

Question

Experiment! $P(E_1) = \frac{|E_1|}{8} = \frac{3}{8}$ 10 people!
Toss a coin three times. How many heads did you get?

- A. 0 $E_0 = \{TTT\} \rightarrow 1/8$
B. 1 **on!** $E_1 = \{HTT, THT, TTH\} \rightarrow 3/8$
C. 2 **off.** $E_2 = \{HHT, HTH, THH\} \rightarrow 3/8$
D. 3 $E_3 = \{HHH\} \rightarrow 1/8$

As the samples go to infinity.
If you do not have a coin, visit justflipacoin.com/

$$P(E_2) = \frac{|E_2|}{8} = \frac{3}{8}$$

5 people had 2 heads.
 $26 \cdot \frac{3}{8} = 9.75$

HHH 1000 people
HHT ≈ 375
HTH
HTT with two heads.
TTH
THT
TTH
TTT

Question

Experiment!

Think of a random number n from one to a thousand. Where was your number n ?

- A. $1 \leq n < 250$
- B. $250 \leq n < 500$
- C. $500 \leq n < 750$
- D. $750 \leq n \leq 1000$.



As the number of samples $\rightarrow \infty$ these numbers go closer to $\frac{1}{4}$.

small numbers

Nada:

$$P(A) = \frac{250}{1000} = \frac{1}{4}$$

$$P(B) = \frac{250}{1000} = \frac{1}{4}$$

Question

Experiment!

Make an educated guess, without actually calculating:

What's the probability that two people in our class share a birthday?

*~ 26
people*

- ~~A.~~ Between 0% and 25%
- B. Between 25% and 50%
- C. Between 50% and 75%**
- D. Between 75% and 100%

Maheen + Chan!

If you have > 365 people then $P=1$.



Question

Five "letters": C O R C M E M E

Suppose that we randomly re-arrange the letters of the word COMMERCÉ. What is the probability that the word MEME appears somewhere in the shuffled letters?

A. $1/5040 = \frac{1}{8!/(2!)^2}$

B. $1/1680 = \frac{4!}{8!}$

C. $1/84 = \frac{5!/2!}{8!/(2!)^3}$

D. $1/60 = \frac{1}{5 \cdot 4!/2!}$

$$P = \frac{\text{MEME ways}}{\text{Total ways}}$$

$$\frac{5!/2!}{8!/(2!)^3}$$

← repeated C

← repeated M, E, C.

oops!
typo.

Question

three 20-sided dice

Suppose that you roll $3d20$. Given that all dice are fair, what is the probability of rolling any specific triple (i, j, k) ?

~~A. $1/20$~~ P of one die

B. $1/400$

C. $1/800$

• i
✓ D. $1/8000$

$(1, 3, 3)$

$$P((i, j, k) = (1, 3, 3)) = P(i=1, j=3, k=3)$$

$$= P(i=1) P(j=3) P(k=3)$$

$$= \frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} = \frac{1}{8000}$$

Question

A chess tournament has twelve players, including Beth and Benny. The players are randomly paired up. What's the probability that Beth and Benny face each other?

- A. $({}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2) / 6!$ in m .
- B. $({}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2) / 5!$ in m .
- C. ${}_{12}C_2$ in m .
- D. 6 in m .

where $m = ({}_{12}C_2 \cdot {}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2) / 6!$

Question

Roll 2d6. Let A be the event that the sum is seven, and let B be the event that **one of the dice** is odd. Which of the following is true?

- A. A and B are independent.
- B. A and B are mutually exclusive.
- C. A and B are both independent and mutually exclusive.
- D. A and B are neither independent nor mutually exclusive.

Question

Roll 2d6. Let A be the event that the sum is seven, and let B be the event that the **first die** is odd. Which of the following is true?

- A. A and B are independent.
- B. A and B are mutually exclusive.
- C. A and B are both independent and mutually exclusive.
- D. A and B are neither independent nor mutually exclusive.

Question

Can we finish early?

4 midterms
next week

Roll 3d6 and record the sum. What is the sample space of this experiment?

A. $\{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), \dots, (6, 6, 6)\}$ ← we write (x, y, z)

~~B. $\{\{1, 1, 1\}, \{1, 1, 2\}, \{1, 1, 3\}, \dots, \{6, 6, 6\}\}$~~

*C. $\{3, 4, 5, \dots, 16, 17, 18\}$ ← we write $x + y + z$.

D. $\{1, 2, 3, \dots, 17, 18\}$

The results of the roll are ordered.
We take the sum of the results.

$$p(3) = \frac{1}{36}$$

$$p(18) = \frac{1}{36}$$

$$p(4) = \frac{3}{36} = \frac{1}{12}$$

Cliff
hanger!

