Question

Hallo!

How many ways can one arrange the letters of the word MATHEMATICS?

A. $39916800 = 11!$

B. $19958400 = 11!/2!$

C. $9979200 = 11!/(2!)^2$

D. $4989600 = 11!/(2!)^3$

AEAI MTHMTCS

$$\frac{11!}{2!\,2!\,2!}$$

Question

3! = ways to arrange 3 objects

How many arrangements of the word ONTARIO have all consonants in alphabetical order from left to right (but not necessarily beside each other)?

A. $840 = \dfrac{7!}{3!} = {}_7P_4$

*B. $\underline{420} = \dfrac{7!}{2!\cdot 3!} = {}_7C_3\dfrac{4!}{2!}$

cons vowels

*C. $72 = \dfrac{4!}{2!}\cdot 3!$

D. $10 = \dfrac{5!}{2!\cdot 3!} = {}_5C_2$

ONTARIO

MNOPQRST

N R T

N R T

{O, A, I, O}

Pick three spots to be consonants.

if they had to be beside each other NRTAAIO

The consonants do not need to be beside each other ONORTIA

Question

You have 12 distinct (physical) textbooks all written by different authors and a new bookshelf that can only hold 7 of them. Let X be the number of ways to pick 7 textbooks and arrange them in a row; and let Y be the number of ways to pick 7 textbooks and arrange them in a row, by author name alphabetically. Then:

A. $X = {}_{12}C_7$ and $Y = {}_{12}C_7$

B. $X = {}_{12}P_7$ and $Y = {}_{12}C_7$

C. $X = {}_{12}C_7$ and $Y = {}_{12}P_7$

D. $X = {}_{12}P_7$ and $Y = {}_{12}P_7$

$\boxed{\begin{array}{l}\text{DBLJIGC} \\ \text{GIJLBDC} \\ \text{Permute}\end{array}}$ X

A B C D E F G H I J K L

$\boxed{\begin{array}{l}\text{BCDGIJL} \\ \text{Choice}\end{array}}$ Y

Question

All arrangements $\frac{8!}{2!2!2!}$ = Ans. D.

How many arrangements are there of the word COMMERCE that contain the word MEME?

A. $15 = 5!/(2!)^3$

B. $30 = 5!/(2!)^2$

C. $60 = 5!/2!$

D. $5040 = 8!/(2!)^3$

O R $\underbrace{\text{M E M E}}$ C C

$5 \cdot \frac{4!}{2!} = \frac{5!}{2!}$ {C, O, R, C} $\frac{4!}{2!}$ ways to arrange

→ First: position MEME: 5 ways

→ Second: arrange {C, O, R, C}

Question

Consider tossing a coin five times. How many outcomes have more heads than tails?

Heads = 3, 4, 5

A. $6 = (5 - 3) \cdot (5 - 2)$

B. $8 = 2^5 - 2^4 - 2^3$

C. $16 = {}_5C_5 + {}_5C_4 + {}_5C_3$

D. $95 = {}_5P_5 - {}_5P_2 - {}_5P_1$

Question

Break extended to Jan 11

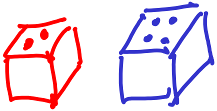
A red d6 and a blue d6 are thrown. How many outcomes have one die show a larger value than the other?

A. $15 = {}_6C_2$

*B. $30 = \boxed{2}\,(5 + 4 + 3 + 2 + 1)$

*C. ~~$35 = 6^2 - 1$~~

D. ~~$240 = 2(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$~~



5

blue > red

blue < red

$6 \cdot 6 = 36$ options

red ↗ ↖ blue

switch roles of red and blue ⇒ double

Red	Blue
1	2 3 4 5 6
2	3 4 5 6
⋮	
5	6

Question $\{Alice, Bob\} = \{Bob, Alice\}$ "unordered"

How many ways can twelve people form six pairs?
Assume that the pairs are unordered, and the collection of all six pairs is unordered.

