NAME (PRINT): ____

Last / Surname   First / Given Name

STUDENT #: ____________________

MAT B41
SUMMER 2018
TERM TEST - VERSION A

Problem	MC	Part II	III-1	III-2	III-3	III-4	Bonus	Total
Points	40	12	12	12	12	12	+5	100
Score								

Tutorial Sections:

- ☐ Christopher Kennedy, TUT3001 (Wed 12:00 MW170)
- ☐ Kaide Ye, TUT3002 (Thur 12:00 SW143)
- ☐ Christopher Kennedy, TUT3003 (Tue 12:00 IC230)
- ☐ Xiucai Ding, TUT3004 (Tue 12:00 IC230)
- ☐ Xincheng Zhang, TUT3005 (Wed 16:00 AC 334)
- ☐ Xiucai Ding, TUT3006 (Fri 11:00 IC300)
- ☐ Kaide Ye, TUT3007 (Thur 15:00 IC302)
- ☐ David Pechersky, TUT3008 (Thursday 17:00 IC326)

INSTRUCTIONS:

- Please make sure your name, student number, and tutorial information are entered ***in ink*** at the top of this page. Select your tutorial section as well.
- Any test that does not contain proper TUT information will receive a *4 point deduction*.
- You have 100 minutes to complete this test. Do not begin until instructed to do so.
- You may use Page 16 for rough work. Your rough work will not be graded.
- Say "woo-hoo" loudly if you have read this instruction.
- This test contains 16 pages. Please ensure they are all there.
- No aids are allowed. No calculators, graphers, smart watches, or cellphones.
- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided. If you choose to write in pencil, you will not be eligible for a re-grade.

Answer Sheet for Multiple Choice Questions - Do Not Detach

Please put your answers to the multiple choice questions from Part I in the table below. Only this page will be looked at when grading, so be careful to transfer your answers correctly.

You may fill this form out in pencil or dark pen.

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Name

1	A	B	C	D	E	6	A	B	C	D	E
2	A	B	C	D	E	7	A	B	C	D	E
3	A	B	C	D	E	8	A	B	C	D	E
4	A	B	C	D	E	9	A	B	C	D	E
5	A	B	C	D	E	10	A	B	C	D	E

Student Number

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9

MAT B41 Midterm 2018 (3742)

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Test Re-Mark Request Form

This form lets you request that your test be re-marked by Parker.
Don't fill it out until after the test.

If you want your test to be re-marked you must:

(1) Complete your test in pen.
(2) Attend the first tutorial after the midterm.
(3) Return your paper to your TA during tutorial.

All re-read requests must include:

(1) A specific question to be re-marked.
(2) Reasons why that question should be re-marked.
(3) Any comments about the question.

For example:
"Please re-mark Question 3 of Part III because the TA did not see the work on the back of Page 13 and that work did not get graded."

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Part I: Multiple Choice. Each question is worth 4 points. No partial credit is given. There is only one correct answer for each question. Place all answers on the answer sheet on Page 2 of the test.

(1) Which of the following pairs of vectors are perpendicular?

A. $(5, 3, 1)$ and $(1, 0, 6)$

B. $(6, 2, -3)$ and $(7, 3, 16)$

C. $(-2, 1, 3)$ and $(7, 10, 2)$

D. $(2, 3, 5)$ and $(4, -2, 1)$

E. None of the above. All pairs are not perpendicular.

(2) Which of the following matrices is NOT invertible?

A. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

E. None of the above. All the matrices listed are invertible.

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(3) Which of the following vectors is normal to the tangent plane of $z = x^2 + y^2$ at $(x, y, z) = (1, 2, 5)$?

A. $(1, 2, 0)$

B. $(1, 2, 5)$

C. $\left(1, 2, -\frac{1}{2}\right)$

D. $(1, 2, 1)$

E. None of the above. No vector listed is normal to that tangent plane.

(4) Which of the following functions has gradient $\nabla f(1, 2, 3) = (2, 4, 6)$?

A. $f(x, y, z) = x^2 + 2yz$

B. $f(x, y, z) = xyz$

C. $f(x, y, z) = x + y^2 + z^3$

D. $f(x, y, z) = 2x + 2y + 2z$

E. None of the above. No function listed has that gradient at that point.

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(5) Here is a new definition for you. Mathematicians say that a function is k-homogeneous if $f(\lambda\vec{x}) = \lambda^k f(\vec{x})$ for all λ and $\vec{x}$. Which of the following is 2-homogeneous?

A. $f(x,y) = x^3 + 5y^3$

B. $f(x,y) = \dfrac{x - 2y^5}{x^3 + 1}$

C. $f(x,y) = \dfrac{xy}{x^4 + y^4}$

D. $f(x,y) = \dfrac{x^5 + 2y^5}{xy^2}$

E. None of the above. No function listed is 2-homogeneous.

(6) Suppose that the following functions are differentiable everywhere:

$$z = f(a,b,c) \quad a = g(x,y) \quad b = h(x) \quad c = k(y)$$

Which of the following is a VALID application of the chain rule?

A. $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial a}\dfrac{\partial g}{\partial x}$

B. $\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial b}\dfrac{\partial b}{\partial y}$

C. $\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial c}\dfrac{\partial k}{\partial y} + \dfrac{\partial f}{\partial a}\dfrac{\partial b}{\partial y}$

D. $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial a}\dfrac{\partial g}{\partial x} + \dfrac{\partial f}{\partial b}\dfrac{\partial h}{\partial x}$

E. None of the above. No equation listed above is valid.

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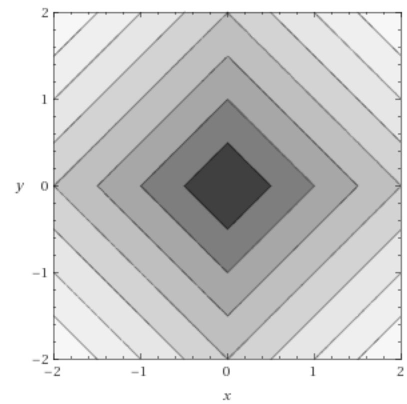
(7) Which of the following functions matches this contour plot?



