NAME (PRINT): _____

Last / Surname First / Given Name

STUDENT #: _____

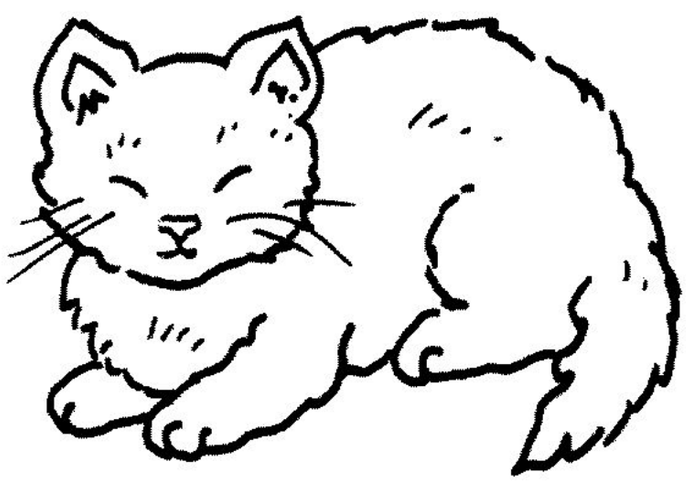
MAT B41

SUMMER 2018
FINAL EXAM

Problem	MC	Part II	III-1	III-2	III-3	III-4	III-5	Bonus	Total
Points	30	20	10	10	10	10	10	+1	100
Score									

INSTRUCTIONS:

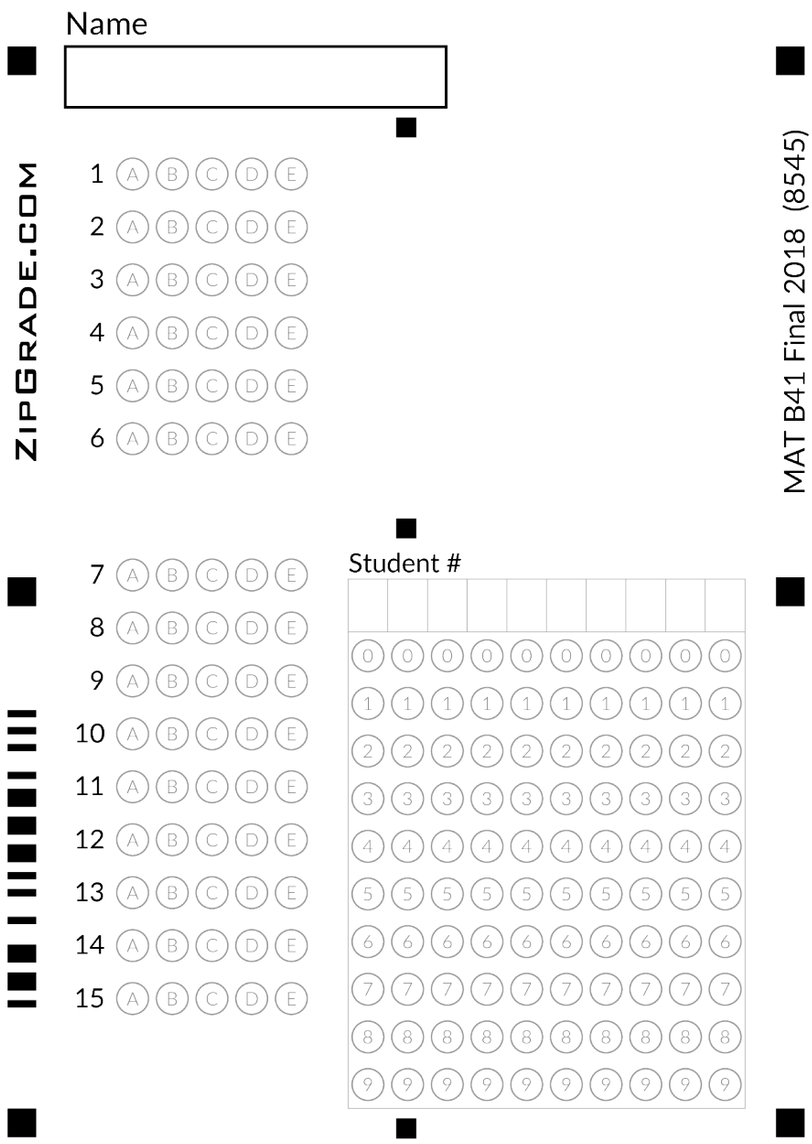
- Please make sure your name, student number, and tutorial information are entered ***in ink*** at the top of this page. Select your tutorial section as well.
- You have 180 minutes to complete this exam. Do not begin until instructed to do so.
- You may use Page 20 for rough work. Your rough work will not be graded.
- Wave to Parker if you have read this instruction.
- This test contains 20 pages. Please ensure they are all there.
- No aids are allowed. No calculators, graphers, smart watches, or cellphones.
- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided. If you choose to write in pencil, you will not be eligible for a re-grade.