A. $161 = {}_{12}C_2 + {}_{10}C_2 + {}_{8}C_2 + {}_{6}C_2 + {}_{4}C_2 + {}_{2}C_2$

*B. $10395 = ({}_{12}C_2)({}_{10}C_2)({}_{8}C_2)({}_{6}C_2)({}_{4}C_2)({}_{2}C_2)/6!$

C. $7484400 = ({}_{12}C_2)({}_{10}C_2)({}_{8}C_2)({}_{6}C_2)({}_{4}C_2)({}_{2}C_2)$

D. $479001600 = {}_{12}P_{12}$

ways to rearrange

Really small: $12 \cdot 11 = 132$ options

Rearrange six pairs → 6! ways

$P_1 P_2 P_3 P_4 P_5 P_6$

What is C counting?

Cheng: The groups are ordered somehow.

"number of ways to schedule presentations given by two people"

What is D counting?

Nada: If pair order mattered.

"presentations by 12 people"

Guadalupe: Grouping people

Oleksander: Christina Stewart:

Breakout rooms / People sitting together at a wedding

Nada: People pairing up for tasks

Question

Megan's Icecream Shoppe offers four flavours of icecream. You may order a single or a double scoop. When ordering a double scoop, you must select two different kinds of icecream.

choose a single flavour

A. $7 = 4 + 3$

*B. $10 = 4 + {}_4C_2$ ← order of scoops does not matter

choose two flavours

C. $12 = 4 \cdot 3$

*D. $16 = 4 + {}_4P_2$ ← order of scoops matters

Extremely important philosophical question: Does order matter here?

Question

How many hands of five cards contain exactly $\boxed{\text{one heart and one jack?}}$

A. $185640 = 13 \cdot \frac{4}{2!} \cdot ({}_{36}C_3)$

*B. $315945 = 12 \cdot 3 \cdot {}_{36}C_3 + {}_{36}C_4$

C. $371280 = 13 \cdot 4 \cdot ({}_{36}C_3)$

*D. $464100 = 13 \cdot 4 \cdot ({}_{36}C_3) + 13 \cdot ({}_{36}C_3)$

J♡ is enough

4 non-Jack non-Heart cards

Case 1: Jack = Heart (J♡)

Noura

${}_{36}C_4$

Choose 4 cards that are not Jacks or Hearts

Case 2: Jack ≠ Heart (No J♡)

13−1 Heart

4−1 Jack

${}_{36}C_3$ Remainder

Question

$_{52-4}C_3$ = all cards three cards from except kings

How many three-card hands have at least one K? king

X A. $_{52}C_3 - 4$ = all possible hands except four

✓B. $_4C_1 \cdot {}_{48}C_2 + {}_4C_2 \cdot {}_{48}C_1 + {}_4C_3 \cdot {}_{48}C_0$ Christine: 4804

✓C. $_{52}C_3 - {}_{48}C_3$ Nada: 4804

X D. $({}_4C_1)({}_{51}C_2)$ = choose a king and two other cards

K♡ {K♢, 3♠} ≠ K♢ {K♡, 3♠}

(B) One king + 2 kings + 3 kings

Nada.

(C) All hands except hands with no kings

Nada.

Challenge Question: What are the values of these expressions?

Noura: 14960

Question

How many three-card hands have at least one K **or** at least one ♡ in them? (Assume that cards can be rearranged in your hand.)

A. ~~7140~~ $= {}_{52}C_3 - {}_{36}C_3$

B. 15555 $= {}_{52}C_3 - {}_{35}C_3$

C. 21675 $= {}_{4}C_1 \cdot {}_{51}C_2 + {}_{13}C_1 \cdot {}_{51}C_2$

D. 22950 $= {}_{4}C_1 \cdot {}_{51}C_2 + {}_{13}C_1 \cdot {}_{51}C_2 - {}_{1}C_1 \cdot {}_{51}C_2$

All hands except hands with no K and no ♡.

all hands − hands without K or ♡.