A. $f(x, y) = \max\{|x|, |y|\}$

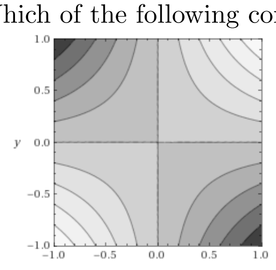
B. $f(x, y) = |x| + |y|$

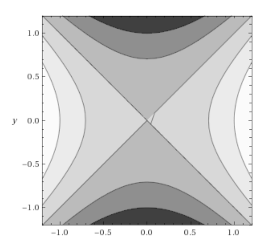
C. $f(x, y) = |x||y|$

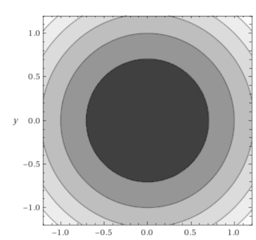
D. $f(x, y) = x^2 + y^2$

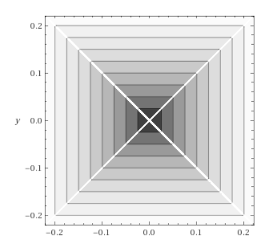
E. None of the above. No function listed matches that contour plot.

(8) Which of the following contour plots represents the function $f(x, y) = x^2 - y^2$?


A.


B.


C.


D.

E. None of the above. No plot listed represents the function.

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(9) What is the coefficient $c_{2,1}$ of x^2y in the Taylor expansion of $e^{2x-y} = \sum_{n,k} c_{n,k}x^n y^k$ at $\vec{x}_0 = (0,0)$?

A. 2

B. 1/2

C. 0

D. $-1/6$

E. None of the above. The coefficient $c_{2,1}$ is not listed above.

(10) We say that a limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists along a path $c(t)$ if the limit $\lim_{t\to 0} f(c(t))$ exists. Consider the function $f(x,y) = \dfrac{xy}{x^4+y^2}$. Along which path $c(t)$ does the limit exist?

A. $c(t) = (2t, 3t)$

B. $c(t) = (t, t^2)$

C. $c(t) = (t^{1/2}, t^{3/2})$

D. $c(t) = (\sqrt{t}, t)$

E. None of the above. The limit does not exist along any listed path.

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Part II: Short Answer. Please calculate the following quantities, and put your answers in the answer box provided. For the short answer question, only final answers will be graded. Each calculation is worth 3 points for a total of $4 \times 3 = 12$ points.

(1) **§2.5Q10c** Find the gradient ∇f where $f(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$.

$\nabla f =$

(2) **§2.5Q36** Let $w = x^2 + y^2 + z^2$ where

$$x = uv \qquad y = u\cos(v) \qquad z = u\sin(v)$$

Find $\dfrac{\partial w}{\partial u}$ at $(u, v) = (1, 0)$.

$\dfrac{\partial w}{\partial u}(1, 0) =$

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(3) **§3.1** Find the second order Taylor polynomial of $f(x, y) = \sin(x - y)$ at $\vec{x}_0 = (0, 0)$

$f(x, y) \approx$

(4) **§1.3Q30** Find an equation for the line that passes through the point $(1, -2, -3)$ and is perpendicular to the plane $3x - y - 2z + 4 = 0$.

$\vec{x}(t) = \vec{p} + t\vec{d} =$

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Part III: Long Answer. Please answer the following questions. You should provide complete solutions, and show your work. Part marks are available, and points will be awarded to questions which are set up correctly. Attempt all problems to the best of your abilities. Correct final answers with little or no work will not receive any credit.

(1) **§2.6Q6:** Find a vector which is perpendicular to the curve $x^3 + xy + y^3 = 11$ at $(x, y) = (1, 2)$.

Place your final answer here:

Continues on the next page →

(2) **§3.1Q12b** Show that $T(x, y, t) = e^{-kt}(\cos(x) + \cos(y))$ satisfies the equation:

$$k(T_{xx} + T_{yy}) = T_t$$

Continues on the next page →

(3) **§2.3Q35b:** Find a specific number $\delta > 0$ such that: if $x^2 + y^2 < \delta^2$ then

$$|x^2 + y^2 + 3xy + 10xy^5| < 10^{-4} = \frac{1}{10\ 000}$$

Warning: Only solutions that include explanations will receive marks.

Show that $\lim_{(x,y)\to(0,0)} x^2 + y^2 + 3xy + 10xy^5 = 0$ using an $\epsilon - \delta$ argument.

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(4) **Term Test 2016** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = \dfrac{x + y}{x^2}$.
Find the equation of the tangent plane to the graph $z = f(x, y)$ at the point $(2, 3, f(2, 3))$.

Place your final answer here:

Find the direction of maximum increase at the point $(2, 3)$.
What it is the rate of maximum increase?

Place your final answer here:

Continues on the next page →

(5) **Bonus:** One of your friends from highschool is getting really interested in video games. They want to become a video game programmer and make lots of games. What are some concepts from this course that you think would be helpful to your friend? What ideas from MAT B41 would be useful for making a video game? Be specific and use concrete examples.

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Rough Work - This page will not be graded!

The End.