# MAT B41 – Homework 6

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These homework exercises are due five minutes after the beginning of tutorial. You must submit them in your usual tutorial. Please write up your solution neatly and clearly. All work must be sumbitted individually.

Hooray! This is the last homework! It is a review of the whole course.

**Question 6.1.** *Consider the paraboloid $z = x^2 + y^2$. In this question, you will write a formula for all the tangent planes of the paraboloid. Express the tangent plane at $(x, y, z) = (a, b, a^2 + b^2)$ in normal form:*

$$\pi(a, b) = \{\vec{x} : \vec{n}(a, b) \cdot (\vec{x} - \vec{p}(a, b)) = 0\}$$

*Write out $\vec{n}(a, b)$ and $\vec{p}(a, b)$ as functions of $a$ and $b$. What do you get for $\pi(1, 1)$? Or $\pi(3, 4)$?*

**Question 6.2.** *What is the geometric relationship between $\vec{u}$ and $\vec{v}$ if $\vec{u} \cdot \vec{v} = \left(\frac{1}{\sqrt{2}}\right) ||\vec{u}||||\vec{v}||$?*

**Question 6.3** (2015 Term Test)**.** *Consider the function $f(x, y) = \dfrac{x^2}{x - y}$. Make a contour plot $f(x, y)$ using the level curves $f(x, y) = k$ where $k \in \{0, 1, 2, 3, 4, 5\}$ and $(x, y) \in [-5, 5] \times [-5, 5]$.*

**Question 6.4.** *We say that a function $f : \mathbb{R}^n \to \mathbb{R}^k$ is $n$-homogeneous if $f(\lambda \vec{x}) = \lambda^n f(\vec{x})$ for all $\vec{x}$ and $\lambda > 0$. Use a $\delta - \epsilon$ argument to show: If $f$ is $n$-homogeneous for $n > 0$ then $f(\vec{x})$ is continuous at $\vec{x} = \vec{0}$.*

**Question 6.5** (2016 Final)**.** *Find the third order Taylor approximation of*

$$f(x, y) = \frac{e^{-xy}}{1 - x^2}$$

**Question 6.6.** *Use the method of Lagrange multipliers to find the critical point of* $f(x, y, z) = \ln(x) + \ln(y) + \ln(z)$ *subject to* $x + y + z = 1$.

**Question 6.7.** *Calculate the integral* $\iiint_R z\, dV$ *over the tetrahedron with vertices:* $(0, 0, 1)$, $(0, 1, 1)$, $(1, 0, 0)$, *and* $(0, 0, 0)$.

**Question 6.8** (§5.5Q2)**.** *Calculate the integral* $\iiint_R \sin(x)\, dxdydz$ *over the region with* $0 \leq x \leq \pi$, $0 \leq y \leq 1$, *and* $0 \leq z \leq x$.

**Question 6.9** (§5.5Q4)**.** *Calculate the integral* $\iiint_R ye^{-xy}\, dxdydz$ *over the region* $R = [0, 1] \times [0, 1] \times [0, 1]$.

**Question 6.10.** *Sketch the region where the integral* $\int_0^1 \int_0^x \int_0^y f(x, y, z)\, dzdydx$ *is calculated. You may use a 3D graphing calculator to produce your sketch.*