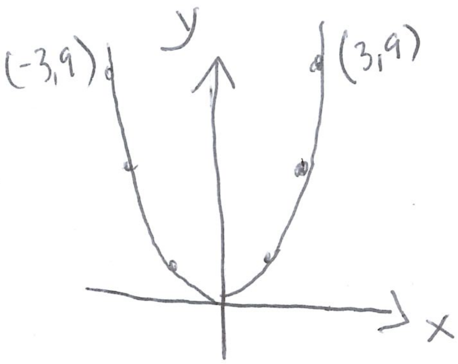
Graphs (M&T p. xxiv)

The graph of a function $f : X \to Y$ is the subset graph $f = \{(x, f(x)) : x \in X\} \subseteq X \times Y$.

Example: Graphs of $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$

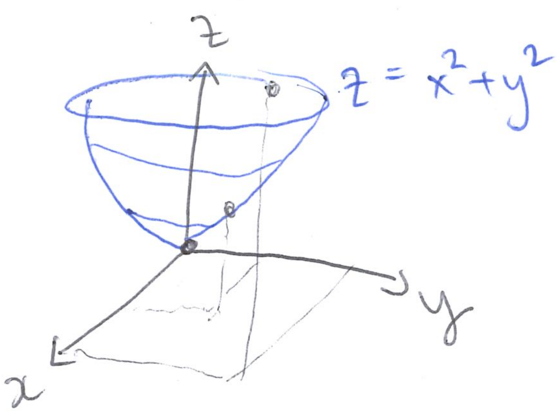
The graph of $f(x) = x^2$ is a subset of $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.
The graph of $g(x, y) = x^2 + y^2$ is a subset of $\mathbb{R}^2 \times \mathbb{R} = \mathbb{R}^3$.
Make a table of values and plot the graph of $g(x, y)$.



