Mat 232 LEC

Sept. 6

Check 232 syllabus for textbook

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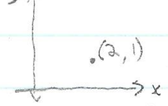
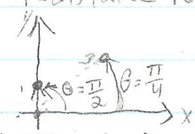
Polar Coordinates (§11.1)



Def^n Cartesian coordinates. The usual coordinates on the plane.

Polar coodinates  (r,θ) θ = angle to x-axis, r = distance to origin.

Ex Plot $(1,\frac{\pi}{2})$ and $(3,\frac{\pi}{4})$ in polar coordinates

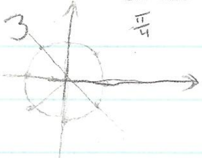


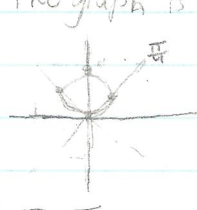
Ex express $(1,\frac{\pi}{2})$ in cartesian corb. Also $(3,\frac{\pi}{4})$. $(1,\frac{\pi}{2}) \Rightarrow (0,1)$ $(3,\frac{\pi}{4}) \Rightarrow$?

observe $x=y \Rightarrow 3^2 = 2x^2 \Rightarrow x = \sqrt{\frac{3^2}{2}} = \frac{3}{\sqrt{2}}$ so $(3,\frac{\pi}{4}) = (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$.

graphing in Polar Coordinates.

Ex: Graph $r=3$ $\frac{\pi}{4}$ The graph is a circle of radius 3.





$r=\sin(0)=0$ $\sin(\frac{5\pi}{4}) = -\frac{1}{\sqrt{2}}$(!)

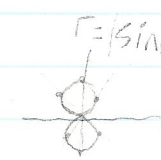
$\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ $\sin(\frac{3\pi}{2}) = -1$

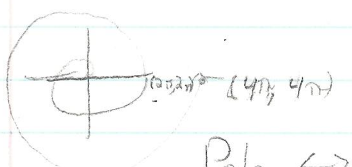
$\sin(\frac{\pi}{2}) = 1$

Ex: Graph $r=\sin(\theta)$ $\sin(\frac{3\pi}{4})\ \frac{1}{\sqrt{2}}$ $\sin(\frac{7\pi}{4}) = -\frac{1}{\sqrt{2}}$

$\sin(\pi)=0$

$r=|\sin(\theta)|$





$\boxed{\text{KEY POINT}}$ Only graph $0 \le \theta < \infty$

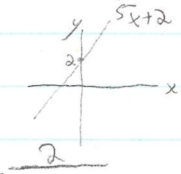
Polar $\leftrightarrow$ Cartesian

Cartesian $\rightarrow$ Polar: Have (x,y) want (r,θ)

$r=\sqrt{x^2+y^2}$ $\theta = \ldots \tan^{-1}(\frac{y}{x})$ (if $x \neq 0$) Polar $\rightarrow$ Cartesian: Have $(r,\theta) \rightarrow (x,y)$ $x = r\cdot\cos(\theta)$

$y = r\cdot\sin(\theta)$

Ex: (§11.1 Q2g) Find the polar formula for $y=5x+2$



\# Sub. in polar coords. $r\sin(\theta) = 5[r\cos(\theta)] + 2$

\# Solve for r in terms of θ $r(\sin(\theta) - 5\cos(\theta)) = 2 \Rightarrow r = \frac{2}{\sin\theta - 5\cos\theta}$

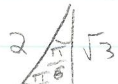
Slope in Polar Form (§11.2)

Recall slope $= \frac{dy}{dx} = \frac{\text{"rise"}}{\text{"run"}}$ $\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$ Ex Determine where $r = 1+\cos(\theta)$ with $0 \le \theta < 2\pi$ has a horizontal or vertical tangent. $\frac{dy}{dx} = 0 = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[(1+\cos(\theta))\sin\theta]}{\frac{d}{d\theta}[(1+\cos\theta)\cos\theta]}$

$= \frac{-\sin\theta\cdot\sin\theta + (1+\cos\theta)\cos\theta}{-\sin\theta\cdot\cos\theta - (1+\cos\theta)\sin\theta} = \frac{-\sin^2\theta + \cos^2\theta + \cos\theta}{-2\sin\theta\cos\theta - \sin\theta} = \frac{2\cos^2\theta + \cos\theta - 1}{-2\sin\theta\cos\theta - \sin\theta}$

