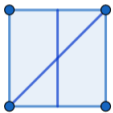
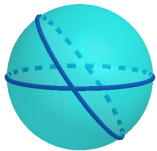
MAT 402: Classical Geometry

Groups

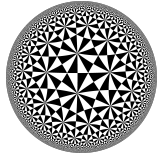


$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$

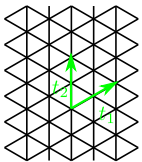
Spherical



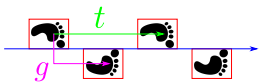
Hyperbolic



Tilings



Friezes

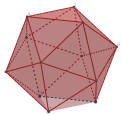


Trigonometry

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

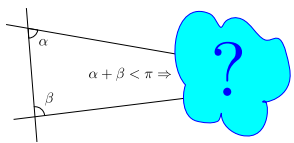
$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

Platonic Solids



Coxeter

Parallels



Questions? Observations about model building? Thoughts?

MAT 402: Monday September 14th 2020

Learning Objectives:

- Calculate products of elements in the dihedral group.
- Find the order of an element in a group.
- Design a colouring with a particular symmetry pattern.
- List the elements of a symmetry group.

Computing in Groups

Task

In $G = \langle r : r^3 = e \rangle$ *compute* r^5.

Computing in Groups

Task

In $G = \langle r : r^3 = e \rangle$ *compute* r^5.

Task (Another Finite Group (5 min))

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

- *What is* a^{-1}*?*
- *Calculate* $abbaaca$.

The Dihedral Group

Definition

The dihedral group D_n is the symmetries of a two-sided regular n-gon.

Task (5 min)

List all symmetries of a two-sided regular 4-gon (square).

Rotational Symmetries

Task (3 min)

Find a two-dimensional shape with only three rotational symmetries.

Rotational Symmetries

Task (3 min)

Find a two-dimensional shape with only three rotational symmetries.

Question

How would you generalize this to n rotational symmetries? (We call this $\mathbb{Z}_n$ symmetry.)

Order

Definition

The order of an element $g \in G$ is the smallest integer $k > 0$ such that $g^k = e$.

Task (5 min)

Find the order of the elements s, r, and r^2 in $D_4 = \mathrm{Symm}(\square)$.

Calculating in Groups

Task

In $D_4 = \mathrm{Symm}(\square)$ *check that* $rs = sr^{-1}$.

Colourings and Symmetry

Task (5 min)

Find a shape and a colouring with only D_3 symmetry
Find an alternative colouring of the same shape with $\mathbb{Z}_n$ symmetry.