NAME (PRINT): _____

Last / Surname First / Given Name

STUDENT #: _____

MAT B41
SUMMER 2018
MOCK TERM TEST - VERSION A

Problem	MC	Part II	III-1	III-2	III-3	III-4	Bonus	Total
Points	40	12	12	12	12	12	+5	100
Score								

Tutorial Sections:

- ☐ Christopher Kennedy, TUT3001 (Wed 12:00 MW170)
- ☐ Kaide Ye, TUT3002 (Thur 12:00 SW143)
- ☐ Christopher Kennedy, TUT3003 (Tue 12:00 IC230)
- ☐ Xiucai Ding, TUT3004 (Tue 12:00 IC230)
- ☐ Xincheng Zhang, TUT3005 (Wed 16:00 AC 334)
- ☐ Xiucai Ding, TUT3006 (Fri 11:00 IC300)
- ☐ Kaide Ye, TUT3007 (Thur 15:00 IC302)
- ☐ David Pechersky, TUT3008 (Thursday 17:00 IC326)

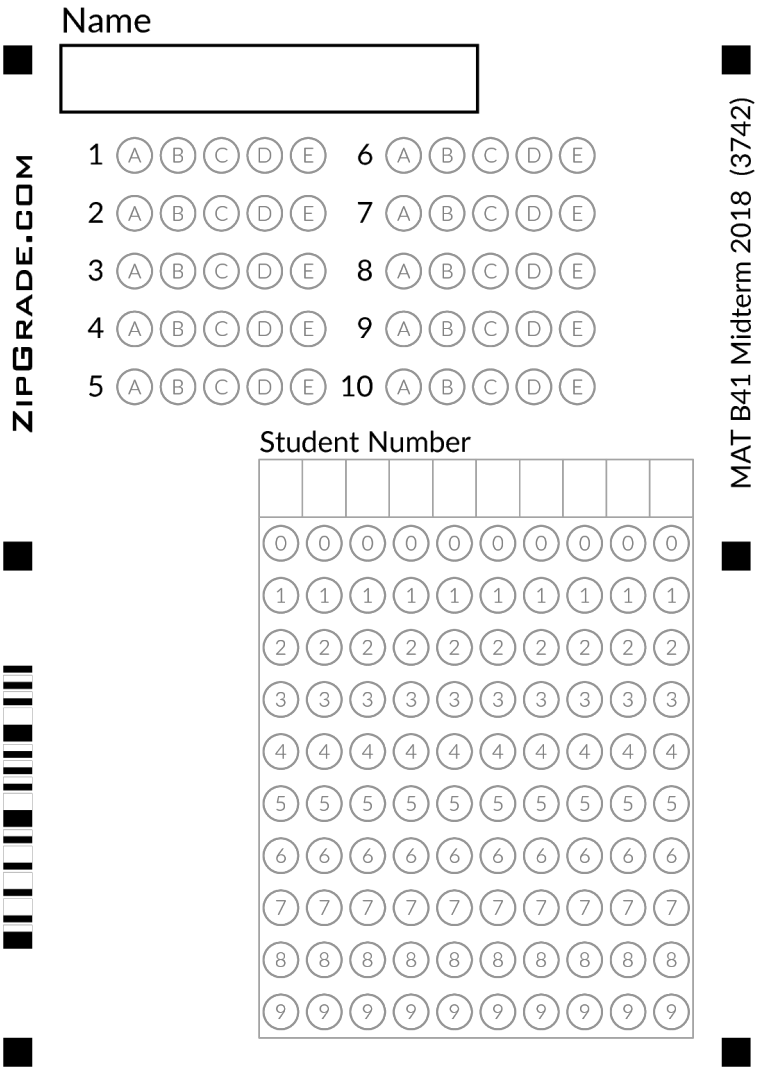
INSTRUCTIONS:

- Please make sure your name, student number, and tutorial information are entered ***in ink*** at the top of this page. Select your tutorial section as well.
- Any test that does not contain proper TUT information will receive a *4 point deduction.*
- You have 100 minutes to complete this test. Do not begin until instructed to do so.
- You may use Page 15 for rough work. Your rough work will not be graded.
- Clap your hands twice if you have read this instruction.
- This test contains 15 pages. Please ensure they are all there.
- No aids are allowed. No calculators, graphers, smart watches, or cellphones.
- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided. If you choose to write in pencil, you will not be eligible for a re-grade.

