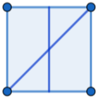
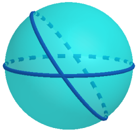
MAT 402: Classical Geometry

Groups

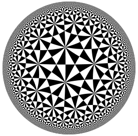


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

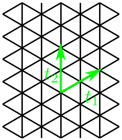
Spherical



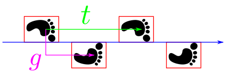
Hyperbolic



Tilings



Friezes

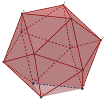


Trigonometry

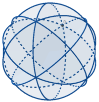
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

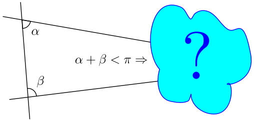
Platonic Solids

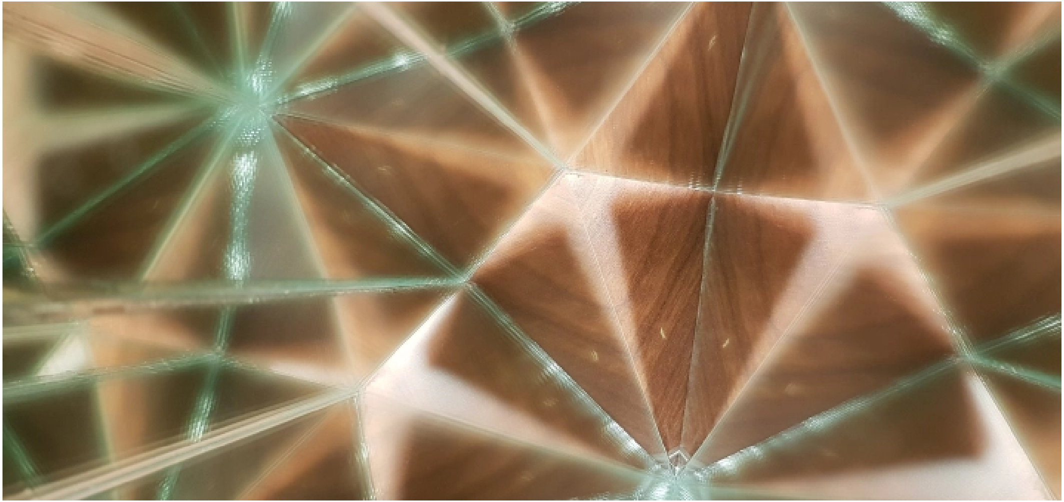


Coxeter



Parallels





A Kaleidoscope by P. Glynn-Adey

Are you able to access Model #4? How are your midterms?

MAT 402: Friday October 30th 2020

Learning Objectives:

- Classify the planar Coxeter geometries.
- Realize a space as a gluing along a map.

Coxeter Geometries

Definition (p. 101)

A planar Coxeter geometry is a geometry generated reflections in the sides of a planar polygon F such that all the angles of F are of the form π/k for $k \geq 2$.

Geometric Lemma

Task

If a polygon has n sides, what is the total of its internal angles?
What is the average angle?

Geometric Lemma

Task

Given that the average angle of an n-gon is $\pi\left(1-\frac{2}{n}\right)$
and the angle at each vertex is of the form $\pi/2$ *for* $k \geq 2$.
What can we conclude about the number of sides?

The Quadrangular Coxeter Geometries

Task

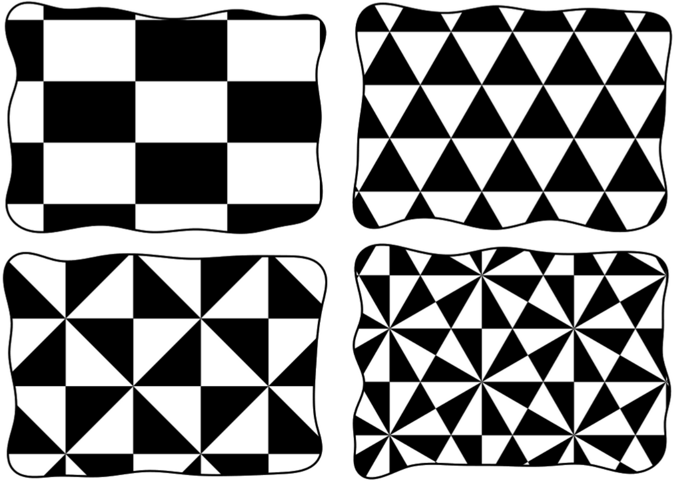
Suppose that a Coxeter polygon has $(n = 4)$-sides.
What can we conclude about the angles?

The Triangular Coxeter Geometries

Task

Suppose that a Coxeter polygon has $(n = 3)$*-sides.*
What can we conclude about the angles?

The Coxeter Geometries (Theorem 5.3.1 p. 102)



The Planar Coxeter Geometries from Sossinsky.

Gluings (for Model #4)

Definition

Given a pair of spaces S_1, S_2, and a map $f : S_1 \to S_2$ we can form a space $S_1 \bigsqcup S_2$ where S_1 is glued to S_2 along the map $f : S_1 \to S_2$.

Gluing a Cylinder

Question

Consider the long thin strip of paper $S = [-10, 10] \times [-1, 1]$.
What function $f : \{-10\} \times [-1, 1] \to \{10\} \times [-1, 1]$ *gives a cylinder?*

Gluing a Möbius Band

Question

Consider the long thin strip of paper $S = [-5,5] \times [-1,1]$.
What function $f : \{-5\} \times [-1,1] \to \{5\} \times [-1,1]$ *gives a Möbius band?*