NAME (PRINT): ____

Last / Surname First / Given Name

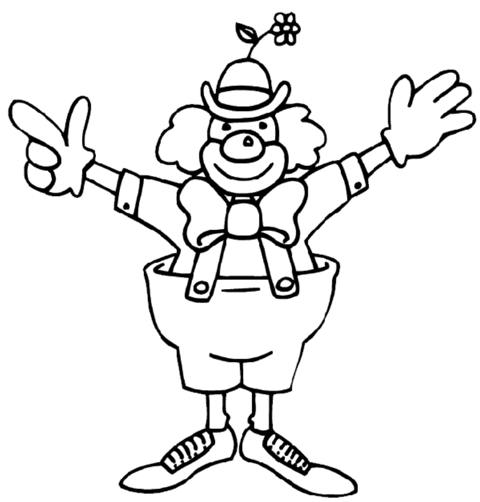
STUDENT #: ____________________

MAT B41
SUMMER 2018
MOCK FINAL EXAM - VERSION A

Problem	MC	Part II	III-1	III-2	III-3	III-4	III-5	Bonus	Total
Points	30	20	10	10	10	10	10	+5	100
Score									

INSTRUCTIONS:

- Please make sure your name, student number, and tutorial information are entered ***in ink*** at the top of this page. Select your tutorial section as well.
- Any test that does not contain proper TUT information will receive a *4 point deduction.*
- You have 100 minutes to complete this test. Do not begin until instructed to do so.
- You may use Page 18 for rough work. Your rough work will not be graded.
- Wave to Parker if you have read this instruction.
- This test contains 18 pages. Please ensure they are all there.
- No aids are allowed. No calculators, graphers, smart watches, or cellphones.
- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided. If you choose to write in pencil, you will not be eligible for a re-grade.



Answer Sheet for Multiple Choice Questions - Do Not Detach

Please put your answers to the multiple choice questions from Part I in the table below. Only this page will be looked at when grading, so be careful to transfer your answers correctly.

You may fill this form out in pencil or dark pen.

Student Name

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MAT B41 Mock Final (0701)

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Student Number

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9

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Part I: Multiple Choice. Each question is worth 2 points. No partial credit is given. There is only one correct answer for each question. Place all answers on the answer sheet on Page 2 of the test. Copy all answers to Page 2 before the end of the test.

(1) Which pairs of vectors for an angle of $\pi/4$?

$$\vec{a} = (1,0) \quad \vec{b} = (0,1) \quad \vec{c} = (-1,1)$$

A. All pairs of the vectors form an angle of $\pi/4$.
B. $\vec{a}$ and $\vec{c}$
C. $\vec{a}$ and $\vec{b}$
D. $\vec{b}$ and $\vec{c}$
E. None of the above. No pair of the listed vectors form an angle of $\pi/4$.

(2) Suppose a pair of vectors of length one satisfy $\vec{u} \cdot \vec{v} = \frac{\sqrt{3}}{2}$.
What is the angle between $\vec{u}$ and $\vec{v}$?

A. $\pi/4$
B. $\pi/2$
C. $\pi/3$
D. $\pi/6$
E. None of the above. The angle is not listed.

(3) Which plane contains the point $(1,0,-1)$?

A. $(1,0,-1) \cdot (\vec{x} - (3,3,0)) = 0$
B. $2x + 3y - 5z - 7 = 0$
C. $x - 2y - 6z - 8 = 0$
D. $(1,1,1) \cdot (\vec{x} - (2,3,4)) = 0$
E. None of the above. No plane listed contains the point $(1,0,-1)$.

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(4) What is the partial derivative $\dfrac{\partial f}{\partial x}$ where $f(x,y) = xe^{x^2+y^2}$?

A. $2x^2e^{x^2}$

B. $2xye^{x^2+y^2}$

C. $(2x^2+1)e^{x^2+y^2}$

D. $e^{x^2+y^2}$

E. None of the above. The partial derivative $\dfrac{\partial f}{\partial x}$ is not listed above.

(5) What is the total derivative of $f(x,y) = (e^{xy}, \sin(xy))$?

A. $\begin{bmatrix} ye^{xy} & xe^{xy} \\ y\sin(xy) & x\sin(xy) \end{bmatrix}$

B. $\begin{bmatrix} ye^{xy} & y\cos(xy) \\ xe^{xy} & x\cos(xy) \end{bmatrix}$

C. $\begin{bmatrix} ye^{xy} & y\sin(xy) \\ xe^{xy} & x\sin(xy) \end{bmatrix}$

D. $\begin{bmatrix} ye^{xy} & xe^{xy} \\ y\cos(xy) & x\cos(xy) \end{bmatrix}$

E. None of the above. The total derivative is not listed above.

(6) What is the tangent plane of $z = e^{2x+3y}$ at $(x,y,z) = (0,0,1)$?

