Question

Suppose that you know A is invertible and you want to solve the system $A\mathbf{x} = \mathbf{b}$. Which of the following strategies would work?

All of these work!

A. Row reduce the augmented matrix $[A|\mathbf{b}]$.

B. Calculate $\mathbf{x} = A^{-1}\mathbf{b}$.

C. Express $\mathbf{b}$ as a linear combination of the columns of A.

D. Ask WolframAlpha to solve it. ← not cheating

Asking Chegg/Reddit/Your weird Roommate are cheating

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \iff x\begin{bmatrix} 1 \\ 3 \end{bmatrix} + y\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Question

What is the determinant of $M = \begin{bmatrix} 3 & 5 & 1 \\ 0 & 5 & 0 \\ 2 & 2 & 3 \end{bmatrix}$?

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

A. 0

B. 35

C. -25

D. 1

Prabhjot:

$0 \cdot 2 - 2 \cdot 5 = -10$

$3\begin{bmatrix} 5 & 0 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix} + 1\begin{bmatrix} 0 & 5 \\ 2 & 2 \end{bmatrix}$

$3(15) - 5(0) + 1(-10)$: Chan

$= 45 - 10 = 35$

Question

What is the determinant of the following matrix?

$$\det \begin{bmatrix} 1 & 3 & 7 & 0 & 0 \\ 0 & 2 & 3 & 1 & 5 \\ 0 & 0 & -1 & 3 & 12 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} = 1\cdot 2\cdot(-1)\cdot 3\cdot 2 = -12$$

$$= 1\cdot\det\begin{bmatrix} 2 & 3 & 1 & 5 \\ 0 & -1 & 3 & 12 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix} - 0 + 0 \cdots$$

$$= 1\cdot\left(2\cdot\det\begin{bmatrix} -1 & 3 & 12 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{bmatrix} - 0 + 0 \cdots\right)$$

- *A. -12
- B. 0
- *C. 74
- ~~D. 133~~ silly.

Question

Suppose the matrix A can be row reduced to the following:

The matrix $\begin{bmatrix} 133 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ has det 133 and RREF:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

What is the determinant of A?

A. -1

B. 0

~~C. 1~~ WRONG

D. It's impossible to determine.

det $B = 4$

$B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

RREF $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

det $C = 9$

$C = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

RREF $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Challenge Question: Find two matrices B and C with the same RREF and different determinants.

RREF give info about det?

Question

Suppose the matrix A can be row reduced to the following:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

What is the determinant of A?

A. -1

B. 0

C. 1

D. It's impossible to determine.

Question

Suppose that A and B are 2×2 matrices satisfying: $\det(A) = 2$ and $\det(B) = 3$. What is $\det(A^T(B^{-1})^2)$?

A. $9/2$

B. 2

C. $2/9$

D. 9

Challenge Question: Produce two matrices A and B with these properties and calculate $A^T(B^{-1})^2$.

Question

Which of the following is an eigenvector of $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ with $\lambda = 2$?

A. $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

B. $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

C. $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

D. $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= 2 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \lambda x$$

Question

Nada: $A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}-3\\0\end{bmatrix}$ $A\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}9\\6\end{bmatrix}=3\cdot\begin{bmatrix}3\\2\end{bmatrix}$:)

Which of the following is an eigenvector of $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$?

A. ~~$\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$~~

B. $\mathbf{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

C. ~~$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$~~

D. ~~$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$~~

$A\vec{x} = 2\vec{x}$

$$\begin{cases} 3x - 3y = 0 \\ 2x - 2y = 0 \end{cases} \Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$A\vec{x} = 3\vec{x}$

$$\begin{cases} 2x - 3y \\ 2x - 3y \end{cases} \Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Chan: $\det\begin{pmatrix} 5-\lambda & -3 \\ 2 & -\lambda \end{pmatrix}$

$= -\lambda(5-\lambda)+6 = (\lambda-3)(\lambda-2)$

Question

Suppose A is an $n \times n$ matrix with eigenvalue λ and associated eigenvector $\mathbf{v}_\lambda$. Which of the following statements is correct?

