MAT B41 – Homework 2

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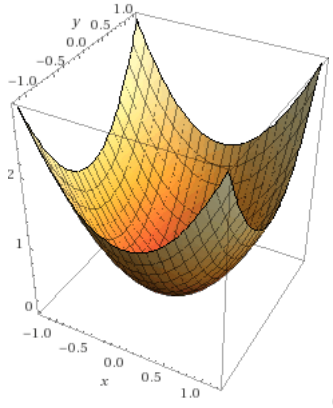
Due: Week of Monday May 28 – Friday June 1st.

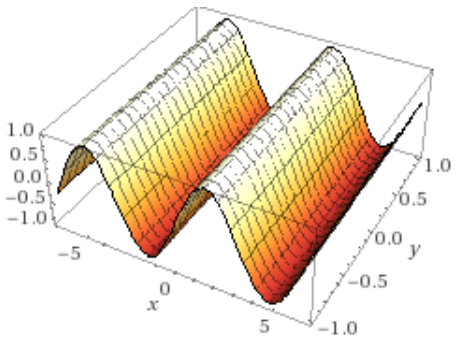
These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Question 2.1. *Consider the parallelogram with edges $\vec{a}$ and $\vec{b}$ emanating from the origin. Using vector methods, show that the diagonals of this parallelogram bisect each other. (Hint: Draw the picture in two dimensions.)*

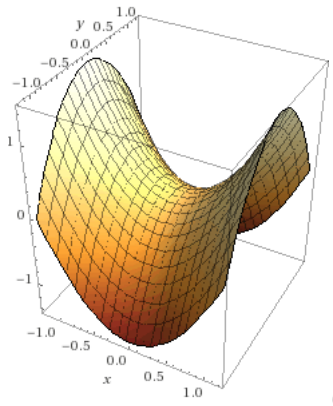
Question 2.2. *Match the following three dimensional graphs A,B,C with their two dimensional contour plots 1,2,3. Give a function $f(x, y)$ for each graph.*



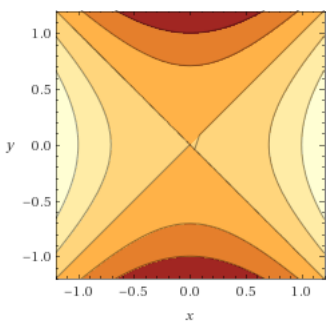
(A)



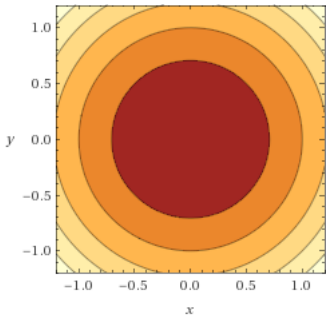
(B)



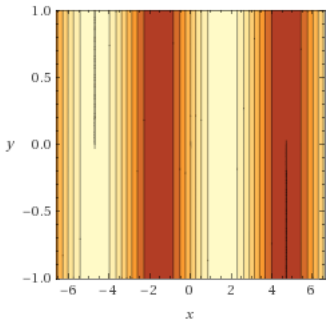
(C)



(1)



(2)



(3)

Question 2.3. *Consider the function* $f(x, y) = \max\{|x|, |y|\}$.

1. *Make a contour plot for* $f(x, y) = k$ *where* $k \in \{0, 1, 2, 3, 4, 5\}$ *and* $(x, y) \in [-5, 5] \times [-5, 5]$.
2. *Show that* $\lim_{(x,y)\to(0,0)} f(x, y) = 0$
3. *Show that* $f(x, y)$ *is continuous.*

Question 2.4. *Compute the following limits if they exist:*

1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{xy} - 1}{y}$
2. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2 + 2}$

Question 2.5. *Suppose that* A *is an* $n \times n$ *matrix and* $A \overset{R_i \leftrightarrow R_j}{\longrightarrow} B$ *is obtained from* A *by switching rows* R_i *and* R_j. *Show that* $\det(B) = -\det(A)$.

Question 2.6. *Show that there is no* 5×5 *matrix* A *such that* $A^2 = -I$.

Question 2.7 (§2.2 Q 33)**.** *Suppose* $f : \mathbb{R}^n \to \mathbb{R}^k$ *is a function such that:*

$$||f(\vec{x}) - f(\vec{y})|| \leq K||\vec{x} - \vec{y}||^\alpha$$

for some constants $K > 0$ *and* $\alpha > 0$. *Show that* f *is continuous.*

Question 2.8 (Bonus)**.** *Pick a mountainous place on Earth: Hawaii, the Matterhorn, Mount Everest, etc. Find a high quality topographic map of that place online. Print the topographic map and identify all mountain peaks on the map. What is the physical significance of a contour that forms a loop? What is the physical significance of two contours on the map getting close to each other? Answer these questions in less than a hundred and fifty words.*