A. $z = 2x + 3y + 1$

B. $z = e^2x + e^3y + 1$

C. $(2,3,1)\cdot((x,y,z) - (0,0,1)) = 0$

D. $z = -2x - 3y + 1$

E. None of the above. The tangent plane is not listed above.

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(7) Along which of the following paths does $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ NOT equal to zero?

A. $(x, y) = (0, t)$

B. $(x, y) = (t^2, t)$

C. $(x, y) = (t, t)$

D. $(x, y) = (t, 0)$

E. None of the above. The limit equals to zero along all listed paths.

(8) Which matrix is the Hessian of $f(x, y) = xe^{xy}$?

A. $\begin{bmatrix} (xy^2+2y)e^{xy} & (x^2y+2x)e^{xy} \\ (x^2y+2x)e^{xy} & x^3e^{xy} \end{bmatrix}$

B. $\begin{bmatrix} y^3e^{xy} & (xy^2+2y)e^{xy} \\ (xy^2+2y)e^{xy} & (x^2y+2x)e^{xy} \end{bmatrix}$

C. $\begin{bmatrix} y^4e^{xy} & (2y+2xy^3)e^{xy} \\ (2y+2xy^3)e^{xy} & (4x^2y^2+2x)e^{xy} \end{bmatrix}$

D. $\begin{bmatrix} (xy^3+2y^2)e^{xy} & (x^2y^2+3xy)e^{xy} \\ (x^2y^2+3xy)e^{xy} & (xy+2x^2)e^{xy} \end{bmatrix}$

E. None of the above. No matrix listed represents the Hessian of $f(x, y)$.

(9) Suppose a critical point $\vec{x} = \vec{c}$ has Hessian matrix

$$Hf(\vec{c}) = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

What does the second derivative test say about the critical point is $\vec{x} = \vec{c}$?

A. The second derivative test is inconclusive.

B. A maximum.

C. A minimum.

D. A saddle point.

E. None of the above.

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(10) What kind of local extrema does $f(x, y) = x^2 + y^2 + z^2 + xy$ have?

A. It has a minimum.

B. It has a maximum.

C. It has a saddle point.

D. The function does not have local extrema.

E. None of the above. None of the statements above it correct.

(11) What is the coefficient $c_{3,2}$ of x^3y^2 in the Taylor expansion of $e^{2x-3y} = \sum_{n,k} c_{n,k}x^n y^k$ at $\vec{x}_0 = (0, 0)$?

A. 3

B. 1/3

C. 1

D. 6

E. 0

F. None of the above. The coefficient $c_{3,2}$ is no listed above.

(12) Which of the following sets is NOT bounded?

A. $\{(x, y) : \max\{|x|, |y|\} \leq 10\}$

B. The triangle with vertices $(0, 0)$, $(1, 10)$, $(3, 5)$.

C. $\{(x, y) : x^2 \leq 1\}$

D. $\{(x, y) : |x| + |y| \leq 1\}$

E. None of the above. All the regions listed are bounded.

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(13) Which of the following optimization problems has a solution?

A. Maximize $f(x, y) = x^2 + y$ on $\{(x, y) : (x + y)^2 < 1\}$.

B. Maximize $f(x, y) = x$ on $\{(x, y) : |x| < 1 \text{ and } |y| \leq 1\}$.

C. Minimize $f(x, y) = 1 - x^2 + y^2$ on $\{(x, y) : x^2 + y^2 < 1\}$.

D. Minimize $f(x, y) = e^x - y$ on $\{(x, y) : x^2 \leq y \leq x\}$.

E. None of the above. No optimization problem listed has a solution.

(14) Which of the following vectors is perpendicular to the surface $x^2 + 3y^2 + 5z^2 = 9$ at $(1, 1, 1)$?

A. $(1, 3, 5)$

B. $(2, 3, 5)$

C. $(1, 1, 1)$

D. $(2, 6, 5)$

E. None of the above. No vector listed is perpendicular to the surface at $(1, 1, 1)$.

(15) What is Parker's favourite number? 😄

A. 1 – The multiplicative identity.

B. 42 – The answer to Life, the Universe, and Everything.

C. 24 – The number of distinct rotational symmteries of the cube.

D. 0 – The additive identity.

E. None of the above. Parker's favourite number is not listed.

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Part II: Short Answer. Please calculate the following quantities, and put your answers in the answer box provided. For the short answer question, only final answers will be graded. Each calculation is worth four points for a total of 5×4 points.

§2.6Q10a: Calculate the gradient of $f(x, y, z) = xy + yz + xz$.

Place your final answer here.

$\nabla f =$

§2Q25: In which direction(s) $\vec{v}$ does the directional derivative $\frac{\partial f}{\partial v}(1, 1) = 0$ where

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

Place your final answer here.

$\vec{v} =$

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§5.3Q3a: Integrate $V = \int_0^1 \int_1^{e^x} (x + y) dy dx$.

Place your final answer here.

