we travel the minor arc then

$$\frac{L_A}{L_S} = \frac{k}{k} = 1$$

Figure: The Physical Ring

Proof Cont'd

If we travel the minor arc then

$$\frac{L_A}{L_S} = \frac{k}{k} = 1$$

If we travel the major arc then

$$\frac{L_A}{L_S} = \frac{N \quad k}{k}$$

Figure: The Physical Ring

Proof Cont'd

If we travel the minor arc then

$$\frac{L_A}{L_S} = \frac{k}{k} = 1$$

If we travel the major arc then

$$\frac{L_A}{L_S} = \frac{N \quad k}{k}$$

This ratio is maximized when

$k = 2$

Figure: The Physical Ring

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Limitations of the Physical Ring

Not practical since relatively poor connectivity

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Not practical since relatively poor connectivity

Even the shortest paths are long

Future Directions

Explore other physical network models

Future Directions

- Explore other physical network models
- Trying to formulate routing algorithm into Markov Chain process or as (sub)martingales and apply related theorems

The End

Thanks for listening!
Any questions?

For Further Reading

To better understand the mechanics of VRR:

M. Caesar, M. Castro, E. B. Nightingale, G. O'Shea, and A. Rowstron. Virtual ring routing: Network routing inspired by DHTs. In *ACM annual conference of the Special Interest Group on Data Communication (SIGCOMM)*, pages 351–362, 2006.

To read about other proofs related to VRR:

Malkhi, D., Sen, S., Talwar, K., Werneck, R. F., and Wieder, U. (2009). Virtual ring routing trends. In *Proceedings of the 23rd international conference on distributed computing*, Berlin, Heidelberg.