A. λ^2 is an eigenvalue of A^2 with eigenvector $\mathbf{v}_\lambda$

B. $\lambda/2$ is an eigenvalue of $\frac{1}{2}A$ with eigenvector $\frac{1}{2}\mathbf{v}_\lambda$

C. $-\lambda$ is an eigenvalue of A with eigenvector $-\mathbf{v}_\lambda$

D. $\lambda = 0$ is an eigenvalue of A, with associated eigenvector $v_\lambda = \mathbf{0}$.

$$\boxed{\begin{aligned} &\left(\tfrac{1}{2}A\right)\left(\tfrac{1}{2}v_\lambda\right) = \tfrac{\lambda}{2}\left(\tfrac{1}{2}v_\lambda\right) \\ \Leftrightarrow\ &\tfrac{1}{4}Av_\lambda = \tfrac{\lambda}{4}v_\lambda \\ \Leftrightarrow\ &Av_\lambda = \lambda v_\lambda \end{aligned}}$$

Eigenvectors cannot be zero.

If $Av = \lambda v$

$\Leftrightarrow \frac{1}{4}Av = \frac{1}{4}\lambda v \Leftrightarrow \left(\frac{1}{2}A\right)\left(\frac{1}{2}v\right) = \left(\frac{1}{2}\lambda\right)\left(\frac{1}{2}v\right)$ This works.

Question

LONG ① Find eigenvalues/vectors.

② Use octave

Consider the matrix: $A = \begin{bmatrix} 7 & 1 \\ 6 & 8 \end{bmatrix}$? → ③ Multiply FAST.

Parker says the matrix has $\mathbf{v}_P = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$ as an eigenvector.

Tyler says the matrix has $\mathbf{v}_T = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ as an eigenvector.

Who is right in this situation?

A. Parker and Tyler are both wrong.

B. Parker and Tyler are both right.

C. Parker is right, but Tyler is wrong.

D. Parker is wrong, but Tyler is right.

v_T

$$\begin{bmatrix} 7 & 1 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix} = 5\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5/2 \\ 5 \end{bmatrix}$$

v_P

$$= 5\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

Question

$S v = 3 v \Rightarrow S \begin{bmatrix} 2 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 6 \\ 27 \end{bmatrix}$

Suppose $S = \begin{bmatrix} -6a & a \\ -6b+3 & b \end{bmatrix}$.
What are the values of a and b such that $\lambda = 3$ is an eigenvalue of S with associated eigenvector $\begin{bmatrix} 2 \\ 9 \end{bmatrix}$?

A. $(a, b) = (0, 0)$
B. $(a, b) = (-2, -7)$
C. $(a, b) = (-1, 4)$
D. $(a, b) = (3, 2)$

$$\begin{bmatrix} -6a & a \\ -6b+3 & b \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 6 \\ 27 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3a \\ -3b+6 \end{bmatrix} = \begin{bmatrix} 6 \\ 27 \end{bmatrix}$$

$$\Rightarrow (a, b) = (-2, -7)$$

Question when does $\det(A-\lambda I)=0$?

Suppose that a 3×3 matrix satisfies $\det(A - \lambda I) = \lambda^3 - 6\lambda^2 + 11\lambda - 6$.
Which of the following is NOT an eigenvalue of A?

A. $\lambda = 1$ $\Rightarrow 1^3 - 6 + 11 - 6 = 0$

B. $\lambda = 2$ $\Rightarrow 2^3 - 6 \cdot 2^2 + 11 \cdot 2 - 6 = 8 - 24 + 22 - 6 = 0$

C. $\lambda = 3$ $\Rightarrow 0$

D. $\lambda = 4$ $\Rightarrow 4^3 - 6 \cdot 4^2 + 11 \cdot 4 - 6 = 6$ Not an eigenvalue

Christina Stewart:

$\det(A - \lambda I) = (\lambda - 1)(\lambda - 2)(\lambda - 3)$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\det\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = xy$

Challenging question: Produce a matrix A with this property. A is 3×3.

Question

Let $A = PDP^{-1}$ where

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The eigenvalues of A are:

A. $-1, 1, 1$

B. $0, 1, 1$

C. $1, -1, -1$ à la Octave.

D. They cannot be determined from the information given.