

his week
 • Differentiation
 • Chain Rule

Last Week
 • $\frac{\partial f}{\partial x_i} = \lim_{h \rightarrow 0} \frac{f(x_i + h e_i) - f(x)}{h}$

• measures rate of change in the direction e_i @ x
 • called partial derivatives of $f(x)$

Ex) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = \sqrt[3]{xy}$ @ $(x,y) = (0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \rightarrow 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt[3]{(0+h) \cdot 0} - \sqrt[3]{0 \cdot 0}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0}{h}$$

$$= 0$$

$\frac{\partial f}{\partial y}(0,0) = 0$ by similar logic

Question 1) what happens along the line $y=x$?

$$\lim_{h \rightarrow 0} \frac{f(0+h, 0+h) - f(0,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt[3]{(0+h)(0+h)} - \sqrt[3]{0 \cdot 0}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt[3]{h^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h^{1/3}} \leftarrow \text{limit does not exist}$$

thus we have a "cusp" or a "fold"

Def: The DIRECTIONAL DERIVATIVE of $f(x)$ in the direction \vec{v} is:

$$\frac{\partial f}{\partial \vec{v}} = \lim_{h \rightarrow 0} \frac{f(x + h\vec{v}) - f(x)}{h}$$

• measures rate of change in the direction \vec{v}

Ex) Find the directional derivative $\frac{\partial f}{\partial \vec{v}}(1,2)$ for $f(x,y) = x^2 + y^2$ and $\vec{v} = (1, -1)$

$$\frac{\partial f}{\partial \vec{v}} = \lim_{h \rightarrow 0} \frac{f((1,2) + h(1,-1)) - f(1,2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(1+h, 2-h) - f(1,2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1+h)^2 + (2-h)^2 - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 + 4 - 4h + h^2 - 5}{h}$$

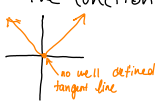
$$= \lim_{h \rightarrow 0} \frac{2 + h - 4 + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 + 2h}{h}$$

$$= -2$$

Differentiability

• The function $y = |x|$ does not have a derivative @ $x=0$



- in several dimension we can fail to have a derivative for separate reasons
 - ↳ imagine a cone @ $x=(0,0)$, that point has no tangent plane
 - ↳ singularities can also have no tangent plane
 - ↳ there can also be a fold thus

