MAT B41 – Homework 4

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These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Question 4.1. *Consider the family of functions* $f_k(x, y) = x^2 + y^2 + kxy$. *For what values of* k *does the shape of the graph change? What happens to the type of critical points as you vary* k*? Illustrate your answer with contour plots.*

Question 4.2 (§3.3Q40)**.** *Consider the curve* $C \subset \mathbb{R}^3$ *defined by the intersection of the cylinder* $x^2 + y^2 = 1$ *and the equation* $x^2 - xy + y^2 - z^2 = 1$. *Find the point or points on* C *closest to the origin.*

Question 4.3 (§3.3Q42)**.** *Find the absolute maximum and minimum values for* $f(x, y) = xy$ *on the rectangle* R *defined by* $-1 \leq x \leq 1$ *and* $-1 \leq y \leq 1$.

Question 4.4. *Using computer software: provide a contour plot for* $f(x, y) = (x^2 + 3y^2)e^{1-x^2-y^2}$ *on the region* $-2 \leq x \leq 2$ *and* $-2 \leq y \leq 2$. *Identify the critical points and classify them visually using your graph. (For the full rigorous classification, see the bonus problem.)*

Question 4.5. *In class we showed: "Of rectangular prisms of volume* V*, the cube has the least surface area." Using similar techniques show the following statement: "Of all recantular prisms of surface area* A*, the cube has the largest volume."*

Question 4.6. *Suppose that a function $f : \mathbb{R}^4 \to \mathbb{R}$ has a critical point at $\vec{c}$ with Hessian:*

$$Hf(\vec{c}) = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$$

Classify the critical point at $x = \vec{c}$.

Question 4.7. *Make a contour plot for $z = f(x, y) = x^2 - y^2$ that includes the level curves $z = -2, -1, 0, 1, 2$. Draw the gradient vector $\nabla f(x, y)$ at various points along the level curves. On a seperate graph, repeat this exercise for $z = g(x, y) = x^2 + y^2$ and $\nabla g(x, y)$. What do you notice? How do the gradients differ?*

Question 4.8 (§3.3Q52)**.** *Consider a pentagon made by placing an isoceles triangle on top of a rectangle. (See p.185 for a diagram.) If the length of the perimeter is fixed at P, find the maximum possible area A.*

Question 4.9 (Bonus)**.** *Classify the critical points from Question 4.4 expicitly by computing the Hessian matrix and applying the second derivative test.*