Question

Welcome! Questions?

Consider the following matrices.

1×4 $\qquad$ 4×4 $\qquad$ 2×4

$$A = \begin{bmatrix} 1 & 1 & 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & -2 & 0 & -4 \\ 1 & 0 & 3 & 5 \\ 0 & 2 & -3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 0 & 1 & 0 \\ 3 & -2 & 0 & 4 \end{bmatrix}$$

Which of the following products is defined?

A. AB

B. BA

C. CA

D. AC

(Are there any unlisted multiplications which are defined?)

Chan:

$n \times k$ and $k \times n$

Nada:

CB

Question

Consider the 2×2 matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$. For which of the below matrices B does $AB = 0$?

A. $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

B. $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

C. $B = \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix}$

D. $B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$

Suppose only A is given. Find AB = 0 with B!=0

Bruno: Make $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Christine L:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\iff \begin{bmatrix} a+c & b+d \\ 2a+2c & 2b+2d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{cases} a+c=0 \\ b+d=0 \\ 2a+2c=0 \\ 2b+2d=0 \end{cases}$$

Shree:

$a = -c$

$b = -d$

Nada:

$B = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$

Question

$A^2 = -I$ $\qquad$ $x^2 = 1$

Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Which of the following statements is correct?

A. ~~There is a non-zero 2×1 vector $\mathbf{v}$ such that $A\mathbf{v} = \mathbf{0}$~~ ?

B. ~~$AB = BA$ for any matrix B~~

C. ~~$A^T = A$~~

D. $A^2 = -I$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ -a & -b \end{bmatrix} \neq \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -b & a \\ -d & c \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question

Suppose $\mathbf{v} \in \mathbb{R}^n$ is a column vector. What are the dimensions of $\mathbf{v}^T\mathbf{v}$?

A. $n \times n$

B. $n \times 1$

C. 1×1

D. We cannot multiply these matrices

$V = 1 \times n$

$V^T = n \times 1$

$V^T V = (n \times 1)(1 \times n)$

$= n \times n$

$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \cdot 1 + 2 \cdot 2$

$= 5$

(For fun: What are the dimensions of $\mathbf{v}\mathbf{v}^T$?)

$VV^T = (1 \times n)(n \times 1) = 1 \times 1$

Question

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } B = \begin{bmatrix} u & v & w \\ x & y & z \end{bmatrix}.$$

What is $(AB)_{21}$?

A. $au + bx$
B. $av + by$
C. $cu + dx$
D. $cy + dy$

Question

Which of the following is NOT necessarily true for $n \times n$ matrices A and B?

A. $(A+B)^T = A^T + B^T$

B. $(kA)^T = k(A^T)$

C. $(AB)^T = (A^T)(B^T)$

D. $(A^T)^T = A$

$A = [A_{ij}] \longrightarrow A^T = [A_{ji}]$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$(A^T)^T = [A_{ji}]^T = [A_{ij}] = A$

$(A+B)^T = [A_{ij} + B_{ij}]^T = [A_{ji} + B_{ji}] = A^T + B^T$

$(kA)^T = [kA_{ij}]^T = [kA_{ji}] = k[A_{ji}] = kA^T.$

Question

Happy Halloween!

Suppose that A is an $n \times n$ matrix and $\mathbf{v}$ is an $n \times 1$ column vector. Which of the following is necessarily true about the vector $\mathbf{w} = A\mathbf{v}$?

A. $\mathbf{v}$ is a linear combination of the columns of A.

B. $\mathbf{w}$ is a linear combination of the columns of A.

C. $\mathbf{v}$ is a linear combination of the rows of A.

D. $\mathbf{w}$ is a linear combination of the row of A.

$$\begin{bmatrix}1&3\\2&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x+3y\\2x+4y\end{bmatrix} = \begin{bmatrix}1x\\2x\end{bmatrix}+\begin{bmatrix}3y\\4y\end{bmatrix}$$

$$= x\begin{bmatrix}1\\2\end{bmatrix}+y\begin{bmatrix}3\\4\end{bmatrix}$$

Question

Suppose $A = \begin{bmatrix} 1 & 3 \\ 7 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Which of the following problems is equivalent to the solving the equation $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$?