Question

$$P(E) = 1 - P(E^c)$$

Roll 3d6. What is the probability that the sum of the dice is at least six?

A. $5/12 = 1 - \frac{7}{12}$

B. $7/12 = 1 - \frac{5}{12}$

* C. $103/108 = 1 - \frac{10}{6^3}$

D. $211/216 = 1 - \frac{5}{6^3}$

What is the prob. that 3d6 is < 6 ? ≥ 6
Outcomes: 3, 4, 5.

③ 111 ④ 112 121 211

⑤ 221 212 122
113 131 311

Ten outcomes give total < 6 .

Enia:

$216 = 6^3$ outcomes

$$1 - \frac{10}{216} = \frac{216 - 10}{216} = \frac{206}{216} = \frac{103}{108}$$

10/15

Question

Suppose that we draw four cards from a deck of fifty-two cards. What is the probability that at least one of the cards is an ace?

A. $\frac{38916}{54145} = \frac{{}_{52-4}C_4}{{}_{52}C_4}$

* B. $\frac{15229}{54145} = \frac{{}_{52}C_4 - {}_{52-4}C_4}{{}_{52}C_4} =$

C. $\frac{38916}{54145} = \frac{{}_{52-4}P_4}{{}_{52}P_4}$

D. $\frac{15229}{54145} = \frac{{}_{52}P_4 - {}_{52-4}P_4}{{}_{52}P_4}$

all
hands

hands w
no ace

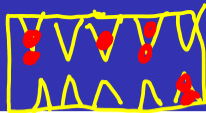
$$\frac{{}_{52}C_4}{{}_{52}C_4} - \frac{{}_{48}C_4}{{}_{52}C_4}$$



Although the fractions are equal,
we have unordered cards.

Question

gr: Tavli
♠ ταβλι



In the game of Backgammon, you roll 2d6. It is often strategically helpful to a pair of consecutive numbers such as 3 and 4. What is the probability of rolling a pair of consecutive numbers?

A. $\frac{6}{36}$

B. $\frac{9}{36}$

C. $\frac{10}{36}$

D. $\frac{18}{36}$

- 12 21
- 23 32
- 34 43
- 45 54
- 56 65

10 possible outcomes.

Unordered:
 $6C_2 + 6$
differ same
 $= 15 + 6 = 21$

$\frac{10}{6^2} \rightarrow 6^2 = 6 \cdot 6 \rightarrow$ ordered
first second

$\frac{5}{18}$

Question



You flip a fair coin, roll a d6, then flip another fair coin. Let E be the event that the number of heads (H) shown is larger than or equal to the number on the d6. Which statement is true?

A. $|E| = 1$

B. $|E| = 2$

C. $|E| = 3$

* D. $|E| = 4$

of heads ≤ 2

	H	die	
H	2	H	$2 \geq 2$
H	1	T	$1 \geq 1$
T	1	H	$1 \geq 1$
H	1	H	$2 \geq 1$

check the events that you write really work!

Question

Important!

Consider the experiment of rolling 2d6, but the first die is fair, while the second follows this distribution:

$$p(y) = \begin{cases} 1/12, & y \text{ is odd} \\ 1/4, & y \text{ is even.} \end{cases}$$



If x denotes the number that shows up on the fair die, while y is the number on the second, compute $P(x \text{ is odd and } y \leq 3)$.

11 12 13

31 32 33 ✓
51 52 53 ✓

A. $\frac{1}{8}$

B. $\frac{5}{24}$

C. $\frac{1}{4}$

D. $\frac{11}{12}$

$$P(E) = \left(\frac{1}{6} \cdot \frac{1}{12} + \frac{1}{6} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{12} \right) \cdot 3 = \frac{3}{6} \left(\frac{1}{12} + \frac{1}{4} + \frac{1}{12} \right) = \frac{5}{24}$$

Question

Suppose that we have two bags of marbles. Bag #1 has two red and five blue marbles. Bag #2 has 97 red and three blue. We pick a bag randomly, and select a marble. What is the probability that it is blue?

A. $\frac{1}{2}$

B. $\frac{8}{107} = \frac{5+3}{7+100}$

C. $\frac{512}{1400} = \frac{15}{27} + \frac{1}{2} \frac{3}{100}$ *

D. $\frac{879}{1400} = \frac{12}{27} + \frac{1}{2} \frac{97}{100}$
= Prob(Red)



$$\begin{aligned} P(\text{Blue}) &= P(\text{Blue}|\#1)P(\#1) + \\ & P(\text{Blue}|\#2)P(\#2) \\ &= \frac{5}{7} \cdot \frac{1}{2} + \frac{3}{100} \cdot \frac{1}{2} \end{aligned}$$

No week 11
on the test!