MAT B41 – Homework 3

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Due: Week of Monday June 4 – Friday June 8.

These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Question 3.1. *Find specific values of δ_1 and δ_2 which make the following $\epsilon - \delta$ limit statements true:*

1. *If* $|x^2 + y^2| < \delta_1$ *then* $|x^3 + 7xy - 100y^3| < 1$
2. *If* $|x^2 + y^2| < \delta_2$ *then* $|x^3 + 7xy - 100y^3| < 1/100$

Question 3.2. *Where are the following functions non-differentiable?*

1. $f(x, y) = \max(x + 2y, x^2 + y^2)$
2. $g(x, y) = \dfrac{1}{1 - x^2 - y^2}$

Question 3.3 (§2.3Q16). *Consider the matrix* $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ *and the function* $f : \mathbb{R}^2 \to \mathbb{R}^2$ *given by* $f(\vec{x}) = A\vec{x}$.

1. *Write* $f(\vec{x}) = f(x, y) = (g_1(x, y), g_2(x, y))$ *where* $g_1 : \mathbb{R}^2 \to \mathbb{R}$ *and* $g_2 : \mathbb{R}^2 \to \mathbb{R}$ *are the component functions of* $f(\vec{x})$.
2. *Find the total derivative of* $f(\vec{x})$.
3. *Show that* $f(\vec{x})$ *is continuous at every point* $\vec{x} \in \mathbb{R}^2$.

Question 3.4. *Compute the matrix of partial derivatives for* $f(x, y) = (\sin(xy), xe^y)$.

Question 3.5 (§2.2Q5)**.** *Find the equation of the tangent plane to the surface* $z = x^2 + y^3$ *at the point* $(3, 1, 10)$. *Express your answer in the normal form:*

$$\pi = \{\vec{x} : (\vec{x} - \vec{p}) \cdot \vec{n}\}$$

Question 3.6 (§2.3Q19b)**.** *Calculate the gradient* ∇f *of* $f(x, y, z) = \frac{xyz}{x^2+y^2+z^2}$.

Question 3.7 (§2.5Q18)**.** *Apply the chain rule to find* $\partial h/\partial x$ *where:*

$$h(x, y) = f(u(x, y), v(x, y)), \quad u(x, y) = e^{-x-y}, \quad v(x, y) = e^{xy}$$

Question 3.8. *Imagine that you are going on a hike in Loopland. You are at camp and you will climb up some mountains. Parker gives you the following information about Looplandic geography: If you hike* x *kilometers east of camp, and* y *km north, then the mountain will be* $h(x, y)$ *km tall where:*

$$h(x, y) = \frac{1}{(x-2)^2 + (y-3)^2 + 1} + \frac{1}{(x-3)^2 + (y-1)^2 + 1}$$

1. *Determine the locations of the mountain peaks and name them.*
2. *Use software*[1] *to graph the mountains and make a contour plot.*
3. *On your contour map, draw a path from camp which always travels in the steepest direction. Which mountain top do you arrive at?*

Question 3.9 (Bonus)**.** *Suppose the mountains of Loopland at called A and B. Let* S_A *be the set of points in the plane such that if you start at a point in* S_A *and always follow the steepest direction, then you will arrive at the top of mountain A. Define* S_B *for mountain B. Find the sets* S_A *and* S_B *explicitly using a contour map and drawing. What is the physical significance of points in* $\mathbb{R}^2 \setminus (S_A \cup S_B)$*?*

[1] Free three-dimensional graphers are available: GeoGebra 3D or WolframAlpha