MAT 133 — Week 3c

Review of Financial Math

Simple Interest:

$P invested with interest rate r% annually for a term of n years

$$S = P + \underbrace{rP + rP + \dots + rP}_{n \text{ years}}$$

$$= P + nrP = P(1+nr)$$

Simple compound Interest (p32)

$P invested with interest rate r% compounded n times

$$S = P\underbrace{(1+r)(1+r)\dots\dots(1+r)}_{n \text{ times}}$$

$$= P(1+r)^n$$

Compound Interest (APR) (p32)

$P invested with annual percentage rate r% for n years compounded t times annually.

$$S = P\underbrace{\left(1+\frac{r}{t}\right)\left(1+\frac{r}{t}\right)\dots\dots\left(1+\frac{r}{t}\right)}_{n \text{ years } t \text{ times each}}$$

$$= P\left(1+\frac{r}{t}\right)^{nt}$$

Question:

If we invest $1^{\$}$ with APR 100% compounded every millisecond for a year, how much money will our investment be worth?

Answer (Bernoulli): $\approx 2.7187^{\$}$

Continuously Compounded Interest (p36)

$P invested with annual percentage rate r% compounded at each instant in time for n years

$$S = e^{rn} \approx (2.718..)^{rn}$$

Def^n (p37) The EFFECTIVE CONTINUOUS RATE of compounding with rate r% compounded t times is R such that

$$e^{R} = \left(1+\frac{r}{t}\right)^{t}$$

$$\Leftrightarrow R = t\ln\left(1+\frac{r}{t}\right)$$

? The idea of R is that it converts discrete compounding in to continuous compounding. It approximates the discrete process continuously.

Ex: Find the ECR for 5% APR compounded monthly

$$R \approx 0.04989 \approx 4.98\%$$

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Annuities (§ 2.2)

Def^n (p37) An **ANNUITY** is a simply compounded investment regularly supplemented by adding principal.

Ex: We invest R dollars with rate r%

Month n	$S(n)$
1	R
2	$(1+r)S(1)+R$
3	$(1+r)S(2)+R$
4	$(1+r)S(3)+R$.

Note:

$$\begin{aligned} S(3) &= (1+r)S(1)+R \\ &= (1+r)[(1+r)S(0)+R]+R \\ &= (1+r)[(1+r)R+R]+R \\ &= (1+r)^2R+(1+r)R+R. \\ &= (1+r)^2R+(1+r)^1R+(1+r)^0R \end{aligned}$$

In general,

$$\begin{aligned} S(n) &= \sum_{k=0}^{n-1}(1+r)^k R \\ &= R\left[\frac{1-(1+r)^n}{1-(1+r)}\right] \\ &= R\left[\frac{(1+r)^n-1}{r}\right] \end{aligned}$$

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Present and Future Value.

Defⁿ (p32) The **FUTURE VALUE** of an investment is the value it will attain in the future.

Defⁿ (p34) The **PRESENT VALUE** of an investment is the amount that must be invested now to achieve a financial target.

Exercise:

Pick a graduation present for yourself. something expensive $5000 ~ 20000..

Decide how to finance it.

What kind of annuity can you afford?