MAT B41 – Homework 5

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These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Question 5.1 (§3.4Q13)**.** *Determine the maximum and minimum values of* $f(x, y) = x^2 + xy + y^2$ *on the unit disk* $x^2 + y^2 \leq 1$.

Question 5.2 (§3.4Q15)**.** *Find the extrema of the* $f(x, y) = 4x + 2y$ *subject to the constraint* $2x^2 + 3y^2 = 21$. *Illustrate your solution by graphing the constraint on a contour plot of* $f(x, y)$.

Question 5.3. *A body of revolution is obtained by rotating a graph* $y = f(x)$ *about the* x*-axis from* $x = a$ *to* $x = b$. *Use Cavalieri's Principle to find a formula for the volume of a body of revolution. Apply your formula to find the volume of:*

1. *The cylinder of base radius* r *and height* h.
2. *The cone of base radius* r *and height* h.
3. *The sphere of radius* r.

For full marks, you must define a function $f(x)$ *for each object and integrate. No marks will be given for formulae without justification.*

Question 5.4. *Consider a solid ball of radius* R. *Cut a cylindrical hole, through the center of the ball, such that the remaining body has height* h. *Call this the donut* $D(R, h)$. *Use Cavalieri's principle to calculate the volume of* $D(R, h)$. *Calculate the volumes of* $D(25, 6)$ *and* $D(50, 6)$.

Question 5.5 (§5.4Q13). *Let R be the triangular region in the plane with vertices $(0,2)$, $(2,0)$, and $(0,0)$. Evaluate the double integral $\iint_R xy dA$.*

Question 5.6. *Sketch the region R in the xy-plane where the integral is calculated. For example, in Question 5.5, the region is a triangle. For each integral, give a region R:*

1. $\displaystyle\int_0^1 \int_2^3 f(x,y) dx dy.$
2. $\displaystyle\int_{-1}^1 \int_{-2|x|}^{|x|} g(x,y) dy dx.$
3. $\displaystyle\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} h(x,y) dy dx.$

Question 5.7. *Let R be the unit disk $R = \{(x,y) : x^2 + y^2 \leq 1\}$. Calculate the volume of the paraboloid $V = \iint_R (1 - x^2 - y^2) dA$ over R.*

Question 5.8. *Calculate the area of the region R bounded by $y = 1$ and $y = x^2$ by integrating the constant function $f(x,y) = 1$ over R.*

Question 5.9 (Bonus). *The torus $T(R,r)$ is the body of revolution obtained by rotating the circle $x^2 + (y - R)^2 = r^2$ about the x-axis, assuming $R > r$. Use Cavalieri's Principle to calculuate the volume of $T(R,r)$. Which cylinder has the same volume as $T(R,r)$?*