Linear Algebra Eigenvectors and Diagonalization Worksheet

This worksheet is to be done in class. Feel free to discuss with other students and ask the instructor for help. You will need to work on fresh *blank paper* to complete these questions. *No answers* will be provided to these questions; please ask in class or office hours.

Question 0.1 *Formulate answers to the following questions. If appropriate, provide examples that show you are correct.*

1. *Can an invertible matrix have $\lambda = 0$ as an eigenvalue?*
2. *How many different eigenvalues can an $n \times n$ matrix have?*
3. *How many distinct eigenvectors can an eigenvalue have?*
4. *Why does every eigenvalue have at least one eigenvector?*

Question 0.2 *Suppose that* $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix}$ *and* $A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$.

1. *Answer without finiding the matrix reprsenting A: What are the eigenvalues and eigenvectors of A?*
2. *What does the linear transformation A do to the unit circle? Draw <u>pictures</u> of the unit circle before and after applying A.*
3. *Find the matrix A.*
4. *Diagonalize the matrix A as $P^{-1}AP = D$.*
5. *How do A, P, and D relate to your pictures?*

Question 0.3 *Complete both parts:*

1. *Diagonalize* $A = \begin{bmatrix}1 & 3\\0 & 2\end{bmatrix}$ *and* $B = \begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$.
2. *Verify that AB is <u>not</u> diagonalizable.*

MORE QUESTIONS ON THE BACK

Question 0.4 *The following question will involve the use of complex numbers and the identity:*

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

1. *Find the eigenvalues of the rotation matrix:* $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
2. *Geometrically, why are there no real eigenvalues for* $\theta \neq \pi$*?*

Question 0.5 *Diagonalize (if possible) the following matrices:*

$$A = \begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

If it is not possible to diagonalize a matrix, clearly state why it is not possible.

Question 0.6 *Using* $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$ *and* $P = \begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix}$ *answer the following:*

1. *Show that* $P^{-1}AP = D$ *for some diagonal matrix* D*.*
2. *Show algebraically, without calculating,* $(P^{-1}AP)(P^{-1}AP) = P^{-1}A^2P$
3. *Convince yourself that* $A^n = PD^nP^{-1}$
4. *Calculate* A^{10}*.*

Short Survey (hand in before leaving)

On a seperate sheet of blank paper, please answer the following:

1. Which questions from homework were easy? (Be specific.)
2. Which questions from homework were really hard?
3. What question would you like to see on the Test 2?