ANSWER SHEET FOR MULTIPLE CHOICE QUESTIONS - DO NOT DETACH

Please put your answers to the multiple choice questions from Part I in the table below. Only this page will be looked at when grading, so be careful to transfer your answers correctly.

You may fill this form out in pencil or dark pen.



Part I: Multiple Choice. Each question is worth 4 points. No partial credit is given. There is (hopefully!) only one correct answer for each question. Place all answers on the answer sheet on Page 2 of the test. Copy all answers to Page 2 before the end of the test.

(1) Which of the following points is NOT contained in the plane

$$\pi = \{\vec{x} : (1,2,3) \cdot (\vec{x} - (1,0,1)) = 0\}$$

A. $(3,-1,1)$

B. $(1,3,-1)$

C. $(4,0,0)$

D. $(1,1,-3)$

E. None of the above. All points listed are contained in the plane.

(2) Which vector is not orthogonal to $\vec{u} = (-5,2,1)$?

A. $\vec{v} = (-2,-10,10)$

B. $\vec{v} = (4,5,8)$

C. $\vec{v} = (3,7,1)$

D. $\vec{v} = (1,2,1)$

E. None of the above. All the vectors listed are orthogonal to $\vec{u}$.

(3) Which of the following is NOT a valid application of the chain rule.

Assume

$$z = f(u,v) \quad u = g(x,y,a) \quad v = h(s,y) \quad s = k(a,b)$$

A. $\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial v}\dfrac{\partial h}{\partial y}$

B. $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial u}\dfrac{\partial g}{\partial x}$

C. $\dfrac{\partial f}{\partial a} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial a} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial s}\dfrac{\partial s}{\partial a}$

D. $\dfrac{\partial f}{\partial b} = \dfrac{\partial f}{\partial v}\dfrac{\partial h}{\partial s}\dfrac{\partial k}{\partial b}$

E. None of the above. All applications listed above are valid.

(4) Along which of the following paths does $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^2}$ exist?

A. $(x,y) = (t,0)$

B. $(x,y) = (t,t^2)$

C. $(x,y) = (0,t)$

D. $(x,y) = (t,t)$

E. All of the above. The limit exists along all paths listed.

Continues on the next page →

(5) What is the coefficient $c_{1,2}$ of xy^2 in the Taylor expansion of $y\sin(xy) = \sum_{n,k} c_{n,k}x^n y^k$?

A. 1/2

B. 1/6

C. 0

D. 1

E. None of the above. The coefficient $c_{1,2}$ is not listed above.

(6) Which of the following functions is continuous at $(x, y) = (0, 0)$?

A. $\sqrt[3]{xy}$

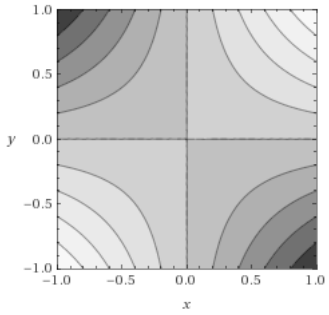
B. $\dfrac{x^2y^2}{x^4 + 3y^4}$

C. $\dfrac{1}{\sqrt[3]{xy}}$

D. $\dfrac{x^3y}{x^6 + y^2}$

E. None of the above. All the functions listed are discontinuous at $(0, 0)$.

(7) Which function $f(x, y)$ matches the following contour plot.



A. $f(x, y) = x^2 + y^2$

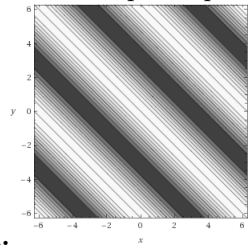
B. $f(x, y) = \min\{|x|, |y|\}$

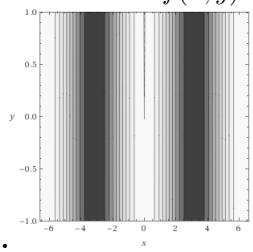
C. $f(x, y) = xy$

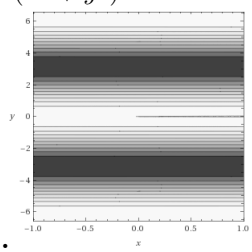
D. $f(x, y) = x^2 - y^2$

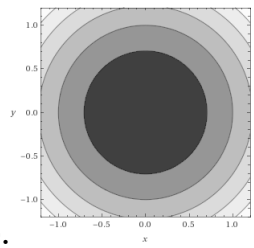
E. None of the above. No function listed matches the contour plot.

