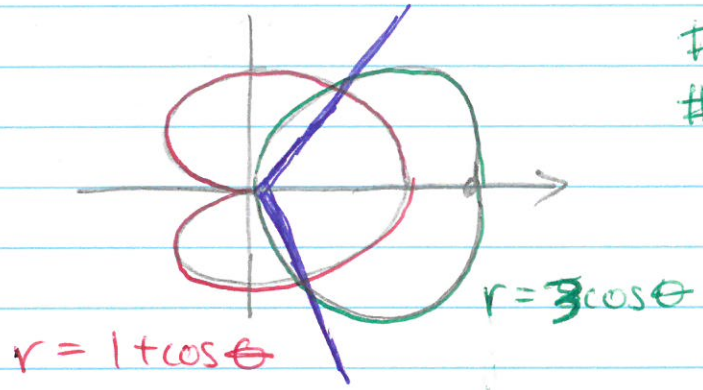
Test Review

Ex: Find the area inside $r = 3\cos(\theta)$ and $r = 1+\cos(\theta)$



Sketch curves
Find points of intersection.

$$1+\cos\theta = 3\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

Thus, $\theta = \pi/6$ and $\theta = -\pi/6$.

We obtain:

$$A = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \frac{(3\cos\theta)^2}{2}\,d\theta + \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{(1+\cos\theta)^2}{2}\,d\theta$$

$$+ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(3\cos\theta)^2}{2}\,d\theta.$$

Ex: Find the tangent plane to

$$f(x,y) = xy\ln(1+x^2-y^2)$$

at the point $(1,1,0)$

Calculate the partials.

$$f_x = y\ln(1+x^2-y^2) + xy\frac{1}{1+x^2-y^2}(2x)$$

$$= y\ln(1+x^2-y^2) + \frac{2x^2y}{1+x^2-y^2}$$

$$f_y = x\ln(1+x^2-y^2) - \frac{2xy^2}{1+x^2-y^2}$$

Determine the slopes.

$$f_x(1,1) = 2 \qquad f_y(1,1) = -2.$$

Write the tangent plane

$$\pi(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$$

$$= 0 + 2(x-1) - 2(y-1) = 2x - 2y$$

Ex: Find the greatest rate of change at $(1,1)$.

$$\|\nabla f(1,1)\| = \|(2,-2)\| = 2\sqrt{2}.$$

Ex: Suppose $f(u,v) = \arctan\left(\frac{u}{v}\right)$ and

$$\begin{cases} u = 2x + 3y \\ v = 5x + 6y^2 \end{cases}$$

Calculate $\frac{\partial f}{\partial y}$ in terms of x and y.

Apply the chain rule.

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$= \frac{1}{1+\left(\frac{u}{v}\right)^2} \cdot \left(\frac{1}{v}\right) \cdot 3 + \frac{1}{1+\left(\frac{u}{v}\right)^2} \cdot \left(-\frac{1}{v^2}\right) \cdot (12y)$$

Ex: Find and classify the critical points of

$$z = y^4 - 2x^2 + x^3 - 12x$$