A. Solve the linear system $\begin{cases} x + 3y + z = 0 \\ 7x + 4y - z = 0 \end{cases}$

B. Solve the linear system $\begin{cases} x + 7y = 1 \\ 3x + 4y = -1 \end{cases}$

C. Determine if $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 & 3 \end{bmatrix}$ and $\begin{bmatrix} 7 & 4 \end{bmatrix}$

D. Determine if $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Question

Consider the following.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\cdot 3 + (-3)\cdot 1 \\ (-1)3 + 2\cdot 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

Which of the following vectors $\mathbf{x}$ is a solution of $A\mathbf{x} = \mathbf{b}$?

A. $\mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

C. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

D. The system $A\mathbf{x} = \mathbf{b}$ has no solution.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + (-3)(3) \\ (-1)\cdot 1 + 2\cdot 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

Question

$(X^T)^T = X$ $Y^T X^T = (XY)^T \neq X^T Y^T$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

Suppose that A, B are square matrices. Which one of the following expressions is equal to $(3AB^T)^T(A^2 + 2B)$?

A. $3(A^3)^T B + 3A^T B$

B. ~~$3BA^{T+2} + 6BA^T B$~~ T+2?!

C. $3BA^T A^2 + 6BA^T B$ *

D. $3BA^T A^2 + 6B^2 A^T$

$(3AB^T)^T(A^2+2B)$

$= 3(B^T)^T A^T (A^2+2B)$

$= 3BA^T(A^2+2B)$

$= 3BA^TA^2 + 6BA^TB$

$\boxed{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}}$

How does the transpose alter products?

$\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\right)^T$

$\begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 6 & 4 \end{bmatrix}$

Question

Suppose that P is a 5×6 matrix. What dimensions should a matrix Q have in order for the following expression to be defined?

$$PQ^T + P$$

Q is $n \times k$

Q^T is $k \times n$

$$\underset{P}{(5 \times 6)}\underset{Q^T}{(6 \times n)} + \underset{P}{(5 \times 6)}$$

$\Rightarrow k = 6$

$$= (5 \times n) + (5 \times 6)$$

$\Rightarrow n = 6$

A. Q must be a 5×5 matrix.
B. Q must be a 5×6 matrix.
C. Q must be a 6×5 matrix.
D. Q must be a 6×6 matrix.

Question $[1\ -2]\begin{bmatrix}4 & 3\\ 2 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = [1\ -2]\begin{bmatrix}4\\ 2\end{bmatrix} = 4 - 2\cdot 2 = 0$

Let $U, \mathbf{x}, \mathbf{y}$ denote the following three matrices.

$$U = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Then $\mathbf{y}^T U \mathbf{x}$ is equal to

$$\underbrace{(1\times 2)}_{y^T}\underbrace{(2\times 2)}_{U}\underbrace{(2\times 1)}_{x} = 1\times 1$$

1×1 matrices

A. 0

B. 1

C. 2

D. 3

What is $y^T U x$ in econ?

U = utility
x = goods
y = costs

Question

Let A be a 3×3 matrix, so that the product $AB = I_3$, where B is the matrix

$$A = \begin{bmatrix} - & r_1 & - \\ - & r_2 & - \\ - & r_3 & - \end{bmatrix} \qquad B = \begin{bmatrix} -0.5 & 2.5 & -0.5 \\ -1.5 & 1.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}. \quad = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then the product $A \begin{bmatrix} 2.5 & -0.5 \\ 1.5 & -1.5 \\ -0.5 & 0.5 \end{bmatrix}$ is: $= \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$

A. $\cancel{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$.

B. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$.

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$.

D. It cannot be determined from the given information.

Question

The matrix $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ is invertible. Which matrix below is its inverse?

A. $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

B. $\begin{bmatrix} \frac{1}{5} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{7} \end{bmatrix}$

* C. $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

D. $\begin{bmatrix} -3 & -7 \\ -2 & -5 \end{bmatrix}$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

If P and Q are invertible $n \times n$ matrices, then all of the following matrices are necessarily invertible, *except*:

A. P^{-1} $\quad P = PP^{-1} = I$

B. Q^T $\quad (Q^{-1})^T = (Q^{-1}Q)^T = I^T = I$

C. QP $\quad (P^{-1}Q^{-1}) = QIQ^{-1} = QQ^{-1} = I$

D. $P + Q$

$I \cdot I = I$

$\Rightarrow I^{-1} = I$

QP is invertible. Why? $\underbrace{(QP)}_{M}\underbrace{(P^{-1}Q^{-1})}_{M^{-1}} = I$

What example shows P + Q is not invertible?

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{invertible}} + \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{invertible}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\text{non-invertible}}$$