Answer Sheet for Multiple Choice Questions - Do Not Detach

Please put your answers to the multiple choice questions from Part I in the table below. Only this page will be looked at when grading, so be careful to transfer your answers correctly.

You may fill this form out in pencil or dark pen.



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Part I: Multiple Choice. Each question is worth 2 points. No partial credit is given. There is only one correct answer for each question. Place all answers on the answer sheet on Page 2 of the test. Copy all answers to Page 2 before the end of the test.

(1) Which point is NOT on the sphere of radius one centered at $(2, 3, 1)$.

A. $(1, 2, 1)$

B. $(2, 2, 1)$

C. $(2, 3, 2)$

D. $(2, 3, 0)$

E. None of the above. All points listed are on the sphere.

(2) Suppose a pair of vectors of length one satisify $\vec{u} \cdot \vec{v} = \frac{1}{\sqrt{2}}$.
What is the angle between $\vec{u}$ and $\vec{v}$?

A. $\pi/6$

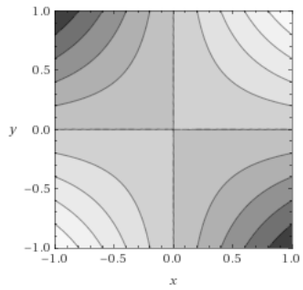
B. $\pi/2$

C. $\pi/4$

D. $\pi/3$

E. None of the above. The angle is not listed.

(3) Which function $f(x, y)$ matches the following contour plot.



A. $f(x, y) = x^2 - y^2$

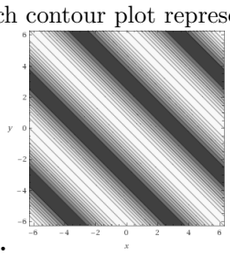
B. $f(x, y) = \min\{|x|, |y|\}$

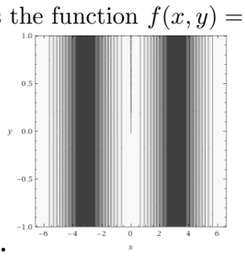
C. $f(x, y) = xy$

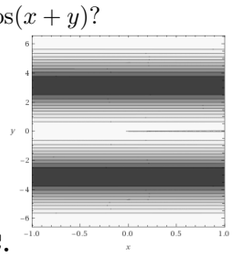
D. $f(x, y) = x^2 + y^2$

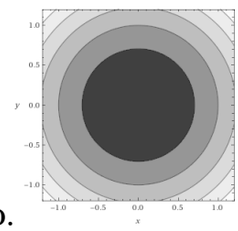
E. None of the above. No function listed matches the contour plot.

(4) Which contour plot represents the function $f(x, y) = \cos(x + y)$?

A. 

B. 

C. 

D. 

E. None of the above. None of plots above represent $f(x, y)$.

(5) What is the partial derivative $\frac{\partial f}{\partial x}$ where $\frac{y}{x+y}$?

A. $\frac{y}{(x+y)^2}$

B. None of the above. The partial derivative $\frac{\partial f}{\partial x}$ is not listed above.

C. $\frac{-y}{(x+y)^2}$

D. $\frac{1-x}{(x+y)^2}$

E. $\frac{x}{(x+y)^2}$

(6) What is the total derivative of $f(x,y) = (\sin(xy), \cos(x+y))$?

A. $\begin{bmatrix} -y\sin(xy) & -x\sin(xy) \\ \cos(x+y) & \cos(x+y) \end{bmatrix}$

B. $\begin{bmatrix} y\cos(xy) & x\cos(xy) \\ -\sin(x+y) & -\sin(x+y) \end{bmatrix}$

C. $\begin{bmatrix} y\sin(xy) & x\sin(xy) \\ \cos(x+y) & \cos(x+y) \end{bmatrix}$

D. $\begin{bmatrix} -y\cos(xy) & -x\cos(xy) \\ \sin(x+y) & \sin(x+y) \end{bmatrix}$

E. None of the above. The total derivative is not listed above.

(7) What is the tangent plane of $z = e^{2x+3y}$ at $(x,y,z) = (0,0,1)$?

A. $z = -2x - 3y + 1$

B. $(2,3,1) \cdot ((x,y,z) - (0,0,1)) = 0$

C. $z = e^2 x + e^3 y + 1$

D. $z = 2x + 3y + 1$

E. None of the above. The tangent plane is not listed above.

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(8) Along which of the following paths does $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ NOT equal to zero?