$\#\ \cos^2\theta + \sin^2\theta = 1$
$-\sin^2\theta = 1 - \cos^2\theta$
$-\sin^2\theta = \cos^2\theta - 1$

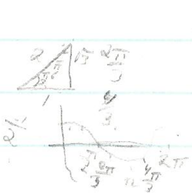
$= \frac{(2\cos\theta - 1)(\cos\theta + 1)}{-\sin\theta(2\cos\theta+1)}$ \# Horizontal tangents $\frac{dy}{dx} = 0 \Leftrightarrow (2\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$



$\Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ Exer. When is $\frac{dy}{dx}$ $+\infty$ or $-\infty$?

$-\sin\theta(2\cos\theta + 1) = 0$, $-\sin\theta = 0$ or $2\cos\theta + 1 = 0$ $\theta = \pi x$, $x \in \mathbb{Z}$ or $\cos\theta = -\frac{1}{2}$

So $-\sin\theta(2\cos\theta + 1) = 0$ when $\theta = \pi x, \frac{2\pi x}{3}$, $\forall x \in \mathbb{Z}$



Sept. 11

LEC. MAT232

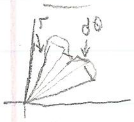
§11.3 - Area in Polar coordinates

Q. What's the area of a circle of radius r? $A = \pi r^2$

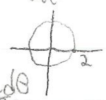
Q. What's the area of an arc of angle θ and radius r

$A = \pi r^2 \left(\frac{\theta}{2\pi}\right) = \frac{r^2\theta}{2}$

Fact: The area of $r = f(\theta)$ between $\theta = a$ and $\theta = b$ $A = \int_a^b \frac{r^2}{2}\, d\theta$



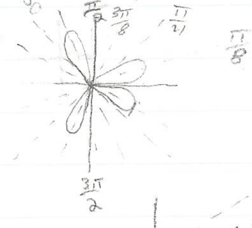
Ex: Find area bounded by $r = 2$

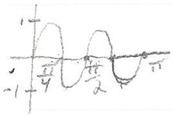


Pick bounds for θ

$A = \int_0^{2\pi} \frac{r^2}{2}\, d\theta = \int_0^{2\pi} \frac{2^2}{2}\, d\theta = \int_0^{2\pi} 2\, d\theta$ $\quad 2\theta\Big]_0^{2\pi} = 4\pi - 0 = 4\pi = r^2\pi = 4\pi$

Ex Sketch the curve $r = \sin(4\theta)$





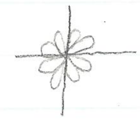
$\sin\left(4 \cdot \frac{\pi}{8}\right) = 1 \quad \sin\left(4 \cdot \frac{3\pi}{8}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

$\sin\left(4 \cdot \frac{\pi}{4}\right) = 0 \ldots$

To summarize

Each arc of size $\frac{\pi}{4}$ will contain a loop.

We should get:



Ex Find area bounded by $r = \sin(4\theta)$. # Use symmetry Total = 8(area one loop)

$A = \int_0^{\frac{\pi}{4}} \frac{r^2}{2}\, d\theta = \int^{\pi/4} \frac{(\sin(4\theta))^2}{2}\, d\theta = \int_0^{\frac{\pi}{4}} \frac{\sin^2(4\theta)}{2}\, d\theta$ # Use trig identity $\sin^2(x) = \frac{1-\cos(2x)}{2}$

$= \int_0^{\pi/4} \frac{1-\cos(8\theta)}{2 \cdot 2}\, d\theta = \frac{\theta - \frac{\sin(8\theta)}{8}}{4}\Bigg]_0^{\frac{\pi}{4}} = \frac{\frac{\pi}{4} - \frac{\sin(2\pi)}{8}}{4} = \frac{\pi}{16}$ = loop area. So, total area $= 8 \cdot \frac{\pi}{16} = \frac{\pi}{2}$

Ex find area of $r = \sin(5\theta)$

$\sin\left(5\, \frac{3\pi}{10}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$