Artificial neural networks: The fundamentals

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• Getting a function to spit out the right vector if we feed it the right numerical input

• Seems straightforward, but...

• I may not know the function, only certain points



An image is an array of numbers (classification problem!)

157 1	153	174	168	150	152	129	151	172	161	155	156	157	153	174	168	150	152	129	151	172	161	155	15
155 1	182	163	74	75	62	33	17	110	210	180	154	155	182	163	74	75	62	33	17	110	210	180	15
180 1	180	50	14	84	6	10	33	48	105	159	181	180	180	50	14	34	6	10	33	48	106	159	18
206 1	109	5	124	131	111	120	204	166	15	56	180	206	109	5	124	131	111	120	204	166	15	56	18
194	68	137	251	237	239	239	228	227	87	71	201	194	68	137	251	237	239	239	228	227	87	n	20
172 1	105	207	233	233	214	220	239	228	98	74	206	172	105	207	233	233	214	220	239	228	98	74	20
188	88	179	209	185	215	211	158	139	75	20	169	188	88	179	209	185	215	211	158	139	75	20	16
189	97	165	84	10	168	134	11	31	62	22	148	189	97	165	84	10	168	134	11	31	62	22	14
199 1	168	191	193	158	227	178	143	182	105	36	190	199	168	191	193	158	227	178	143	182	106	36	19
205 1	174	155	252	236	231	149	178	228	43	95	234	205	174	155	252	236	231	149	178	228	43	96	23
190 3	216	116	149	236	187	85	150	79	38	218	241	190	216	116	149	236	187	86	150	79	38	218	24
190 3	224	147	108	227	210	127	102	36	101	255	224	190	224	147	108	227	210	127	102	36	101	255	22
190 2	214	173	66	103	143	95	50	2	109	249	215	190	214	173	66	103	143	96	50	2	109	249	21
187 1	196	235	75	1	81	47	0	6	217	255	211	187	196	235	75	1	81	47	0	6	217	255	21
183 2	202	237	145	0	0	12	108	200	138	243	236	183	202	237	145	0	0	12	108	200	138	243	23

Image credit: [2]

- Even words can be converted to numbers
 - This is how you make chat bots!

The quick brown fox jumped over the brown dog



Image credit: [3]

Artificial Neurons

- Building blocks of neural networks
 - Inputs
 - Weights
 - Summation
 - Activation function
 - Ex: threshold, sigmoidal



Artificial Neural Networks

- Chain multiple neurons together
 - Series of linear and nonlinear transformations allows for input to be 'converted' to correct output



Forward Propagation Walkthrough



Image credit: [5]

DEF: Two sets of points A and $B \subseteq \mathbb{R}^n$ are linearly separable if there exist n + 1 points w_0 , ..., w_n such that for any point a in A, $\sum_{i=1}^{n} w_i a_i < w_0$, and for any point b in B, $\sum_{i=1}^{n} w_i b_i > w_0$

• Effectively, two sets are linearly separable if there exists some hyperplane that separates the two sets

Why this is necessary: Linear separability

• What an individual node without an activation function does is place an input above/below a hyperplane

- What if data is not linearly separable?
 - Drawing a line will not help me separate these two groups
 - So the network won't be able to tell the difference in its output!



• Multiple nodes, layers, and nonlinear activation transform our data to something linearly separable



[8]; Image credit: [8]

Part 1 Summary: Artificial Neural Networks Work!

- Given an input, so long as the weights are correct, an ANN will return an output with a numerical value that either:
 - Represents the 'class' of the input
 - Is the image of the input, as transformed by some unknown (approximated) function

REGARDLESS OF WHETHER OR NOT THE INPUT CAN BE LINEARLLY SEPARATED INTO OUTPUT CATEGORIES

The next problem: Weights?

- How do we determine weights?
 - Training!

11111111111111111 222222222222222 6666666666666666 7777777777777777777 9999999999999

Image credit: [9]

- Backpropagation is how we train ANNs
 - Weight-update algorithm

- Basic concept
 - 1) Forward propagate training data
 - 2) Calculate error
 - 3) Update weights to minimize error
 - 4) Repeat

- STEP 1: Forward propagate the data
 - This is just a normal forward pass
 - We get a result, which may or may not be 'correct'

• STEP 2: Calculate error

DEF: for some piece of training data x, let y_t be the expected (correct) output, and let y_e be the actual output of the network

- Define some differentiable error function $E(y_t, y_e)$ that represents the error between the expected and actual outputs
 - We call the error of the output the "loss"
 - We want to MINIMIZE LOSS

- STEP 3: Update weights to minimize loss
 - Use the backpropagation algorithm, which employs the chain rule

- Recall the gradient of a function is a vector of the partial derivatives of the function wrt each component
 - If the gradient is positive in a direction, the function is increasing in that direction

- Remember, we want to minimize E
 - Get the gradient of E to zero

- E is a function of the output, and thus a function of the input and the weights $\{w_1, \dots, w_k\}$
 - If $\frac{\partial E}{\partial w_i}$ is positive, E is increasing as w_i does, so we want to decrease w_i

• Weight update rule: $w_i = w_i - (l \frac{\partial E}{\partial w_i})$

l is the learning rate of the network

• How do we find the partial derivative wrt w_i? Chain rule!

- E is a function of $y_{\rm e}$
- y_e is a function of the layer's input, z
- z is a function of the previous layer's output and the weights



- How do we find the partial derivative wrt w_i? Chain rule!
- E is a function of $y_{e,i}$
- y_e is a function of the layer's input, z

 $\frac{\partial y_{e,i}}{\partial z}$

 ∂E

 $\partial y_{e,i}$

• z is a function of the previous layer's output $\frac{\partial z}{\partial w_i}$



• How do we find the partial derivative wrt w_i? Chain rule!

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial y_{e,i}} \frac{\partial y_{e,i}}{\partial z} \frac{\partial z}{\partial w_i}$$

[10]

- In summary:
 - 1) Forward pass
 - 2) Calculate loss
 - 3) Backpropagate and weight update
 - 4) Rinse and repeat until loss has been minimized

- So that's how you train a neural net!
 - There are some other tweaks you can make
 - There are some issues
 - Local minima
 - Fine-tuning learning rate



Intuitively, what's happening?

- By adjusting weights, ANNs pick out which inputs are important in certain circumstances
 - The weights are chosen to notice patterns in inputs, and produce different outputs with the same weights, depending on the presence/absence of certain patterns (features)
 - Far more powerful than a simple regression
 - Robust to small changes in the input, so long as general patterns are present (ie: different species of cats are still cats)

Time for a quick demo!

Advanced ANNs

- Recurrent ANNs allow for time-dependent data
 - Output of previous timestep becomes an input
 - Good for chat bots

Advanced ANNs

• Convolutional neural networks are good for image classification



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