A. $(x, y) = (t, t)$

B. $(x, y) = (t, 0)$

C. $(x, y) = (t^2, t)$

D. $(x, y) = (0, t)$

E. None of the above. The limit equals to zero along all listed paths.

(9) Which matrix is the Hessian of $f(x, y) = e^{xy}$?

A. $\begin{bmatrix} xye^{xy} & (xy+1)e^{xy} \\ (xy+1)e^{xy} & x^2e^{xy} \end{bmatrix}$

B. $\begin{bmatrix} y^2e^{xy} & xye^{xy} \\ xye^{xy} & x^2e^{xy} \end{bmatrix}$

C. $\begin{bmatrix} e^{xy} & (xy+1)e^{xy} \\ (xy+1)e^{xy} & e^{xy} \end{bmatrix}$

D. $\begin{bmatrix} y^2e^{xy} & (xy+1)e^{xy} \\ (xy+1)e^{xy} & x^2e^{xy} \end{bmatrix}$

E. None of the above. No matrix listed represents the Hessian of $f(x, y)$.

(10) Suppose a critical point $\vec{x} = \vec{c}$ has Hessian matrix

$$Hf(\vec{c}) = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

What does the second derivative test say about the critical point is $\vec{x} = \vec{c}$?

A. A saddle point.

B. A maximum.

C. A minimum.

D. The second derivative test is inconclusive.

E. None of the above.

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(11) What is the conclusion of the second derivative applied to $f(x, y) = x^2 + 2xy + y^2$?

A. It has a minimum.

B. It has a maximum.

C. It has a saddle point.

D. The function does not have local extrema.

E. None of the above. None of the statements above it correct.

(12) What is the coefficient $c_{2,1}$ of x^2y^1 in the Taylor expansion of $y\sin(x^2) = \sum_{n,k} c_{n,k}x^n y^k$ at $\vec{x}_0 = (0, 0)$?

A. 0

B. 3

C. 1

D. 1/2

E. None of the above. The coefficient $c_{2,1}$ is no listed above.

(13) Which of the following sets is NOT bounded?

A. $\{(x, y) : 0 \leq x \leq y \leq 1\}$.

B. $\{(x, y) : 0 \leq xy \leq 1\}$

C. $\{(x, y) : 0 \leq |x| + |y| \leq 1\}$

D. $\{(x, y) : 0 \leq x^2 + y^2 \leq 1\}$

E. None of the above. All the regions are bounded.

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(14) Which of the following optimization problems has a solution?

A. Minimize $f(x,y) = \sin(x)e^y$ on $\{(x,y) : x^2 + y^2 \leq 1\}$.

B. Maximize $f(x,y) = x$ on $\{(x,y) : |y| \leq 1\}$.

C. Minimimze $f(x,y) = 1 - x$ on $\{(x,y) : |x| < 1\}$.

D. Maximize $f(x,y) = xe^y$ on $\{(x,y) : y - x < 1\}$.

E. None of the above. No optimization problem listed has a solution.

(15) Which of the following vectors is perpendicular to the surface $2xy + 3yz = 5$ at $(1,1,1)$?

A. $(5,3,2)$

B. $(2,5,3)$

C. $(2,3,5)$

D. $(5,2,3)$

E. None of the above. No vector listed is perpendicular to the surface at $(1,1,1)$.

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Part II: Short Answer. Please calculate the following quantities, and put your answers in the answer box provided. For the short answer question, only final answers will be graded. Please copy every final answer over to Page 13. Each calculation is worth four points for a total of 5×4 points.

§2Q10: Find the tangent plant to $f(x, y) = x^2 e^{-xy}$ at $(x, y) = (1, 2)$.
Express your final answer in the form $\{\vec{x} : \vec{n} \cdot (\vec{x} - \vec{p}) = 0\}$.

Place your final answer on Page 13.

§2Q5: Let $f(u, v) = (\cos(u), v + \sin(u))$ and $g(x, y, z) = (x^2 + \pi y^2, xz)$.
Compute $D(f \circ g)$ at $(0, 1, 1)$ using the chain rule.

Place your final answer on Page 13.

§5.3Q16: Integrate $V = \int_0^1 \int_1^2 \int_2^3 \cos\left[\pi(x+y+z)\right] dxdydz$

Place your final answer on Page 13.

§5.4Q1b: Change the order of integration of $\int_0^9 \int_0^{\sqrt{y}} f(x,y)dxdy$.

Place your final answer on Page 13.

§6.2Q11: Find the mass M of a solid cube of side length 5 with density given by

$$\delta(x, y, z) = 2x^2 + 2y^2 + 2z^2 + 1$$

assuming the cube is centered at the origin.

Place your final answer on Page 13.

ANSWER SHEET FOR SHORT ANSWER QUESTIONS - DO NOT DETACH

Please put your answers to the short answer questions from Part II in the table below. Only this page will be looked at when grading, so be careful to transfer your answers correctly.