36 = 52 − 13 all hearts
− 3 other kings ♢ ♠ ♣

Question

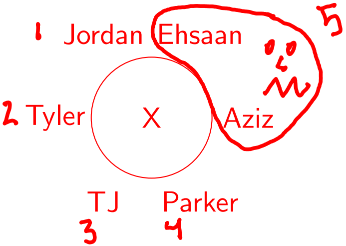
To arrange n objects in a circle : $\frac{n!}{n}$

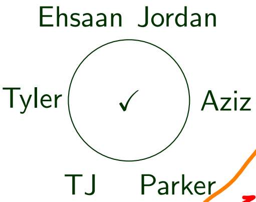
Aziz, Ehsaan, Jordan, Parker, TJ, and Tyler are sitting at a circular table. But Aziz secretly hates Ehsaan and **does not** want to sit next to him. How many ways are there for everyone to seat themselves?

$\frac{6!}{6} - 2 \cdot \frac{5!}{5}$

all - bad

in the bad case: EA are one.





order: EA ≠ AE

all possible arranges

E and A beside.

A. $120 = 6!/6$

B. $96 = \frac{6!}{6} - \frac{5!}{5}$

C. $72 = \frac{6!}{6} - 2 \cdot \frac{5!}{5}$

D. $48 = 2 \cdot \left(\frac{6!}{6} - \frac{5!}{5}\right)$

Question

2♡ 2◇ 5♡ 5♣ 5◇

How many five card hands have a pair and a three-of-a-kind of different ranks?

number.

A. $3744 = 13 \cdot {}_4C_2 \cdot 12 \cdot {}_4C_3$

B. $44982 = 52(4-1)(52-4)(4-1)(4-2)(4-3)$

C. $79872 = {}_{13}C_2(4^2)(4^3)$

D. $22934496 = ({}_{52}C_2)({}_{52-2}C_3)$

$$13 \cdot {}_4C_2 \cdot 12 \cdot {}_4C_3$$

rank of pair, suits, rank of three-of-kind, suits

Question

1001

There are fifteen people in the Music Club, and you want to make two 4-person bands. Each band will have a guitarist, a drummer, and two backup singers. How many ways can you make the two bands?

A. $2702700 = {}_{15}C_2 \cdot {}_{13}C_2 \cdot {}_{11}C_4$

B. $5405400 = 2 \cdot {}_{15}C_8 \left(\frac{8!}{4!2!2!}\right)$

C. $32432400 = \frac{1}{2!} \cdot {}_{15}C_4 \cdot {}_{11}C_4 \cdot \left(\frac{4!}{2!}\right)^2$

D. $64864800 = 15 \cdot 14 \cdot {}_{13}C_2 \cdot 11 \cdot 10 \cdot {}_9C_2$

g D S S

$\frac{1}{2!}$ switch bands

${}_{15}C_4$ band 1

${}_{11}C_4$ band 2

$\left(\frac{4!}{2!}\right)$ $\left(\frac{4!}{2!}\right)$ arrange all four people with switch of singers.

Question

We had "or" before.

How many three-card hands have at least one K **and** at least one ♡ in them? (Assume that cards can be rearranged in your hand.)

A. $1455 = {}_1C_1 \cdot {}_{51}C_2 + {}_3C_1 \cdot {}_{12}C_1 \cdot {}_{50}C_1$

B. $2805 = {}_{52}C_3 - ({}_{48}C_3 + {}_{39}C_3 - {}_{36}C_3)$

C. $3266 = 4 \cdot 13 \cdot 50 + 1 \cdot ({}_{52-15}C_2)$

D. $14960 = {}_{52}C_3 - {}_{36}C_3$

:)

B = all hands − (hands without either K or ♡)

$= {}_{52}C_3 - (\underbrace{{}_{48}C_3}_{\text{no king}} + \underbrace{{}_{39}C_3}_{\text{no heart}} - \underbrace{{}_{36}C_3}_{\text{no K and no ♡}})$