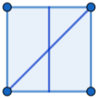
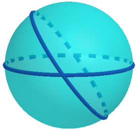
# MAT 402: Classical Geometry

Groups

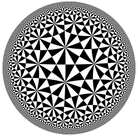


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

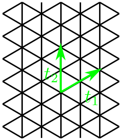
Spherical



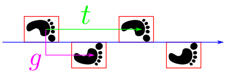
Hyperbolic



Tilings




Friezes

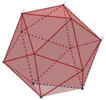



Trigonometry

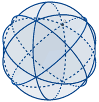
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

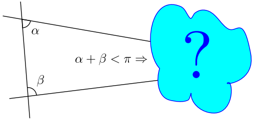
Platonic Solids

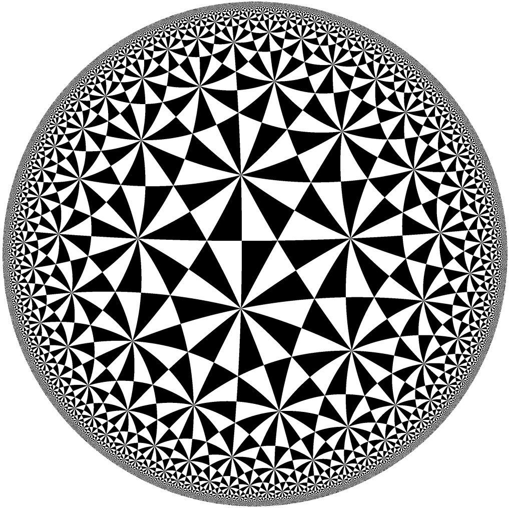


Coxeter



Parallels





**Welcome to non-Euclidean Geometry. How is your Euclid?**

# MAT 402: Monday November 9th 2020

**Learning Objectives:**

- Prove theorems about hyperbolic geodesics using euclidean geometry.
- Compare and contrast euclidean and hyperbolic geometry.

# Inversion

## Definition (7.1)

The inverse of a point $M$ in a circle of radius $r$ centered at $O$ is the point $N$ on the ray $OM$ such that: $|OM||ON| = r^2$

## Task

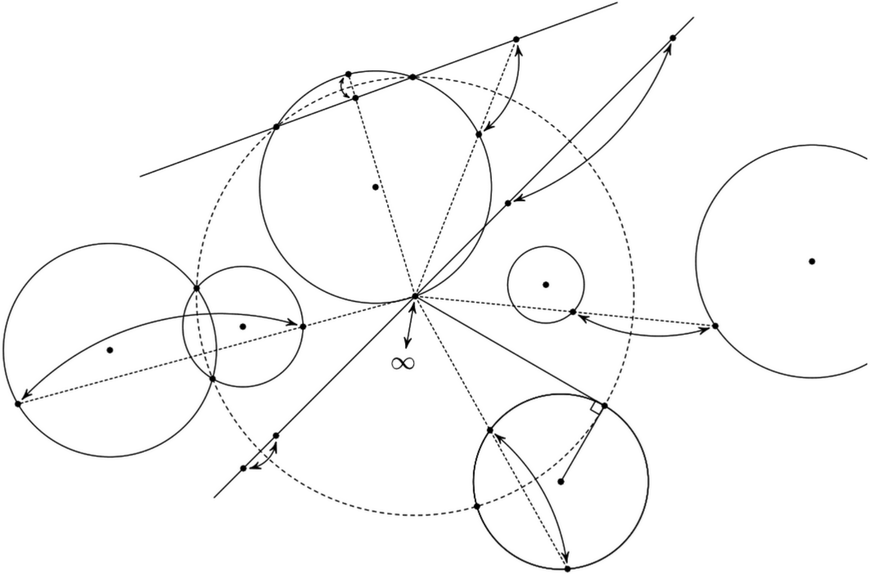
*Which points $M$ are fixed by this operation? If we apply inversion twice, what happens?*

# Construction of Inverses

**Task (7.1.3)**

*Show that the following construction works: If $M$ is inside the circle $C$, then we can obtain $N$ in the following way. Draw the ray $OM$ and extend it indefinitely. The line perpendicular to $OM$ at $M$ will meet the circle $C$ at $T$. The tangent line to $C$ at $T$ meets $OM$ at $N$.*

# Properties of Inversion (Sossinsky 7.3)



# Properties of Inversion

## Theorem (Euclid)

*If $C$ is a circle centered at $O$, $AB$ is a diameter of $C$, and $P$ is on $C$, then $\angle APB = \pi/2$.*

# Properties of Inversion

## Theorem (Euclid)

*If $C$ is a circle centered at $O$, $AB$ is a diameter of $C$, and $P$ is on $C$, then $\angle APB = \pi/2$.*

## Task

*Let $C$ be a circle centered at $O$. If $L$ is line which does not intersect $C$ then the inverse of $L$ is a circle inside $C$ passing through $O$.*

# Properties of Inversion

## Theorem (Euclid)

*Consider a pair of secants $AB$ and $EF$ to a circle $C$ which, when extended, intersect at a point $P$. We have:* $|PA||PB| = |PE||PF|$.

# Properties of Inversion

## Theorem (Euclid)

*Consider a pair of secants $AB$ and $EF$ to a circle $C$ which, when extended, intersect at a point $P$. We have:* $|PA||PB| = |PE||PF|$.

## Task

*Let two circles $C$ and $D$ be orthogonal at their points of intersection.*
*Inversion in $C$ maps $D$ to itself.*