$\{\vec{x} : \vec{n} \cdot (\vec{x} - \vec{p}) = 0\} =$
$D(f \circ g) =$
$\int_0^1 \int_1^2 \int_2^3 \cos[\pi(x+y+z)]\, dxdydz =$
$\int_0^9 \int_0^{\sqrt{y}} f(x,y) dxdy =$
$M =$

Part III: Long Answer. Please answer the following questions. You should provide complete solutions, and show your work. Part marks are available, and points will be awarded to questions which are setup correctly. Attempt all problems to the best of your abilities. Correct final answers with little or no work will not receive any credit.

(1) **§3.4Q13** Consider the function $f(x, y) = x^2 + xy + y^2$ defined on the region

$$D = \{(x, y) : x^2 + y^2 \leq 1\}$$

(a) Use the second derivative test to classify the critical points of $f(x, y)$ on $\mathbb{R}^2$.

(b) Use Lagrange multipliers to find the max and min of $f(x, y)$ on $x^2 + y^2 = 1$.

(c) Using parts (a) and (b): determine the absolute max and min of $f(x, y)$ on D.

Place your final answer to part (c) here:

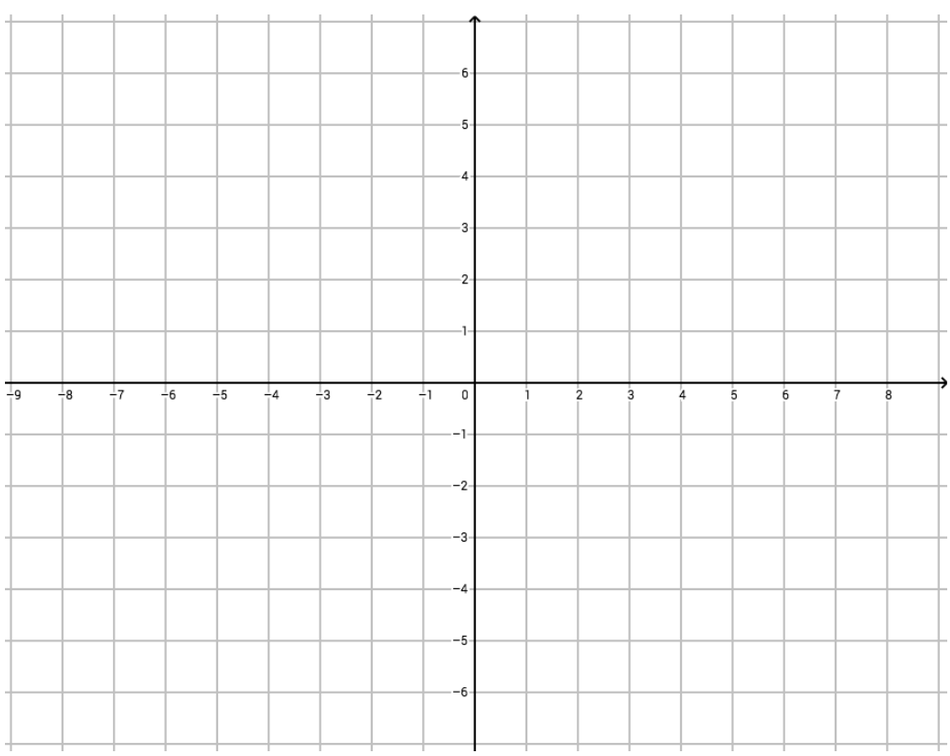
(2) **§3.3Q18** Let $f(x, y, z) = x^2 + y^2 + z^2 + kyz$. Check that $(x, y, z) = (0, 0, 0)$ is a critical point for all values of k. Find all values of k such that this critical point is a minimum.

Place your final answer, in terms of k, here:

(3) **§3.2Q7** Find the second order Taylor polynomial of $f(x, y) = \cos(x)\sin(y)$ at the point $(x, y) = (0, 0)$.

Place your final answer here:

(4) Draw a contour plot of $f(x, y) = x^2 + 4y^2$. Sketch the contours: $f(x, y) = 0, 4, 16, 64$. Label a point (x, y) on each contour.



(5) **§6.2Q4** Let D be the region bounded by $0 \leq y \leq x$ and $0 \leq x \leq 1$. Evaluate

$$\iint_D (x+y)dxdy$$

by making the change of variable $(x, y) = (u + v, u - v)$.

NO marks will be given to solutions which do not use change of variables.

Place your final answer here:

Bonus: Describe your experience with this course. How does it compare to other math courses? What would you do to improve it? Consider topics like: lecture format, office hours, homework, term test, tutorial, and the website. Is there anything that you want Parker to know about your experience of MAT B41? Be honest, Parker looks forward to reading your thoughts about the course.

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Rough Work - This page will not be graded!

The End. Thanks for the great course!