$V =$

Continues on the next page →

§5.5Q16: Calculate the iterated integral: $W = \int_0^1 \int_0^x \int_0^y (y + xz) dz dy dx.$

Place your final answer here.

$W =$

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Homework 4.6: Suppose that a function $f : \mathbb{R}^4 \to \mathbb{R}$ has a critical point at $\vec{c}$ with Hessian:

$$Hf(\vec{c}) = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$$

Classify the critical point at $x = \vec{c}$.

Place your final answer here.

The critical point is a ______________________________

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Part III: Long Answer. Please answer the following questions. You should provide complete solutions, and show your work. Part marks are available, and points will be awarded to questions which are setup correctly. Attempt all problems to the best of your abilities. Correct final answers with little or no work will not receive any credit.

(1) **§3.4Q16** Use Lagrange multipliers to find the distance from the point $(2, 0, -1)$ to the plane $3x - 2y + 8z + 1 = 0$.

Place your final answer here:

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(2) Find and classify the critical point(s) of $f(x, y) = 4 + x^3 + y^3 - 3xy$.

Place your final answer here:

Continues on the next page →

(3) **Final 2016** Find the fourth order Taylor polynomial of $f(x, y) = \dfrac{e^{-xy}}{1 - x^2}$.

Place your final answer here:

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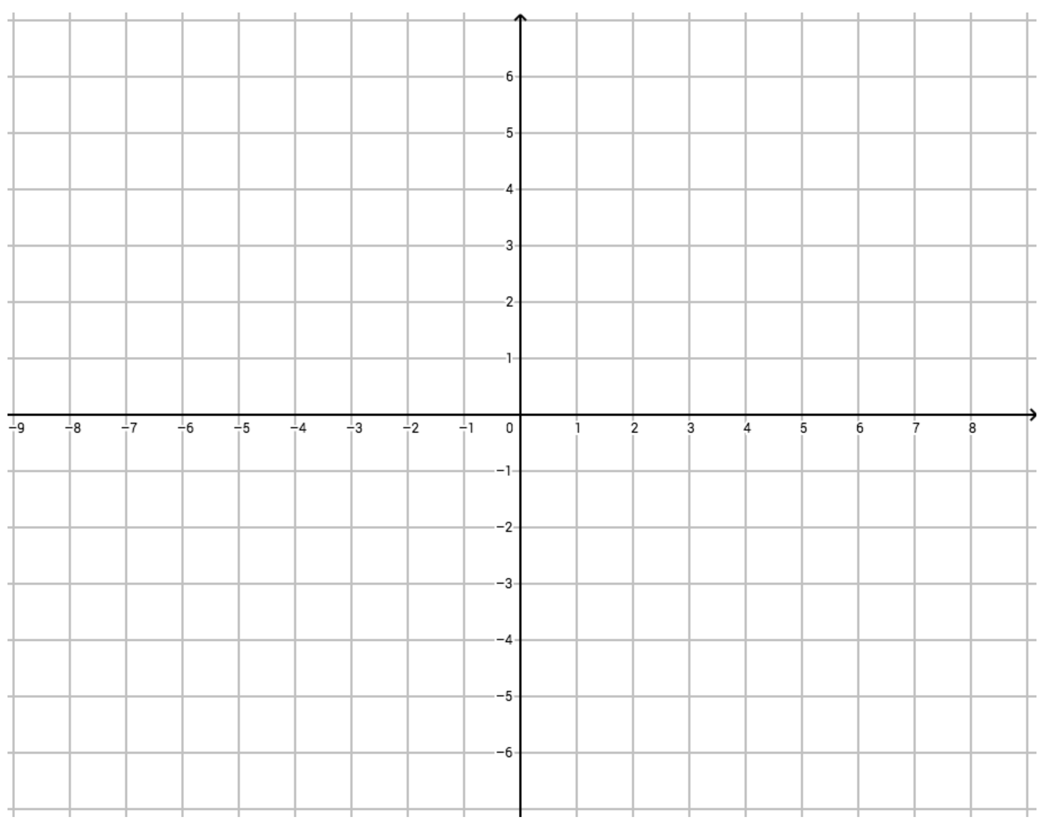
(4) Draw a contour plot of $f(x, y) = x^2 - 2x - y^2 + 2y + 2$.
Your contour plot must include at least five contours.



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(5) **§6.2Q10** Calculate $\iint_R \frac{1}{x+y} dydx$ where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, and $x + y = 4$, by changing coordinates using the map $T(u, v) = (u - uv, uv)$. [*Comment on Mock Final*: You might not yet have the tools to do this question, as it contains material from Week 12. You are still encouraged to try it out. Try to sketch the regions R and $T(R) = \{(u, v) : (u, v) = T(x, y) \text{ for } (x, y) \in R\}$.]

Place your final answer here:

Continues on the next page →

Bonus: Using pictures, words, and equations, describe how you could use multivariate calculus to design a castle. Points will be awarded for originality, and creativeness.

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Rough Work - This page will not be graded!

The End.