

# MAT223 Exam Practice Sheet

April 3, 2018

Please solve these problems on a separate sheet of paper. No aids permitted.

**Question 1 (Lines and Planes)** • Find the equation  $ax + by = c$  of the line passing through the points:  $(3, 2)$  and  $(9, -3)$ .

- Find the vector equation  $\vec{p} = \vec{v}_0 + t\vec{v}_1$  of the line passing through  $(1, 2, 3)$  in the direction  $\vec{v} = (4, 5, 6)$ .
- Find the vector equation  $\vec{p} = \vec{v}_0 + s\vec{v}_1 + t\vec{v}_2$  for the plane passing through the points:  $(2, 2, 5)$ ,  $(3, 4, 5)$ , and  $(4, 5, 7)$ . Write this plane in the format  $ax + by + cz = d$  as well.
- Find two 2-dimensional planes in  $\mathbb{R}^4$  that intersect in single point. Could two planes intersect in a point in  $\mathbb{R}^3$ ?

**Question 2 (Rotations)** Find the  $2 \times 2$  matrix  $R_\theta$  representing a counter-clockwise rotation of  $\mathbb{R}^2$  by  $\theta = \pi/6$  about the origin.

- What does  $R_\theta$  do to the line  $\vec{p} = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} t$ ?
- Does  $R_\theta$  have any eigenvectors? Explain.

**Question 3** Consider the system:

$$\begin{cases} x + Ay = -1 \\ 5x - By = 6 \end{cases}$$

For what values of  $A$  and  $B$  does this system have: (i) a unique solution, (ii) no solutions, (iii) infinitely many solutions?

**Question 4** Find all the solutions of  $(\bar{z})^3 + 2 = 0$ .  
If  $v = 1 + i$  and  $w = 1 - 2i$  find  $\bar{v}w - iw^2$ .

**Question 5** Find the equation of a plane perpendicular to the line:

$$\vec{p} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

and containing the point  $p = (6, 6, 6)$ .

**Question 6** Let  $\vec{e}_1$  and  $\vec{e}_2$  be the standard basis vectors of  $\mathbb{R}^2$ . Suppose  $T$  is a linear transformation such that:  $T(\vec{e}_1 + \vec{e}_2) = 2\vec{e}_1$  and  $T(-\vec{e}_1 + \vec{e}_2) = \vec{e}_1 + 3\vec{e}_2$ . Find the  $2 \times 2$  matrix representing  $T$ .

**Question 7** Express the vector  $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$  in the basis:

$$\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} \right\}$$

**Question 8** Consider the matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. Find a basis for the nullspace of  $A$ .  
What is the dimension of the nullspace?
2. Find an orthogonal basis for the nullspace of  $A$ .  
Find an orthonormal basis for  $\text{null}(A)$ .
3. What is the orthogonal complement of the nullspace of  $A$ ?
4. Find a basis for the column space of  $A$ .
5. Find a basis for the row space of  $A$ . How is this related to (4)?
6. What is the rank of  $A$ ? How is this related to (1)?

GOOD LUCK!