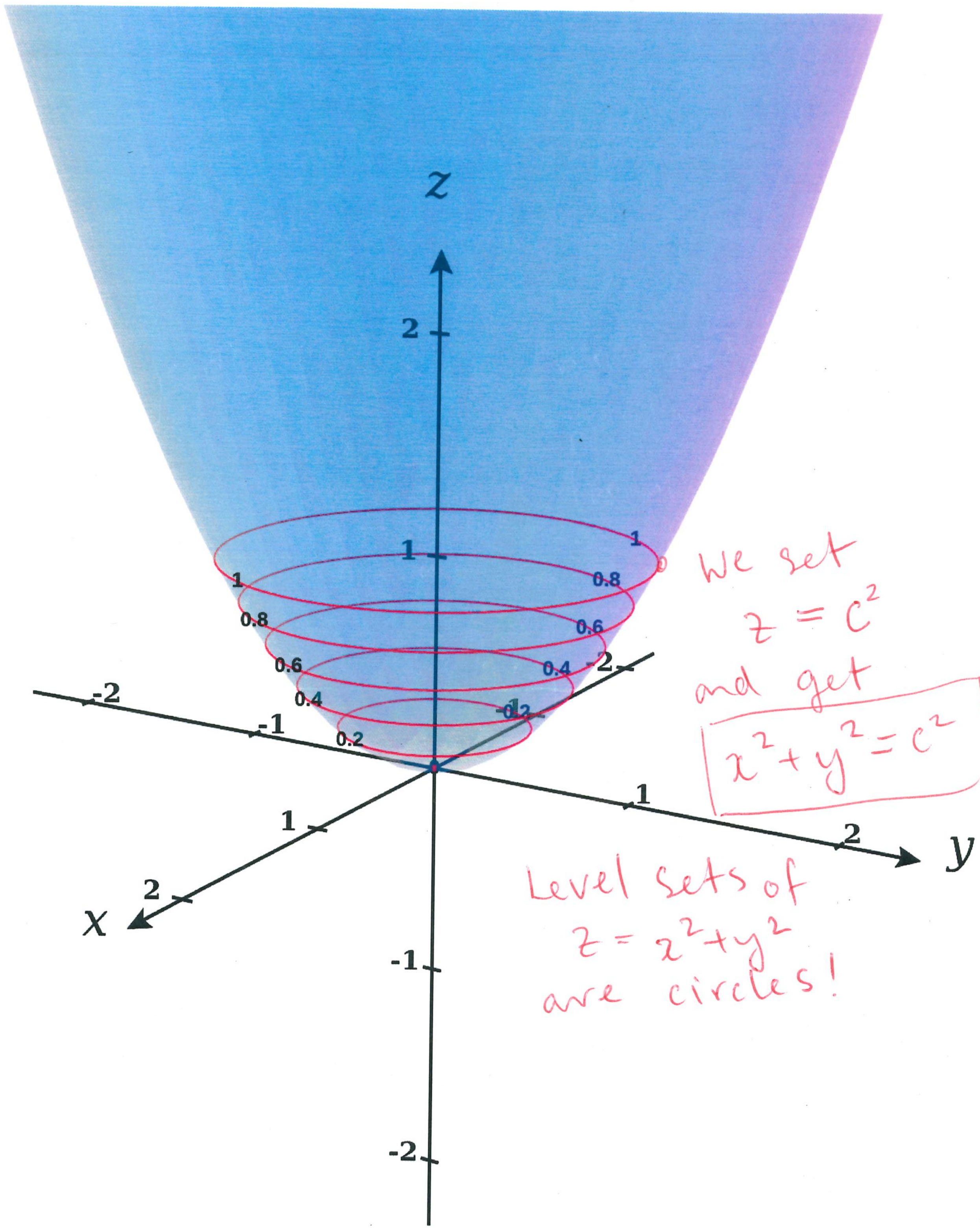
$f(x) = x^2$

x	$f(x)$
± 1	1
± 2	4
± 3	9

we draw the points $(x, f(x))$ for all values of x.



(x,y)	$g(x,y) = x^2 + y^2$	
$(0,0)$	$0^2 + 0^2 = 0$	$(0,0,0)$
$(\pm 1, \pm 1)$	$1^2 + 1^2 = 2$	$(1,1,2)$
$(\pm 2, \pm 2)$	$2^2 + 2^2 = 8$	$(2,2,8)$
$(1,2)$	5	
$(2,1)$	5	



We set $z = c^2$ and get $\boxed{x^2+y^2=c^2}$

Level sets of $z = x^2+y^2$ are circles!

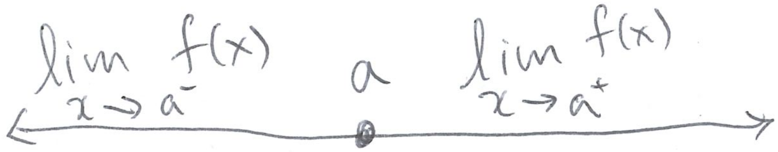
The Paraboloid $z = x^2 + y^2$ in CalcPlot3D

Example: Limits in Multiple Dimensions

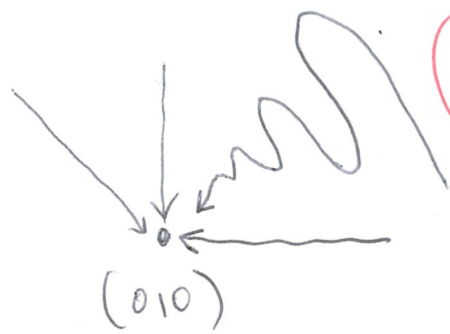
In one dimension, there are only two sides to a limit.

$$\lim_{x\to a^-} f(x) \quad \text{and} \quad \lim_{x\to a^+} f(x)$$

How does this compare the situation in multiple variables?



There are infinitely many paths of approach.



(!) The path that we follow influences the value of the limit.

A Calculation.

Consider $f(x,y) = \dfrac{xy^2}{x^2+y^3}$ This is not defined at the point (0,0).

We can limit to (0,0) along various paths.

$c(t) = (t,t)$

$$\lim_{t\to 0} f(c(t)) = \lim_{t\to 0} \frac{t^3}{t^2+t^3}$$

$$= \lim_{t\to 0} f(t,t) = 1$$

We need to fix this calculation.

$c(t) = (0,t)$

$$\lim_{t\to 0} f(0,t)$$

$$= \lim_{t\to 0} \frac{0\cdot t^2}{0^2+t^3} = 0$$

Along which directions does it blow up?

$$\lim_{t \to 0} \frac{t^3}{t^2 + t^3} = \lim_{t \to 0} \frac{1}{\frac{1}{t} + 1}$$ # Factor out the t^3

This gets very large.

$= 0$ This also goes to zero.

For t small this is $\frac{1}{\text{HUGE} + 1} \longrightarrow$ small.