(8) Which contour plot represents the function $f(x, y) = \cos(x^2 + y^2)$?

A. 

B. 

C. 

D. 

E. None of the above. None of plots above represent $f(x, y)$.

(9) Which of the following is NOT in the total derivative of $f(x, y) = (xy^3, xy + 3xy^2)$?

A. $4xy$

B. y^2

C. $x + 6xy$

D. $3xy^2$

E. None of the above. All functions listed are in the total derivative of $f(x, y)$.

(10) Which of the following is differentiable at $(x, y) = (1, 1)$?

A. $\sqrt{xy - x - y + 1}$

B. $f(x, y) = \dfrac{1}{2 - x^2 - y^2}$

C. $f(x, y) = \max\{|x|, |y|\}$

D. $f(x, y) = |x| + |y|$

E. None of the above. All functions listed are not differentiable at $(x, y) = (1, 1)$.

Part II: Short Answer. Please calculate the following quantities, and put your answers in the answer box provided. For the short answer question, only final answers will be graded. Each calculation is worth 3 points for a total of $4 \times 3 = 12$ points.

§2.6Q10b: Find the gradient ∇f where $f(x, y, z) = xy + yz + xz$.

$\nabla f =$

§2.5Q17c: Find the partial derivative $\dfrac{\partial h}{\partial x}$ where $h(x, y, z) = f(u(x, y, z), v(x, y), w(x))$.

$\dfrac{\partial h}{\partial x} =$

§3.1Q1b: Find the third order Taylor polynomial of $f(x, y) = e^{x+y}$ at $(0, 0)$

$f(x, y) \approx$

§1.3Q16b: Find the equation of the plane that passes through: $(1, 2, 0), (0, 1, -2)$, and $(4, 0, 1)$.

$0 = \vec{n} \cdot (\vec{x} - \vec{p}) =$

Part III: Long Answer. Please answer the following questions. You should provide complete solutions, and show your work. Part marks are available, and points will be awarded to questions which are setup correctly. Attempt all problems to the best of your abilities. Correct final answers with little or no work will not receive any credit.

(1) Find the equation of the tangent plane to $f(x, y) = x^2 + 3xy - y^3$ at $(x, y) = (1, 2)$.

Place your final answer here:

(2) Provide a complete $\epsilon - \delta$ proof for the following limit calculation: $\lim_{(x,y)\to(1,2)} 2x+3y = 8$. Place your choice for δ in the box provided. Your choice of δ must involve ϵ.

Given $\epsilon > 0$ we choose: $\boxed{}$

Thus, we know that the following limit is true: $\lim_{(x,y)\to(1,2)} 2x + 3y = 8$.

(3) **§2.3Q33:** Let $f : \mathbb{R}^n \to \mathbb{R}^m$ satisfy $||f(\vec{x}) - f(\vec{y})|| \leq K||\vec{x} - \vec{y}||^\alpha$ for some values $K > 0$ and $\alpha > 0$. Show that f is continuous everywhere. You must use an $\epsilon - \delta$ argument to prove any limits that you use in this question.

(4) **Term Test 2015** Determine the rate of change of $f(x, y, z) = x^2 + y^2 - z$ as you move from the point $(1, 0, 1)$ towards $(2, 1, 2)$. What is the direction of fastest increase from $(1, 0, 1)$?

Place your final answer here:

(5) **Bonus** Suppose that a friend from highschool has been studying mathematics and computer science at the University of Waterloo. You meet this friend over reading week. They ask you what you have been studying at UTSC. They are very curious about MAT B41 and ask: "What have you learned in MAT B41? What is that course about?" Explain to your highschool friend what you have learned and what the course is about. Provide examples and discuss specific topics in the class.

Rough Work - This page will not be graded!