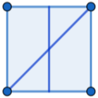
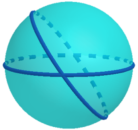
MAT 402: Classical Geometry

Groups

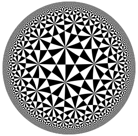


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

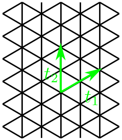
Spherical



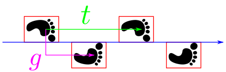
Hyperbolic



Tilings



Friezes

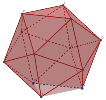


Trigonometry

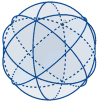
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

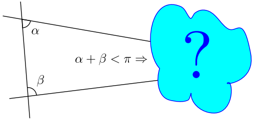
Platonic Solids

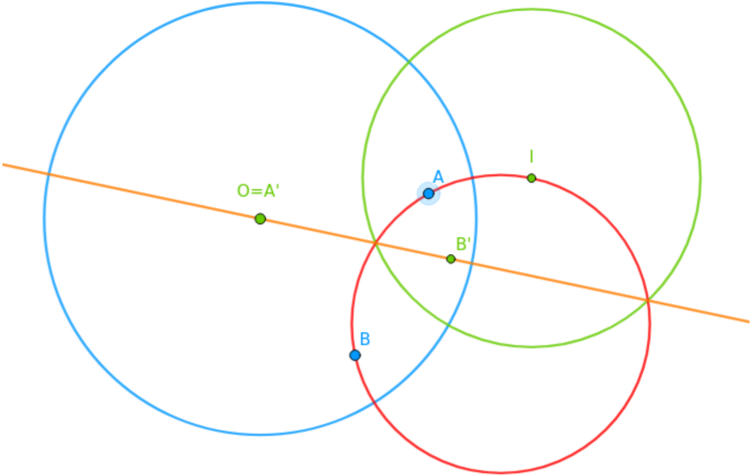


Coxeter



Parallels





Homework #4 was extended to Saturday @ 11:59am. Any questions?

MAT 402: Friday November 13th 2020

Learning Objectives:

- Prove theorems about hyperbolic geodesics using euclidean geometry.
- Construct orthogonal circles.
- Construct geodesics in the hyperbolic plane.

The Poincaré Disk Model

Definition (7.2.1)

The Poincaré Disk model is $\mathbb{H} = \{(x, y) : x^2 + y^2 < 1\}$. We call the circle $\mathbb{A} = \{(x, y) : x^2 + y^2 = 1\}$ the absolute. The transformation group is:

$$\mathcal{M} = \{\text{reflections in circles or lines perpendicular to } \mathbb{A}\}$$

An Example of a Circle Orthogonal to $\mathbb{A}$

Task

Check that the points $(0,0)$, $\left(\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$, $(\sqrt{2},0)$ *form a square.*

Task

Check that the circle C *centered at* $(\sqrt{2},0)$ *of radius* $r = 1$ *is orthogonal to* $\mathbb{A}$.

Finding the Image of a Segment

Task

Find the image of the segment from $A = (0, 0)$ *to* $B = (1/10, 0)$ *under the hyperbolic isometry given by inverting in* C *a circle centered at* $(\sqrt{2}, 0)$ *of radius* $r = 1$*.*

A Targeted Inversion

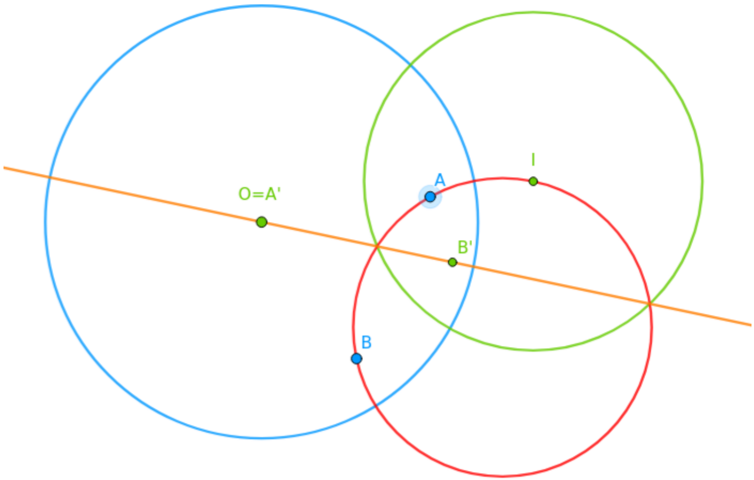
Task (7.1.3)

Given a point A in circle C centered at O there is a circle D orthogonal to C such that inversion in D takes A to O.

Key Facts about Inversion

Lemma

- *We can map any A in C to O (previous slide).*
- *Given a line not through O, its inverse is a circle through O.*
- *Inversion preserves angles.*



Pull up GeoGebra! https://www.geogebra.org/m/mk9preaj2,

The Construction of Hyperbolic Geodesics

Theorem (7.1.4(i) $\Longleftrightarrow$ 7.3.2)

Given $A, B \in \mathbb{H}^2$ there is a hyperbolic geodesic A and B which is orthogonal to $\mathbb{A}$.