MAT B41 – Homework 1

Parker Glynn-Adey

July 20, 2018

These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Please write your solutions on lined paper. Parker encourages you to learn LaTeX and typeset your solutions. If you are curious about typesetting math, ask Parker. Clarity and precision are highly valued in this course.

1 Pre-Requisite Check

These questions are meant to remind you of material from the pre-requisite courses. We did not cover this material in lecture. Review it on your own.

Question 1.1. *Find the derivative of* $f(x) = x^2 + 3x$ *using the definition:*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

Question 1.2. *Find the indefinite integral:* $\int x^2 \sin(x) dx$.

Question 1.3. *Find the the value of the definite integral:* $\displaystyle\int_4^9 \cos(\pi\sqrt{x}) dx$.

Question 1.4. *Find the tangent line to* $x^2 + 4y^2 = 5$ *at* $(x, y) = (1, 1)$.

Question 1.5. *Find the volume of a sphere of radius* R *by rotating the curve* $f(x) = \sqrt{R^2 - x^2}$ *about the x-axis and integrating from* $x = -R$ *to* $x = R$.

2 New Material

Question 1.6. *Consider the sphere* $S^2 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$. *You may assume without proof that the tangent plane of* S^2 *at* (x, y, z) *has normal vector* (x, y, z)*. Write down the tangent planes at the following point in normal form:*

- $(x, y, z) = (1, 0, 0)$.
- $(x, y, z) = (-1, 0, 0)$.
- $(x, y, z) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Question 1.7. *Consider two vectors* $\vec{u}$ *and* $\vec{w}$*.*
If $\vec{u} \cdot \vec{v} = \vec{w} \cdot \vec{v}$ *for all vectors* $\vec{v}$ *is it true that* $\vec{u} = \vec{w}$*?*

Question 1.8. *Using techniques from calculus, find the point on the line* $\vec{x}(t) = (1, 2, 0) + t(1, 0, -1)$ *which is closest to the origin* $(0, 0, 0)$*.*

Question 1.9. *Find the two vectors of length one which are orthogonal to* $\vec{u} = (1, 0, 2)$ *and* $\vec{v} = (0, 3, -1)$*. Find the angle between* $\vec{v}$ *and* $\vec{u}$*.*

Question 1.10. *Express the standard basis vectors* $\vec{i}, \vec{j}, \vec{k}$ *as linear combinations of the vectors:* $\vec{a} = (1, 2, 3)$, $\vec{b} = (0, 4, 5)$, $\vec{c} = (0, 0, 6)$*. For example,* $\vec{j} = 0\vec{a} + 0\vec{b} + \frac{1}{6}\vec{c}$*. Express the vector* $\vec{i} - 2\vec{j} + 4\vec{k}$ *as a linear combination of* $\vec{a}$, $\vec{b}$*, and* $\vec{c}$*.*

Note: There was a minor typo in the homework. Orginally, the question asked for $\vec{i} - 2\vec{j} + 4\vec{j} = \vec{i} + 2\vec{j}$. This has been changed to: $\vec{i} - 2\vec{j} + 4\vec{k}$. Thanks, typo-spotters!

Question 1.11 (§1.1Q29)**.** *Describe the set of points* $\vec{x} \in \mathbb{R}^3$ *which form the parallelpiped having the vectors* $\vec{a}$,$\vec{b}$,$\vec{c}$ *as edges emanating from the origin. Your answer must be of the form:*

$$P = \left\{ \vec{x} \in \mathbb{R}^3 : \vec{x} = \text{ something involving } \vec{a},\ \vec{b},\ \vec{c} \right\}$$

*(*Hint*: Find a picture of a parallelpiped online.)*

Question 1.12 (§1.2Q15)**.** *What is the geometric relationship between* $\vec{u}$ *and* $\vec{v}$ *if* $\vec{u} \cdot \vec{v} = -||\vec{u}||||\vec{v}||$*?*

Question 1.13 (Bonus)**.** *Find an application of the dot product in computer science or programming. Read some Wikipedia pages related to computer graphics. Briefly describe what you found in less than one hundred and fifty words.*